CALIFORNIA INSTITUTE OF TECHNOLOGY
PHYSICS MATHEMATHEMATICS AND ASTRONOMY DIVISION

Sophomore Physics Laboratory (PH005/105)

Analog Electronics
Basics on Oscillators

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Chapter 7

Basics on Oscillators

7.1 Introduction

Waveform generators are circuits which provide a periodic signal with constant frequency, phase, and amplitude. The quality of these devices are measured by the frequency stability, amplitude stability, and absence of distortion. The last characteristics is essentially cleanness of the spectrum signal. For example, the spectrum of a perfect sinusoidal oscillator must be a delta of Dirac at the oscillating frequency. Practically, sinusoidal oscillators has a sharp narrow peak at the oscillation frequency, and other less taller peaks at different frequencies, mainly at multiples of the oscillation frequency (harmonics).

In this chapter we will study the criterion to sustain a sinusoidal oscillation with a positive feedback amplifier, the so-called **Barkhausen criterion**, and some simple circuit to produce different waveforms.

Direct Digital Synthesis[1], a more versatile and effective technique to produce arbitrary waveforms, is out of the scope of these simple notes.

7.2 Barkhausen Criterion

Let's consider an ideal amplifier with a positive feedback network as show in figure 7.1. Considering that the summation point output is

$$V_i + \beta(\omega) V_o,$$

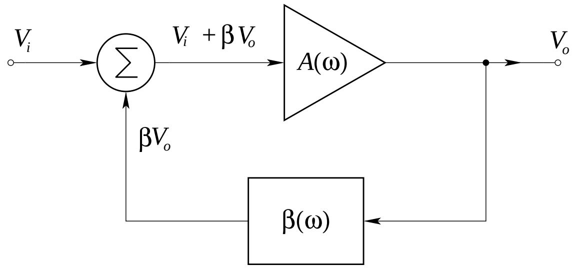

Figure 7.1: Amplifier with positive feedback

and the amplifier gain is $A(\omega)$, the output voltage will be

$$V_o = A(V_i + \beta V_o),$$

Collecting V_o we will finally have

$$V_o = \frac{A}{1 - \beta A} V_i.$$

For

$$|\beta(\omega)A(\omega)| = 1, \qquad \arg[\beta(\omega)A(\omega)] = 0, 360, ...$$

the output V_o diverges. If the previous condition is satisfied for the angular frequency ω_0, any excitation at the frequency ω_0 will make the output to oscillate at the frequency ω_0 with infinite amplitude. If $V_i$is equal to zero then the output will oscillate at the frequency ω_0 with the amplitude A.

The previous condition which can be rewritten as

$$\Re[\beta A] = 1, \qquad \Im[\beta A] = 0 \tag{7.1}$$

is the so called **Barkhausen criterion** for the oscillation.

The term βA is called the **open loop gain** or simply **loop gain** since that is exactly the gain of the loop in the feedback amplifier network when the loop is open at the summing point.

7.2.1 Practical Considerations

Oscillators with exactly unitary loop gain at a given frequency and input V_i equal to zero at any time are just a mere mathematical abstraction.

For example, external perturbations, drifts due to temperature, and aging would make this condition impossible to keep.

Practically, it is necessary to have a loop gain βA somewhat larger than unity to start and sustain the oscillation. This can lead to a slow drift of the oscillation amplitude, and in the worst case, the oscillation can even saturate or stop.

To properly sustain the oscillation in case of temperature drifts, we need to add to the positive feedback path another feedback loop this time negative to stabilize the gain. This path often called Automatic Control Circuit (AGC) can be done using temperature sensitive components. For example, semiconductor diodes or transistors whose resistivity decreases with the temperature can be used in the AGC.

It is worthwhile to notice that large values of the amplifier gain A produce saturation at the output, and therefore can be used to generate squares or pulse waves. Moreover, cascading a proper filtering stage, one can select just one frequency and make a quite amplitude stable sinusoidal generator.

We don't have to provide an initial kick to start the oscillation. This is true, because every time we switch a circuit on a step propagates through the circuit providing an initial excitation at the right frequency. Moreover, the probability to have a small signal fluctuation at the right frequency are usually quite high.

The frequency stability of the oscillator is a quite complex topic of study. Here we can simply say that it depends mainly on the ability of the circuit to maintain the loop gain phase constant to 0°or to multiples of 360°.

In the discussion of the oscillator circuits, we will assume that the amplifier is able to deliver the required positive or negative gain without adding any additional phase. In the general case, this is clearly a crude approximation, but it is used just to simplify the study of the circuits.

7.3 Phase Shift Oscillator

The phase shift oscillator exemplifies the concepts set forth above. Referring to figure 7.2, we can distinguish the JFET amplifier stage and the positive feedback network made of three cascaded RC phase shifting filters.

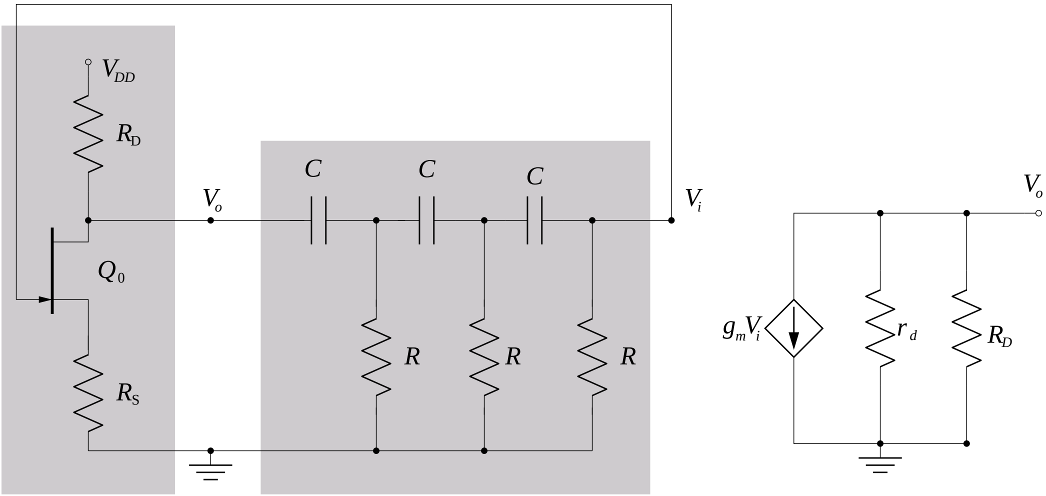

Figure 7.2: Phase shift oscillator using a JFET as amplification stage (left gray rectangle) and a phase shift network (right gray rectangle). The circuit on the left represents the low frequency model of the JFET amplifier.

Supposing that the amplifier load Z_L is negligible, i.e. $|Z_L| \gg R_D||r_d$ then, the amplifier will just change sign (180°) to any signal injected in the gate. The network feedback will provide additional phase shift to satisfy the Barkhausen criterion at a given angular frequency ω_0.

It can be proved that

$$\beta(\omega) = \frac{V_i}{V_o} = \frac{1}{1 - \frac{5}{(\omega\tau)^2} + j\left(\frac{1}{(\omega\tau)^3} - \frac{6}{\omega\tau}\right)} \qquad \tau = RC\,, \tag{7.2}$$

The amplifier gain, supposed to be constant is $A = -g_m R_D$, where g_m is the JFET amplifier gain.

Imposing the condition $\Im[\beta A] = 0$, we get

$$\omega_0 = \frac{1}{\sqrt{6}}\frac{1}{\tau}.$$

Replacing the previous expression in the open loop gain $A\beta$ and using the second condition $\Re\,[\beta A] = 1$, we get

$$g_m R_D = 29$$

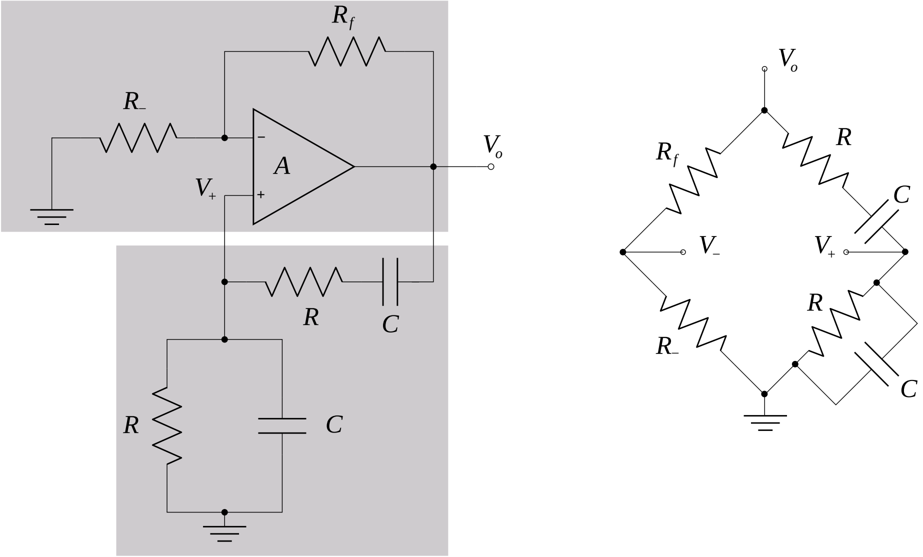

Figure 7.3: Wien bridge oscillator, and components rearrangement to show the bridge topology.

To sustain the oscillation, the amplifier must have a gain of at least $29/R_D$.

7.4 The Wien Bridge Oscillator

The Wien Bridge Oscillator show in figure 7.3, uses a differential amplifier to provide positive and negative feedback to satisfy the two condition of oscillation.

Referring to figure 7.3 , setting $Y_C = 1/\left(j\omega C\right)$, and thanks to the voltage divider equation we can write

$$V_+ = \frac{\frac{RY_C}{Y_C+R}}{R + Y_C + \frac{RY_C}{Y_C+R}} V_o = \frac{1}{\frac{(Y_C+R)^2}{RY_C} + 1} V_o = \frac{1}{\frac{Y_C}{R} + \frac{R}{Y_C} + 3} V_o$$

and

$$\beta(\omega) = \frac{V_+}{V_o} = \frac{1}{3 + j\left(\omega\tau - \frac{1}{\omega\tau}\right)} \qquad \tau = RC.$$

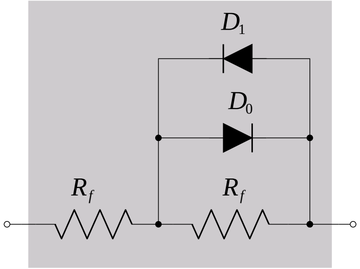

Figure 7.4: Automatic gain control circuit for the Wien bridge oscillator negative feedback.

The oscillation will happen where the phase shift is zero, i.e. for

$$\omega\tau - \frac{1}{\omega\tau} = 0, \qquad \Rightarrow \qquad \omega_0 = \frac{1}{\tau}\,.$$

The angular oscillation frequency ω_0 depends on the inverse of the resistance R and the capacitance C.

Because the attenuation at the resonant frequency is

$$\frac{V_+}{V_o} = \frac{1}{3}\,.$$

the negative feedback must have a theoretical gain of $A(\omega_0) = 3$. The resistances R_- and R_f must be given by the usual equation

$$\frac{V_{o'}}{V_+} = 1 + \frac{R_f}{R_-}\,.$$

The oscillation frequency can be continuously tuned using coupled variable resistors.

To minimize distortions due to the Op-amp saturation when the gain is larger than one, it is required to provide a circuit with variable gain. Essentially, we need an overall gain larger than one for small signal to sustain the oscillation and gain of about 1 or less for large signal to avoid distortion. The negative feedback path shown in figure 7.4 does the job. For large signals one of the diodes becomes forward biased reducing the feedback resistance and the Op-Amp gain. For smaller signal the gain is not affected by the diodes.

Practically, Wien Bridge oscillators are used in the kilohertz region with a variable range up to ~10 times ω_0.

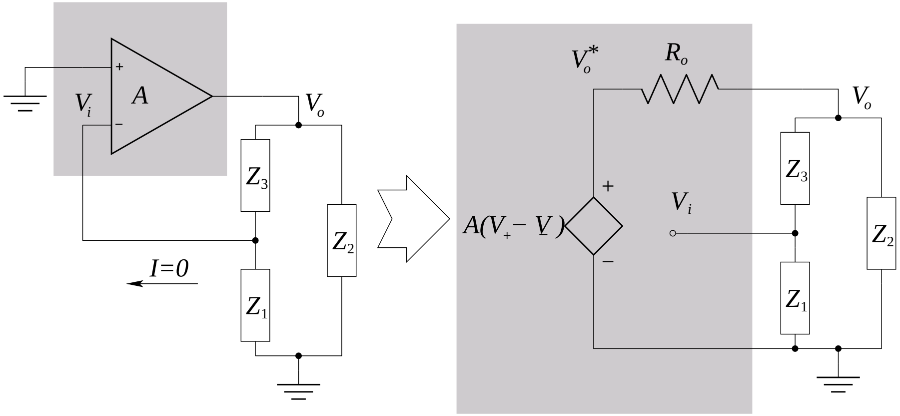

Figure 7.5: LC Oscillator circuit using an ideal Op-Amp with non zero output impedance R_o and its equivalent ideal circuit. Note that the feedback loop is connected to the negative input of the amplifier, and therefore to get a positive loop feedback the feedback network has to flip the signal phase by 180° .

7.5 LC Oscillator

A quite general form of oscillator circuits is depicted in figure 7.5. In this case it is not straightforward to separate the oscillating feedback network and the amplifier itself. Let's suppose that the amplifier is ideal but has a non zero output resistance R_o. Referring to figure 7.5 we have

$$\beta = \frac{V_i}{V_0^*} .$$

Applying the voltage divider equation twice we have the two equations

$$V_i = \frac{Z_1}{Z_1 + Z_3} V_o$$

and

$$V_o = \frac{Z}{Z + R_o} V_0^* , \qquad Z = Z_2 || \left(Z_1 + Z_3 \right) ,$$

or

$$\frac{1}{V_o^*} = \frac{Z}{Z + R_o} \frac{1}{V_0} .$$

After some algebra we finally get

$$\beta = \frac{Z_1 Z_2}{R_o \left(Z_1 + Z_2 + Z_3\right) + Z_2 \left(Z_1 + Z_3\right)} . \tag{7.3}$$

Let's consider the case of the LC tunable oscillators, i.e. the impedances are purely reactive (real part equal to zero)

$$Z_i \;=\; jX_i \;, \qquad X_i > 0 \qquad \text{for } i = 1, 2, 3$$

Then the previous eq. (7.3) becomes

$$\beta = \frac{-X_1 X_2}{jR_o \left(X_1 + X_2 + X_3\right) - X_2 \left(X_1 + X_3\right)} .$$

For β to be real

$$X_1 + X_2 + X_3 = 0 ,$$

and

$$\beta(\omega_0) = \frac{X_1}{X_1 + X_3} ,$$

where ω_0 is the oscillation frequency. Using the two previous equation we finally get

$$\beta(\omega_0) = -\frac{X_1}{X_2} \qquad \Rightarrow A_{OL} = -A\left(-\frac{X_1}{X_2}\right) .$$

Since A_{OL} must be positive and $A > 0$, then X_1 and X_2 must have same sign. For example they have be both capacitors or inductors. From the condition of imaginary part equal to zero we find that if X_1 and X_2 are capacitors, then X_3 must be an inductor, and vice versa. Here is the oscillator circuit name depending on the choice of the reactance:

- **Colpitts Oscillator**: X_1 and X_2 capacitive reactances and X_3 an inductive reactance ($X_{1,2} = -1/(\omega C_{1,2})$, $X_3 = \omega L_3$).
  The oscillator angular frequency and the gain in this case are

$$\omega_0 = \sqrt{\frac{1}{L_3 \left(\frac{C_1 C_2}{C_1 + C_2}\right)}} , \qquad \beta(\omega_0) = \frac{C_2}{C_1}$$

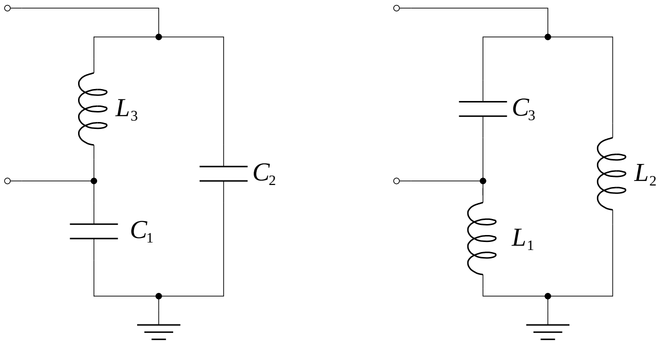

Figure 7.6: Colpitts (left) and Hartley (right) feedback circuits $\beta(\omega)$ for the LC oscillator circuit of Figure 7.5.

- **Hartley oscillator**: X_1 and X_2 inductive reactances and X_3 a capacitive reactance ($X_{1,2} = \omega L_{1,2}$, $X_3 = -1/(\omega C_3)$).
  The oscillator angular frequency and the gain in this case will be

$$\omega_0 = \sqrt{\frac{1}{C_3\,(L_1 + L_2)}}, \qquad \beta(\omega_0) = \frac{L_1}{L_2}$$

Using a BJT amplifier we can usually obtain higher oscillating frequency than using standard operational amplifiers. In this case the high frequency hybrid-π model[2] must be used to properly model the transistor behavior. Moreover, the BJT amplifier low input impedance makes the design more complicated.

7.6 Crystal Oscillator

Crystal oscillators are based on the property of piezoelectricity[1] exhibited by some crystals and ceramic materials. Piezoelectric materials change size when an electric field is applied between two of its faces. Conversely, if we apply a mechanical stress, piezoelectric materials generate an electric field. Some crystals have internal mechanical resonances with very high

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[1] Piezoelectricity was discovered by Jacques and Pierre Curie in the 1880's during experiments on quartz.

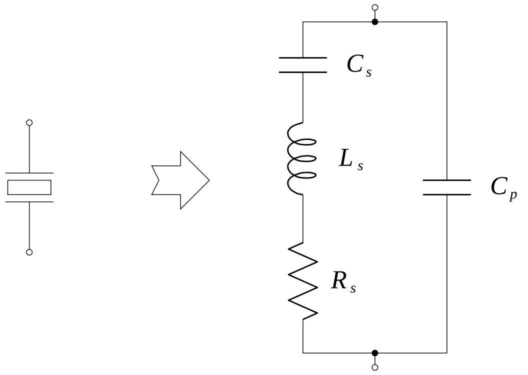

Figure 7.7: Circuit symbol for a piezoelectric oscillator (or quartz oscillator) and the equivalent electronic circuit. The LCR series circuit accounts for the sharp mechanical resonance The capacitor C_p in parallel describes the capacitance of the crystal for frequency far for the resonance.

quality factors (quartz can reach quality factors of 10^4) [2] and can be indeed used to generate very stable oscillators.

Figure 7.7 shows the circuit symbol for a piezoelectric component and the equivalent circuit modeled using ideal components.

Usually, to apply an electric field to a crystals is necessary to make a conductive coating on two parallel faces, and this process creates a capacitor with an interposed dielectric. This explain the presence of the capacitor of capacitance C_p in the model. The LCR series circuit accounts for the particular mechanical resonance we want to use to build the oscillator.

To design a crystal oscillator it is important to study the reactance (the imaginary part of the impedance) whose qualitative behavior is shown in figure 7.8. Where the reactance is essentially inductive and very close to the resonance, the crystal behaves as a simple equivalent inductor. We can indeed replace the inductor L_s of the LC oscillator of figure 7.5 with the piezoelectric crystal to build a simple oscillator.

Crystal oscillators using a Colpitts configuration and a BJT in common-emitter or common-collector configuration, can work from few kHz up to ~100MHz.

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[2] Mechanical resonance stability depends mainly on the fact that the resonance value is determined by the crystal geometry. It the crystal size slightly depends on the temperature we can have very stable resonators. Active temperature stabilization can clearly improve frequency stability.

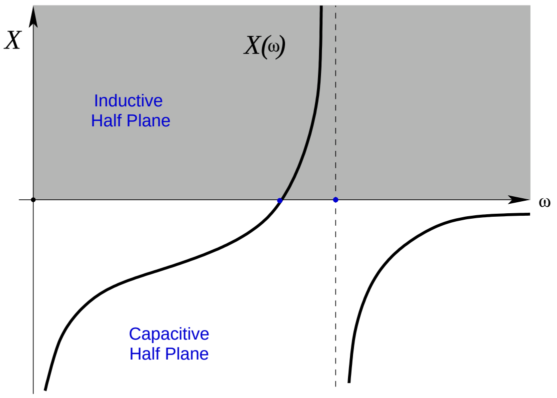


Figure 7.8: Qualitative behavior of the crystal reactance versus frequency.

7.7 Relaxation Oscillators

Relaxation oscillators include a wide class of non-linear systems, many of them in different fields such as mechanical, biological chemical, electrical fields to just mention a few.

Relaxation oscillators are characterized by the following properties:

- a non linear mechanism that provides a bistable state,
- a relaxation process that creates the transition from one stable state to the other,
- a period of oscillation characterized by a relaxation phenomena, i.e. by the time constant of the relaxation process,

The canonical example of relaxation oscillator is the seesaw with one bucket on one end and a weight on the other, with the bucket continuosly filled by a constant water flow. When the bucket is filled it changes the equilibrium of the seesaw and the system transitions to the new state. In the new state, the bucket is tilted enough to be emptied and therefore the system transition back to the older state. The seesaw + waterflow is clearly the bistable nonlinear system, and the relaxation process is the emptying of the bucket.

Vander Pol was one of the first to analize a relaxation oscillator system.

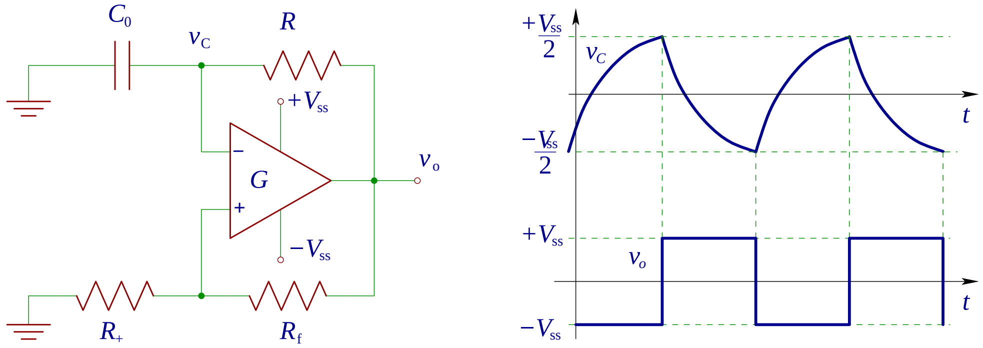


Figure 7.9: The Op-Amp version of the RC Charge Discharge Oscillator.

7.7.1 The RC Relaxation Oscillator

A very simple relaxation oscillator is the RC charge discharge oscillator show in figure 7.9. The ideal Op-Amp is configured as a Schmitt trigger which provides the non-linear bistable states. The negative feedback provides the relaxation mechanism.

To qualitatively understand the circuit, let's suppose that the Schmitt trigger output rails down to the Op-Amp power supply voltage $-V_{ss}$. The capacitor will start to charge down and its voltage V_C will swing down to reach $-V_{ss}$ with a characteristic time constant $\tau = RC$. Once $V_C \leq -V_{ss}/2$, the Schmitt trigger output will switch to $+V_{ss}$and the the capacitor voltage V_C will start swinging to $+V_{ss}$ with the same characteristic time constant τ. This cycle will keep repeating generating a square wave at the Schimtt trigger output.

The Period of the oscillation can be computed considering the time for exponential decay with time constant τ to go from $V_{ss}/2$ to $-V_{ss}/2$. After some algebra one obtains

$$T = 2\log(3)\,\tau$$

which shows in this case (same resistors on the positive feedback loop) that the period does not depend on the voltage limits but only on τ. In a more general case when the positive feedback loop resistors R_0 are different, T will depend also on the values of those resistors.

7.8 Problems Preparatory to the Laboratory

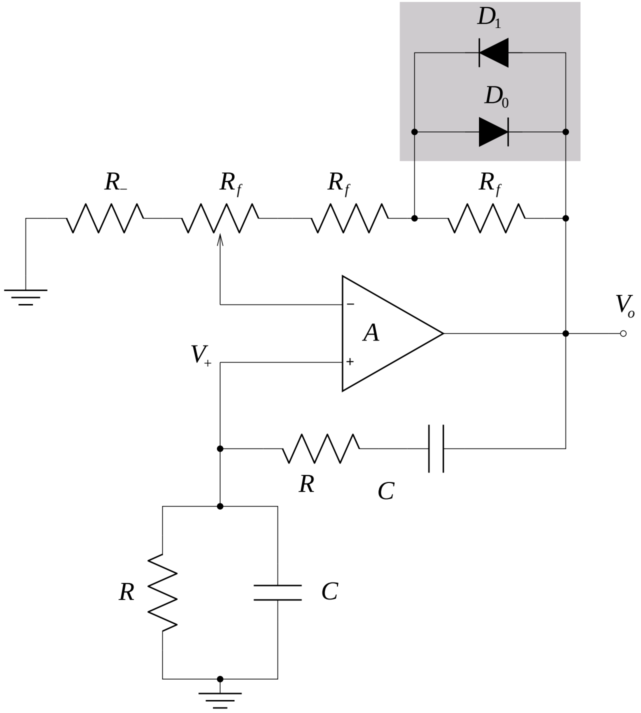
1. Replace a JFET amplifier of the phase shift oscillator with an Op-amp. Hint: the amplifier configuration must provide 180° of phase shift and the virtual ground can be used to simplify the feedback network and amplifier.
   Find the components values to satisfy the Barkhausen criterion for an oscillating frequency $\nu = 5$kHz.
2. Design a Wien bridge oscillator with a frequency of 1 kHz, 5kHz and 10kHz.
3. Using the expression of β, derive the resonance frequencies formulas of the Colpitts and the Harley for LC oscillators.
4. Determine the expression of the RC relaxation oscillator period T when the positive feedback loop resistors of figure 7.9 are different. Let's rename the feedback resistor R_f and the resistor from ground to the positive input R_+.
5. Design a RC relaxation oscillator with a frequency of 10 kHz, 100kHz and 500kHz.

7.9 Laboratory Procedure

Read carefully the entire procedure before starting the experiment.
Consult the data-sheet to properly connect the devices pin-out.
Before powering your circuit up, cross-check the power supply connections.
It is always a good practice to turn on the dual power supply at the same time to avoid potential damages of circuit components.
Note on your log book all the unpredicted behavior you experience in the circuits response.

1. Build a Wien bridge sinusoidal oscillator with a frequency ν between 1 kHz and 10 kHz with a μ741 Op-Amp. Use a potentiometer to match the resistances of the positive feedback network. Arrange more than one capacitor to obtain two capacitances with the same value. Neglect first the automatic gain control circuit highlighted in

the gray box in the figure below



(a) Measure the open loop transfer function $A(\omega)\beta(\omega)$ to verify the gain and phase around the resonance.

(b) Compare the measured oscillator frequency $\langle\omega_0\rangle_{Exp}$with the theoretical value ω_0.

(c) Check the behavior of the circuit when the open loop gain $A\beta$ is greater than one or smaller than one.

(d) Add the AGC circuit and tune the gain to properly sustain the oscillation.

(e) Verify that the oscillator can be tuned by changing the resistors or the capacitors pair.

(f) Measure the spectrum of your oscillator using the FFT math function of your oscilloscope or a laboratoty spectrum analyzer, and compute the total harmonic distortion

$$THD = \frac{1}{V_0^2} \sum_{n=1}^{N} V_n^2 \, ,$$

where V_n is the amplitude of the *nth*-harmonic frequency, V_0 is the amplitude of the fundamental frequency. N is determined by the required precision, and practically by the resolution of the instrument.

2. Build a RC relaxation oscillator with a frequency ν between 10 kHz and 500 kHz using an AD711. Verify that
   (a) the Schmitt trigger triggers properly,
   (b) the time constant $\langle \tau \rangle_{Exp}$ of the relaxation process equals RC,
   (c) oscillation frequency if matches the design.

Bibliography

[1] http://www.analog.com/UploadedFiles/Tutorials/450968421DDS_Tutorial_rev12-2-99.pdf

[2] Microelectronics, Jacob Millman, and Arvin Grabel , Mac-Graw Hill