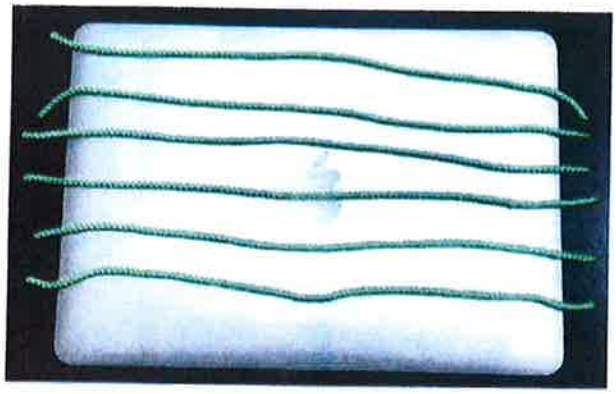
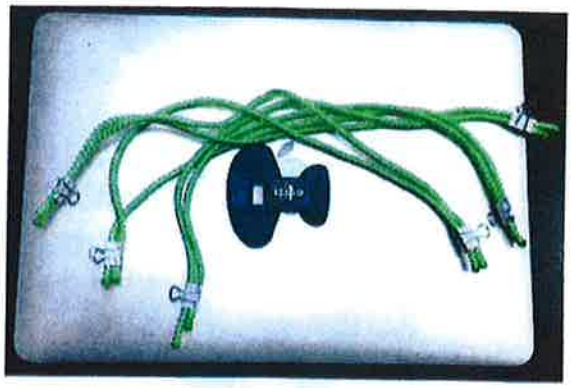
It's KNOT fun...

(The SIX-string version)





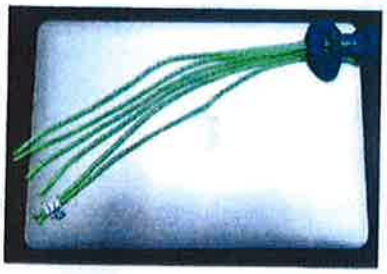
Materials: Six pieces of string, about 1 foot long. *(See pictures above)*
Hints:

a) To prevent the ends from unraveling, I heated the ends
b) Use just ONE color of string
c) I started using 'mini' binder-clips instead of having students tie knots!
(Some students will have difficulty tying knots and it is much quicker to release the binder-clips verses undo the knots!)

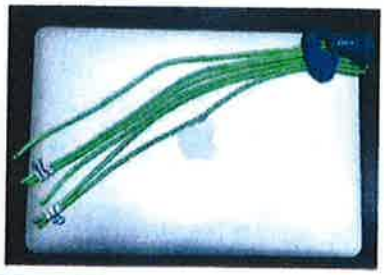
Procedure: This activity works best when working in pairs.
(But one person can do this activity by covering/hiding the other end of the strings).

- First person holds the tops of the six strings in their hand while the second person randomly ties (or uses the mini binder-clip to attach) any 2 strings together. (*Pic. 1a*)
- Repeat with another 2 strings (*Pic. 1b*) until there are NO single strings left at the bottom end (*or you see 3 binder-clips/knots, Pic. 1c*)

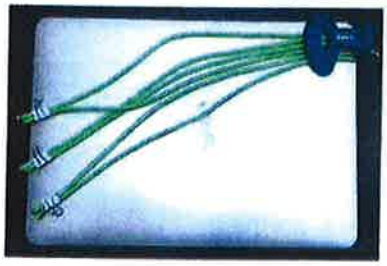
1a



1b



1c



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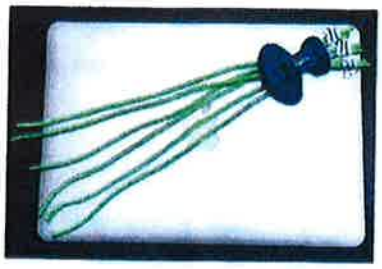
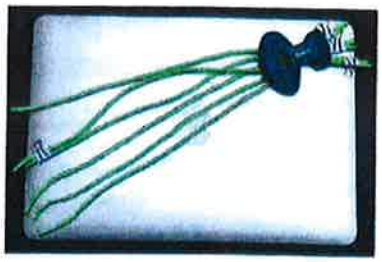
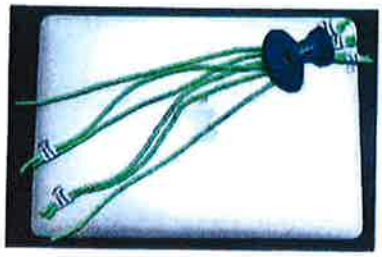
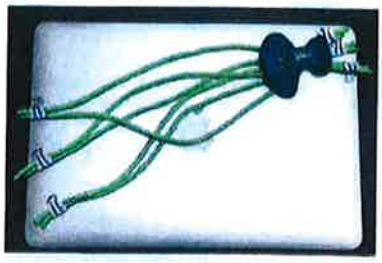
[1] LIGO-SEC/ T.Huynh-Dinh 2011 (Modified June 2015) *** For a complete electronic copy email: tien@ligo-la.caltech.edu

- Second person now holds and 'hides' the 3 knots inside their fist (*pic. 2a*) while the first person randomly ties 2 strings together from the bottom/loose end. Repeat until you have 3 more knots on the bottom. *(pic. 2b, pic. 2c, and pic. 2d)*

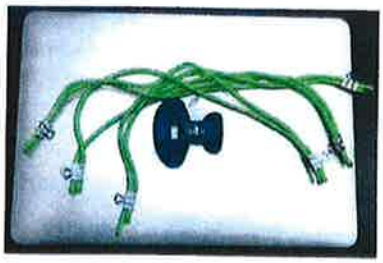
2a 

2b 

2c 

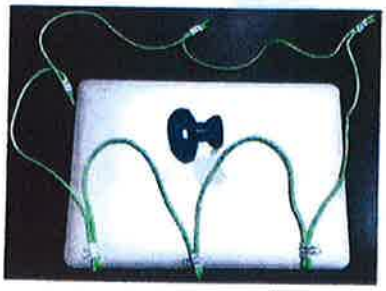
2d 

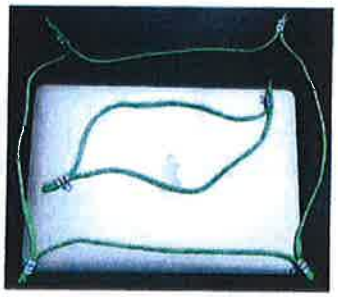
- Try to predict what you will get when you release the bundle of six strings that now should have a total of 6 knots, 3 knots on the top & 3 knots on the bottom. *(Pic. 3)*

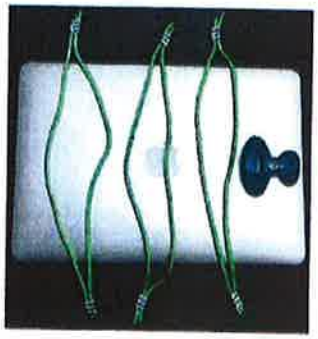
3 

Results: *For simplicity I ONLY considered the "un-attached" loops.*

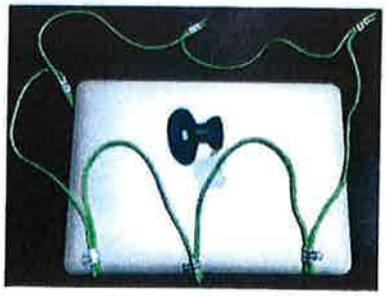
- If you followed the procedures, you should end up with <u>one of three possibilities</u>:
  - 1 large loop consisting of 6 strings (*Pic. 4a*)
  - 2 loops; 1 small consisting of 2 strings & 1 medium consisting of 4 strings (*Pic. 4b*)
  - or 3 small loops consisting of 2 strings each (*Pic. 4c*)

4a 

4b 

4c 

In the procedure above (*Pic. 3*), the 6-strings resulted in <u>1 large loop of 6 strings</u> (*Pic. 5*)

5 

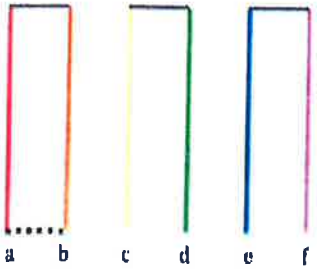
What's going on? *(Solving the probabilities for each outcome...)*

This problem can be solved 2 different ways (at least), depending on student's comfort level with math:

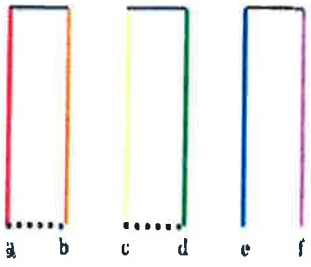
a) Solving the problem with a series of diagrams
(Easier for upper elementary and middle school students to understand)
b) Solving the problem mathematically
(For students with good math skills)

Using diagrams to solve the 6-string KNOT problem:
(See *handout 1701a* for a more complete worksheet)

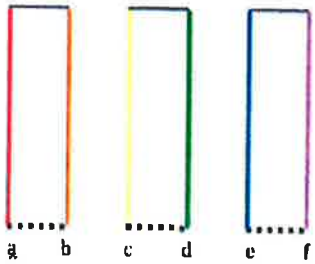
The top knots (black lines connecting the colored strings) do NOT figure into the calculations as they are the starting points.



Start by connecting string (a) to string (b).



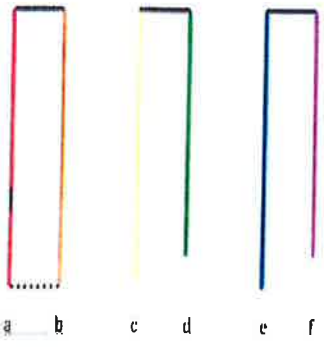
...followed by string (c) to string (d).



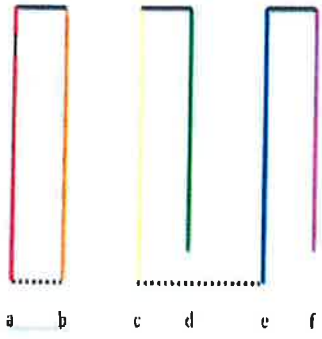
...leaving string (e) no choice but to connect to string (f).

As you can see, this results in **three loops of 2-strings each!** How often do you think this will occur?

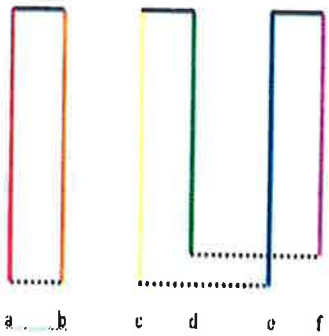
If we start with strings (a) & (b) connected, how many other combinations can we come up with? *In order to make the diagrams less confusing when the strings are connected, the length of some strings will be changed to help see what is actually going on.*



String (c) will connect with either string (e) or (f), but NOT (d) as before.



... in this example, string (c) is connected to string (e).

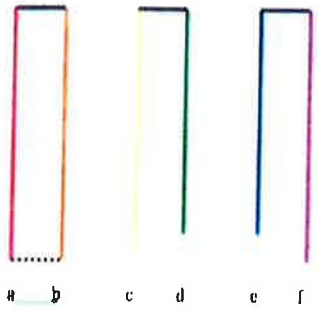


...and string (d) is left to connect with string (f).

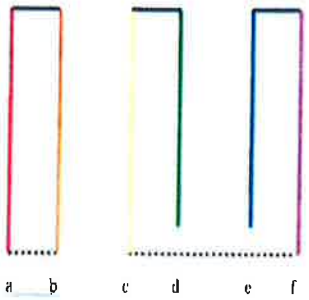
This results in **two loops (1 loop of 2-strings and 1 loop of 4-strings).** What else can occur?

With strings (a) and (b) connected, we:

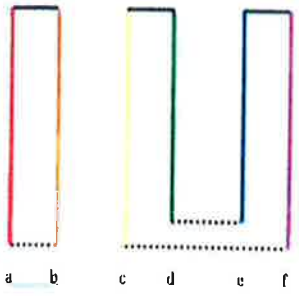
a) first connected string (c) to (d) and (e) to (f), resulting in **3 loops** (2-strings each)
b) then connected string (c) to (e) and (d) to (f), resulting in **2 loops** (2 and 4-strings each)
c) there remains only ONE more choice for string (c) and that is to connect with string (f)!



String (c) will connect with string (f), and NOT (d) or (e) as in the previous examples.



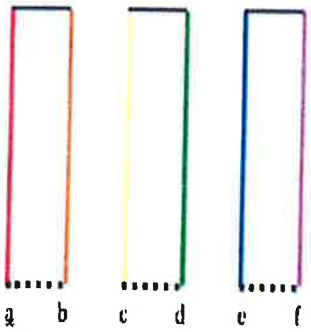
...string (c) is connected to string (f).



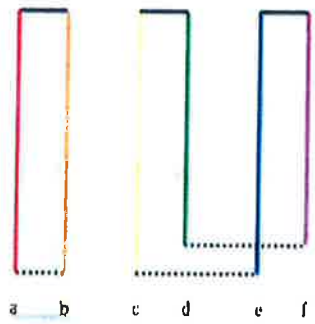
...and string (d) is left to connect to string (e).

This also results in **two loops (1 loop of 2-strings and 1 loop of 4-strings).**

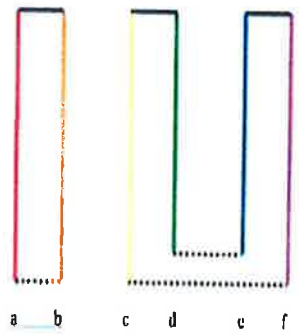
So when we started with strings (a) and (b) connected, using all the possible combinations left for string (c), we will end up with the three results seen below:



3-loops



2-loops



2-loops

The next logical step in solving this "KNOT" problem would be to start with strings (a) and (c) being connected, yielding **three more results**. Then on to strings (a) and (d), followed by (a) and (e) and finally (a) and (f). Since each starting combination produces **three possible results,** and there are FIVE different starting combinations: (ab), (ac), (ad), (ae), and (af)... there will be a total of **15 outcomes/results** for all the possible combinations!

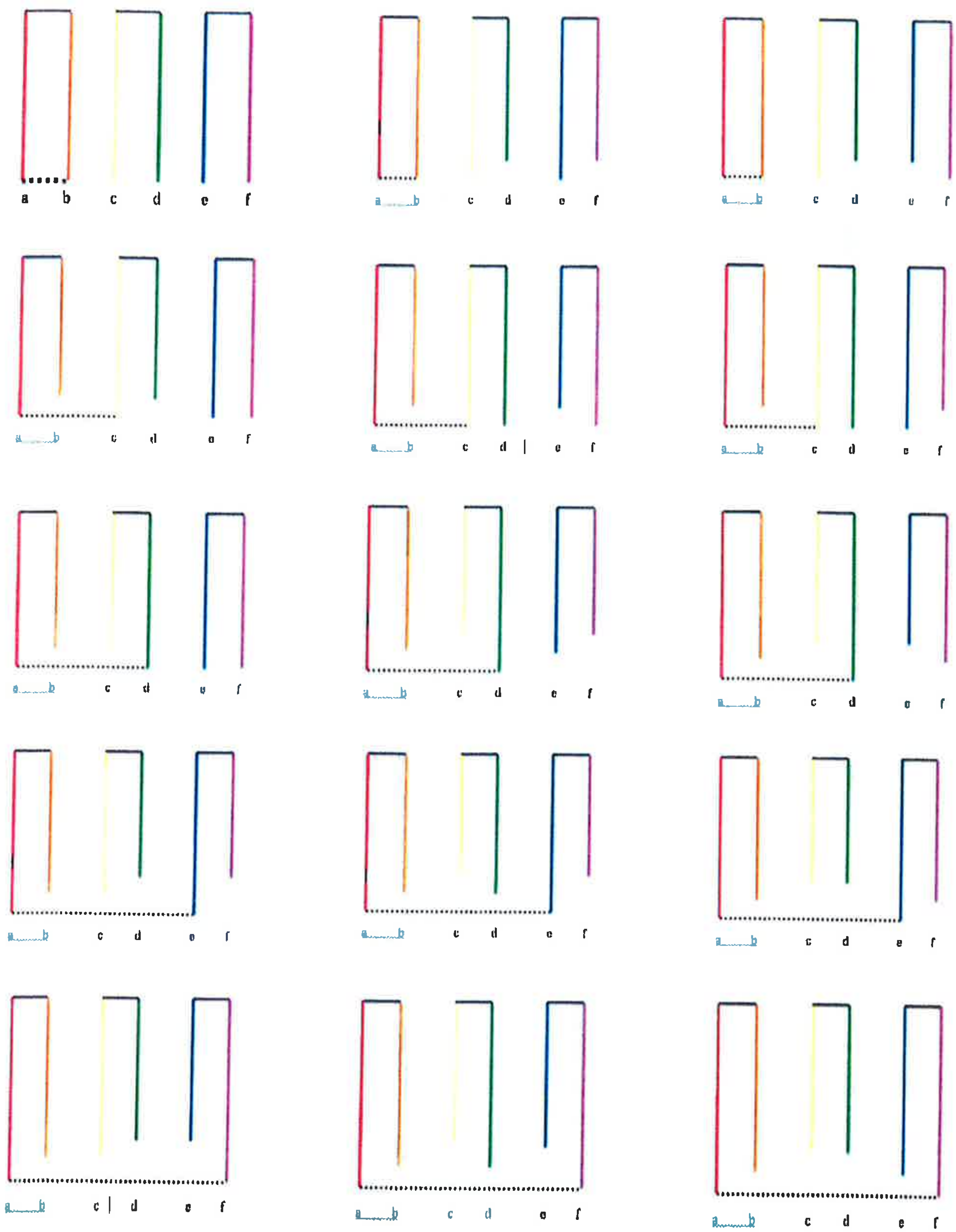
When *Handout 1701a* is filled-in correctly, the results should yield:

- 1 large loop consisting of six-strings **8 times out of 15**
- 2 loops (1 with two-strings and 1 with four-strings) **6 times out of 15**
- 3 small loops (two-strings each) occurring just **ONCE out of 15!!**

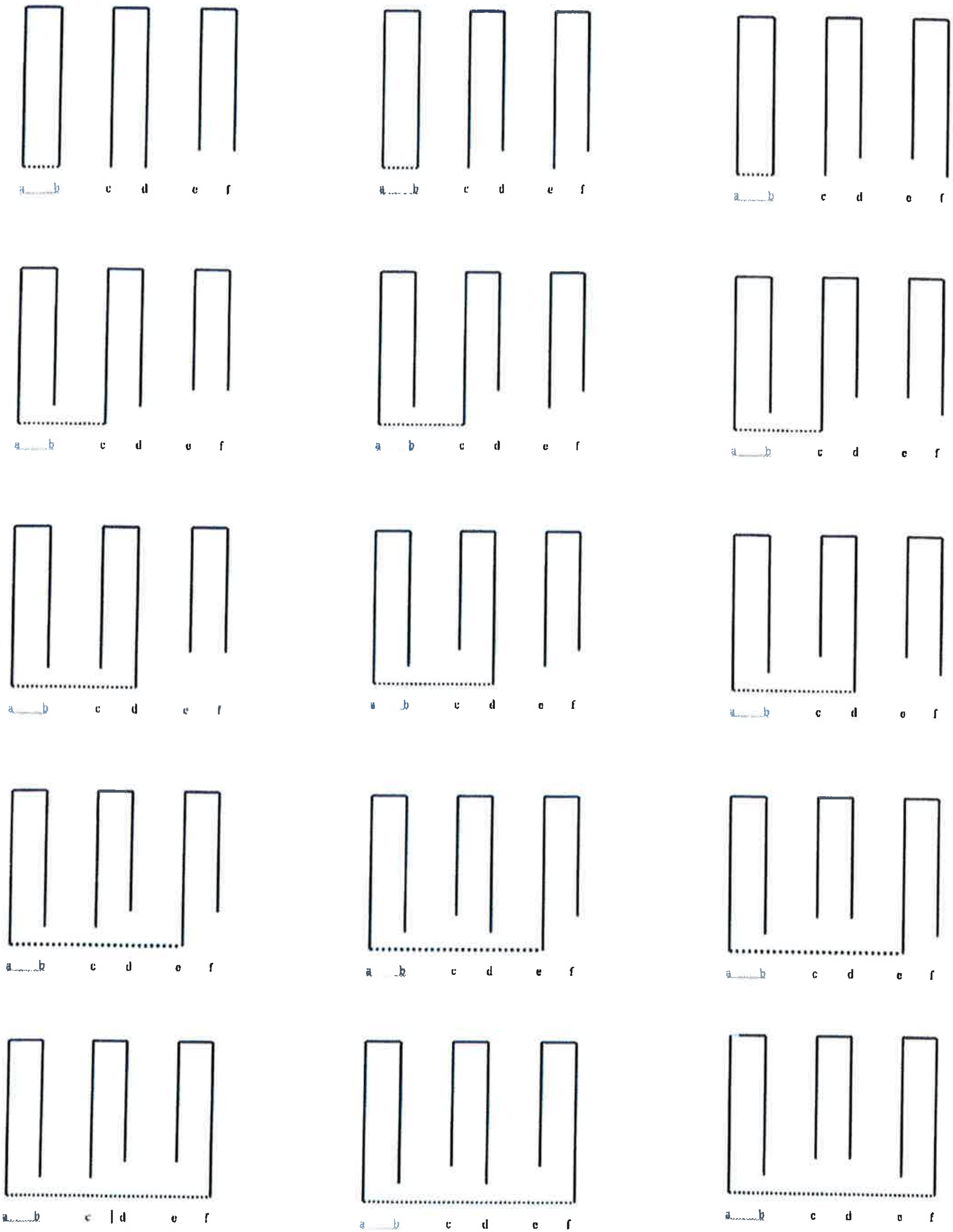
Large loop = 8/15, 2 loops = 6/15, and the 3 small loops = 1/15

Handout 1701a: Determining the probability for each outcome using diagrams below:

1 large loop: ________ 2 loops: ________ 3 small loops: ______

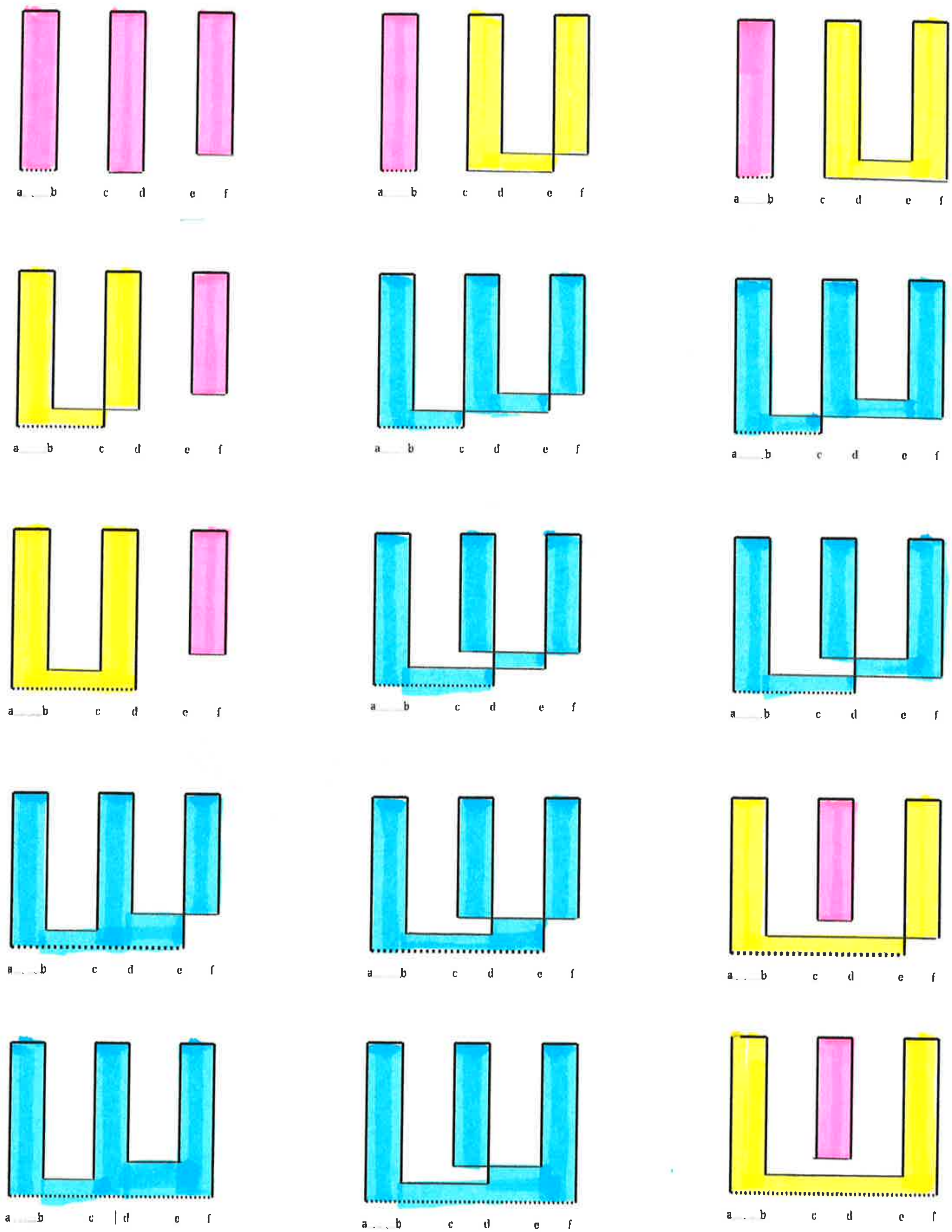


Form 1701a' 1 large loop: ______ 2 loops: ______ 3 small loops: ______



"KEY"

Form 1701a' 1 large loop: 8/15 2 loops: 6/15 3 small loops: 1/15



Solving for 2 loops (1 small and 1 medium sized loop)

The simplest method is to subtract the probability of *3 small loops* (**1/15**) and *1 large loop* (**8/15**) from 1 or (**15/15**) to get **6/15.**

So the probability of ending up with **2 loops** (1 small loop of 2-strings and 1 medium loop of 4-strings) is **6/15.** This answer is the same as if we solved it using Form 1701a.

The more "tricky"/challenging method of solving this problem is as follows:

Since there are TWO ways (can you guess what they are?) you can end up with *2 loops,* we have to calculate the probability of each and then ADD the two probabilities together.

1st case: Tying a small loop of 2-strings first (then the medium loop of 4-strings)

2nd case: Starting with the medium loop of 4-strings (and then the small loop)

Since I've gone through the first 2 examples (*3 small loops* of 2-strings and *1 large loop* of 6-strings), see if you can follow this explanation without using ~~as many~~ diagrams of the strings.

1st case:

To start 1 small loop, you have a *1 in 5 chance* of attaching string (a) to string (b) = **1/5**
To end up with a medium loop of 4-strings you have a *2 in 3 chances* = **2/3**
So the probability of the 1st case is:

$$1/5 \quad x \quad 2/3 \quad = \quad \mathbf{2/15}$$

2nd case:

To start with a medium loop of 4-strings, you have *4 in 5 chances* (**4/5**) to make a loop larger that 2-strings, followed by a *1 in 3 chance* (**1/3**) of closing that medium loop. The other 2 possibilities will yield a large loop of 6-strings!!! (We don't want that either...)
So the probability of the 2nd case is:

$$4/5 \quad x \quad 1/3 \quad = \quad \mathbf{4/15}$$

The probability of **2 loops** (1 small loop of 2-strings and 1 medium loop of 4-strings) is the sum of the 1st and 2nd case probabilities.

$$2/15 \quad + \quad 4/15 \quad = \quad \mathbf{6/15}$$ <<< That's all folks! ☺

Extensions: For remediation, you could do this problem with just 4 strings.
For further enrichment, try this problem using **8-strings**!
(If you need assistance with 8-strings, email me for hints!)

Feel free to send comments, suggestions, and/or questions to tien@ligo-la.caltech.edu