



## **Sophomore Physics Laboratory** (PH005/105)

# **Analog Electronics Alternating Current Network The- ory**

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# Chapter 1

## Alternating Current Network Theory

In this chapter we will study the properties of electronic networks propagating sinusoidal voltages and currents (alternate current/AC regime). In other words, voltage or current sources connected to the networks produce electromagnetic waves whose frequency can be ideally changed from 0 to  $\infty$ .

Considering an electromagnetic wave whose frequency  $f$  is 1MHz, (typical maximum frequency  $f$  used in this course of analog electronics) and the electromagnetic field propagates at a speed  $c = 3 \cdot 10^8$  m/s (1' / ns), the wavelength  $\lambda = c/f$  will be usually much greater than several hundred feet, hundreds to thousands of times greater than the physical sizes of our electronic circuits. Consequently, each individual circuit component will have at any instant, to a high degree of accuracy, zero net total current flowing in or out of all its connections and components. Such components are known as a *lumped elements*.

When fields wavelength becomes comparable to the size of the circuit components, fields and currents can vary across the element, making it a *distributed element*. Examples of distributed elements include antennas, microwave waveguides, and the electrical power distribution grid.

For two-terminal lumped elements, we can conclude that at any instant, the current flowing out of one terminal must equal the current flowing into the other, so we can simply refer to the current flowing through the element and the potential difference (voltage difference or voltage drop) between the two terminals.

In general, if we have a sinusoidal signal (sinusoidal voltages, or currents) applied to a circuit having at least one input and one output, we will expect a change in the amplitude and phase at the output. The determination of these quantities for quite simple circuits can be very complex. It is indeed important to develop a convenient representation of sinusoidal signals to simplify the analysis of circuits in this AC regime.

The AC analysis of such circuits is valid once the network is at the steady state, i.e. when the transient behavior (such as those ones produced by closing or opening switches) is extinguished.

## 1.1 Symbolic Representation of a Sinusoidal Signals, Phasors

A sinusoidal quantity (a sinusoidal current or voltage for example),

$$A(t) = A_0 \sin(\omega t + \varphi),$$

is univocally characterized by the amplitude  $A_0$ , the angular frequency  $\omega$ , and the initial phase  $\varphi$ . The phase  $\varphi$  corresponds to a given time shift  $t^*$  of the sinusoid ( $\omega t^* = \varphi \Rightarrow t^* = \varphi/\omega$ ).

We can indeed associate to  $A(t)$  an applied vector  $\vec{A}$  in the complex plane with modulus  $|A| = A_0 \geq 0$ , rotating counter-clockwise around the origin  $O$  with angular frequency  $\omega$  and initial angle  $\varphi$  (see figure 1.1). Such vectors are called *phasors*.

The complex representation of the phasor is<sup>1</sup>

$$\vec{A} = A_0 e^{j(\omega t + \varphi)}, \quad j = \sqrt{-1},$$

or

$$\vec{A} = x + jy, \quad \begin{cases} x = A_0 \cos(\omega t + \varphi) \\ y = A_0 \sin(\omega t + \varphi) \end{cases}$$

Extracting the real and the imaginary part of the phasor, we can easily compute its amplitude  $A_0$  and phase  $\varphi$ , i.e.

$$|A| = \sqrt{\Re[\vec{A}]^2 + \Im[\vec{A}]^2} \quad \varphi = \arg[A] = \arctan\left(\frac{\Im[\vec{A}]}{\Re[\vec{A}]}\right) \quad (t = 0),$$

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<sup>1</sup>To avoid confusion with the electric current symbol  $i$ , it is convenient to use the symbol  $j$  for the imaginary unit.

## 1.1. SYMBOLIC REPRESENTATION OF A SINUSOIDAL SIGNALS, PHASORS 11

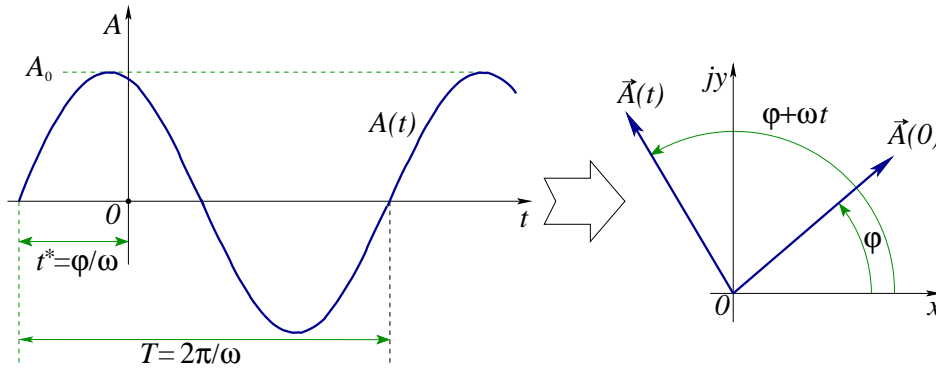


Figure 1.1: Sinusoidal quantity  $A(t)$  and its phasor representation  $\vec{A}$  at the initial time  $t = 0$  and at time  $t$ .

and reconstruct the real sinusoidal quantity. It is worth noting that in general, amplitude  $A_0$  and phase  $\varphi$  are functions of the frequency.

The convenience of this representation will be evident, once we consider the operation of derivation and integration of a phasor.

### 1.1.1 Derivative of a Phasor

Computing the derivative of a phasor  $\vec{A}$ , we get

$$\frac{d\vec{A}}{dt} = j\omega A_0 e^{j(\omega t + \varphi)} = j\omega \vec{A},$$

i.e. the derivative of a phasor is equal to the phasor times  $j\omega$ .

### 1.1.2 Integral of a Phasor

The integral of a phasor  $\vec{A}$  is

$$\int_{t_0}^t \vec{A} dt' = \frac{1}{j\omega} \left[ A_0 e^{j(\omega t + \varphi)} - A_0 e^{j(\omega t_0 + \varphi)} \right] = \frac{1}{j\omega} \vec{A} + \text{const.},$$

i.e. the integral of a phasor is equal to the phasor divided by  $j\omega$  plus a constant. For the AC regime we can assume the constant to be equal to zero without loss of generality.

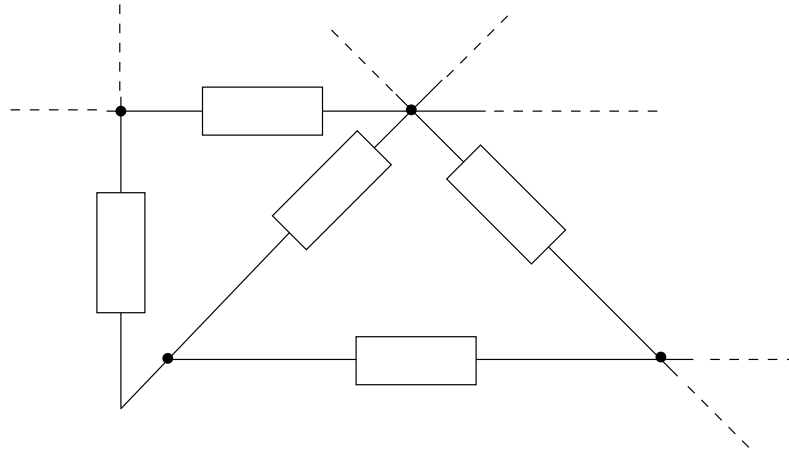


Figure 1.2: Generic representation of a network

## 1.2 Network Definitions

To make easier the understanding of network basic theorems, some more or less intuitive definitions must be stated.

An *electronic network* or circuit is a set of electronic components/devices connected together to modify and transmit/transfer energy/information. This information is generally called an *electric signal* or simply a signal.

To graphically represent a network, we use a set of coded symbols with terminals for the *network lumped elements* and lines for connections. These lines propagate the signal among the elements without changing it. The elements change the propagation of the electric signals.

A *network node* is a point where more than two network lines connect.

A *network loop, or mesh*, is any closed network line. To determine a mesh it is sufficient to start from any point of the circuit and come back running through the network to the same point without passing through the same point.

Quantities defining signal propagation are voltages  $V$  across the elements and currents  $I$  flowing through them.

Solving an electronic network means determining the currents or the voltages of each point of it.

Figure 1.2 shows a generic portion of a network with 4 visible nodes, and 3 visible meshes. The empty boxes are the electronic elements of the

network and their size is just for convenience and don't correspond to any physical dimensions. These are the points in the network where voltages and currents are modified.

### 1.2.1 Series and Parallel

Let's consider the two different connection topologies shown in figure 1.3, the parallel and the series connections.

A set of components is said to be in series if the current flowing through them and anywhere in the circuit is the same .

A set of components is said to be in parallel if the voltage difference between them is the same.

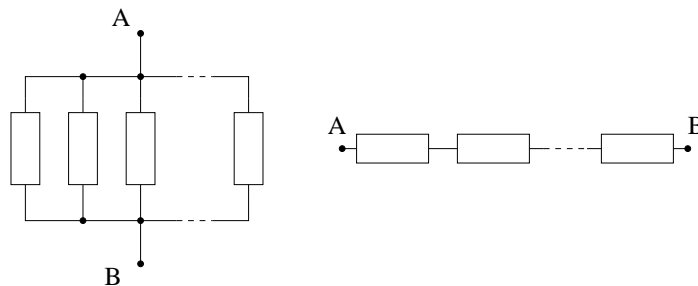


Figure 1.3: Considering the points A, and B, the components of the left circuit are in parallel, and those in the right circuit are in series.

### 1.2.2 Active and Passive Components

Circuit components can be divided into two categories: active and passive components. Active components are those devices that feed energy into the network. Voltage and current sources are active components. Amplifiers are also active components.

Passive components are those components, that do not feed energy to the network. Resistors, capacitors, inductors are typical passive components.

In general, both active and passive components dissipate energy.

### 1.3 Kirchhoff's Laws

Kirchhoff's laws, are fundamental for the solution of an electronic circuit. They can be derived from Maxwell's equations in the approximation of slowly varying field.

**Kirchhoff's Voltage Law (KVL):** The algebraic sum of the voltage difference  $v_k$  at the time  $t$  around a loop must be equal to zero at all times, i.e.

$$\sum_k v_k(t) = 0$$

**Kirchhoff's Current Law (KCL):** The algebraic sum of the currents  $i_k$  at the time  $t$  entering and leaving a node must be equal to zero at all times, i.e.

$$\sum_k i_k(t) = 0.$$

These laws hold for phasors as well.

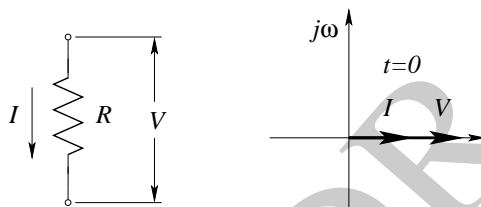
### 1.4 Passive Ideal Components with Phasors

Let's rewrite the I-V characteristic for the passive ideal components using the phasor notation. For sake of simplicity, we remove the arrow above the phasor symbol. To avoid ambiguity, we will use upper case letters to indicate phasors, and lower case letters to indicate a generic time dependent signal.

#### 1.4.1 The Resistor

For time dependent signals, Ohm's law for a resistor with resistance  $R$  is

$$v(t) = Ri(t).$$



Introducing the phasor  $I = I_0 e^{j\omega t}$  (see figure above), we get

$$v(t) = RI_0 e^{j\omega t},$$

and in the phasor notation

$$V = RI.$$

The frequency and time dependence are implicitly contained in the phasor current  $I$ .

### 1.4.2 The Capacitor

The variation of the voltage difference  $dv$  across a capacitor with capacitance  $C$  due to the amount of charge  $dQ$ , is

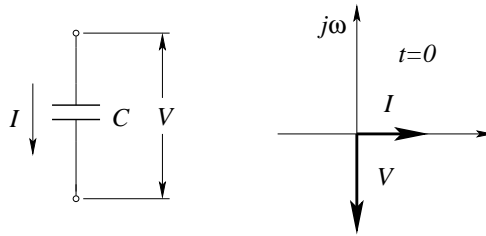
$$dv = \frac{dQ}{C}.$$

If the variation happens in a time  $dt$  and

$$i(t) = \frac{dQ}{dt},$$

we will have

$$\frac{dv(t)}{dt} = \frac{1}{C}i(t), \quad \Rightarrow \quad v(t) = \frac{1}{C} \int_0^t i(t') dt' + v(0).$$



For sinusoidal time dependence, we introduce the phasor  $I = I_0 e^{j\omega t}$  (see figure above), and get

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$$v(t) = \frac{1}{C} \int_0^t I_0 e^{j\omega t'} dt' + v(0),$$

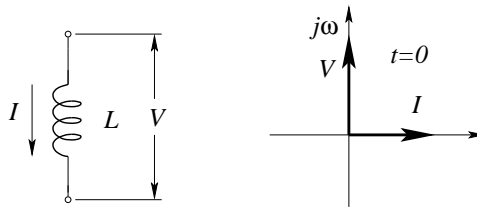
Using the phasor notation and supposing that for  $t = 0$  the capacitor is discharged, we finally get

$$V = \frac{1}{j\omega C} I, \quad , v(0) = 0.$$

### 1.4.3 The Inductor

The induced voltage  $v(t)$  of an inductor with inductance  $L$ , is

$$v(t) = L \frac{di(t)}{dt}.$$



Introducing the phasor  $I = I_0 e^{j\omega t}$  (see figure above), we get

$$v(t) = L \frac{d}{dt} I_0 e^{j\omega t},$$

and in the phasor notation

$$V = j\omega L I.$$

## 1.5 The Impedance and Admittance Concept

Let's consider a generic circuit with a port, whose voltage difference and current are respectively the phasors  $V = V_0 e^{j(\omega t + \varphi)}$ , and  $I = I_0 e^{j(\omega t + \psi)}$ . The ratio  $Z$  between the voltage difference and the current

$$Z(\omega) = \frac{V}{I} = \frac{V_0}{I_0} e^{j(\varphi - \psi)}.$$

is said to be the *impedance of the circuit*.

The inverse

$$Y(\omega) = \frac{1}{Z(\omega)}$$

is called the *admittance of the circuit*.

For example, considering the results of the previous subsection, the impedance of a resistor, a capacitor, and an inductor are respectively

$$Z_R = R, \quad Z_C(\omega) = \frac{1}{j\omega C}, \quad Z_L(\omega) = j\omega L,$$

and the admittances are

$$Y_R = \frac{1}{R}, \quad Y_C(\omega) = j\omega C, \quad Y_L(\omega) = \frac{1}{j\omega L}.$$

In general, the impedance or admittance of a circuit port is a complex function, which depends on the angular frequency  $\omega$ . Quite often, they are graphically represented by plotting the magnitude  $|Z(\omega)|$  or  $|Y(\omega)|$  in a double logarithmic scale and the phase  $\arg [Z(\omega)]$  or  $\arg [Y(\omega)]$  in a logarithmic scale.

For completeness, let's introduce some other definitions:

- The real part  $R$  of the impedance  $Z(\omega)$  is called *resistance*.
- The imaginary part  $X$  of the impedance  $Z(\omega)$  is called *reactance*. If  $X > 0$ , then  $X$  is an *inductive reactance* (from the fact that the reactance of an inductor is  $\omega L \geq 0$ ). If  $X < 0$  then  $X$  is a *capacitive reactance* (the reactance of a capacitor is  $-1/\omega C \leq 0$ ).
- The real part  $G$  of the admittance  $Y(\omega)$  is called *conductance*.
- The imaginary  $B$  part of the admittance  $Y(\omega)$  is called *susceptance*. If  $B < 0$ , then is  $B$  an *inductive susceptance*. If  $B > 0$  then  $B$  is a *capacitive susceptance*.

### 1.5.1 Impedance in Parallel and Series

It can be easily demonstrated that the same laws for the total resistance of a series or a parallel of resistors hold for the impedance

$$Z_{tot} = Z_1 + Z_2 + \dots + Z_N, \quad (\text{impedances in series})$$

$$\frac{1}{Z_{tot}} = \frac{1}{Z_1} + \frac{1}{Z_1} + \dots + \frac{1}{Z_N}, \quad (\text{impedances in parallel})$$

It is left as exercise to derive the homologue laws for the admittance.

## 1.5.2 Ohm's Law for Sinusoidal Regime

Thanks to the impedance concept, we can generalize Ohm's law and write the fundamental equation (*Ohm's law for sinusoidal regime*)

$$V(\omega) = Z(\omega)I(\omega).$$

## 1.6 Two-Port Networks

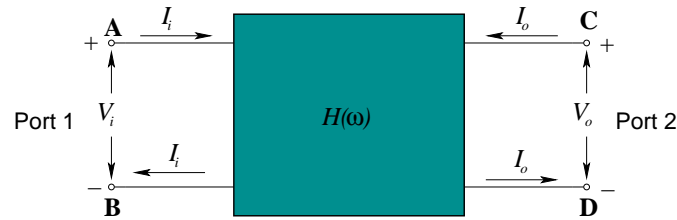


Figure 1.4: Two-port network circuit representation. The voltage difference signs and current directions follow standard conventions.

A *terminal port* is a terminal pair of a linear circuit whose input current equals the output current. A linear circuit with one pair of input terminals **A**, **B** and one pair output terminals **C**, **D** is called *two-port network* (see figure 1.4). The electronic circuits we will consider here, are two-port network with terminals **B** and **C** connected together[2]. In this case, to characterize the behavior of a two-port network, we can study the response of the output  $V_o$  as a function of the angular frequency  $\omega$  of a sinusoidal input  $V_i$ .

In general, we can write

$$V_o(\omega) = H(\omega)V_i(\omega), \quad \text{or} \quad H(\omega) = \frac{V_o(\omega)}{V_i(\omega)},$$

where the complex function  $H(\omega)$  is called the *transfer function* or *frequency response of the two-port network*. The transfer function contains the information of how the amplitude and the phase of the input changes when it reaches the output. Knowing the transfer function of this particular two-port network, we characterize the circuit<sup>2</sup>. The definition of  $H(\omega)$  suggests a way of measuring the transfer function. In fact, exciting the input with a sinusoidal wave, we can measure at the output the amplitude and the phase lead or lag respect to the input signal.

### 1.6.1 Bode Diagrams

To graphically represent  $H(\omega)$ , it is common practice to plot the magnitude  $|H(\omega)|$  (gain) in a double logarithmic scale, and the phase  $\arg [H(\omega)]$  using a logarithmic scale for the angular frequency. These plots are called Bode diagrams. Units for  $\omega$  are normally rad/s or Hz. The magnitude is quite often expressed in decibels dB (see appendix A)

$$X_{\text{dB}} = 20 \log_{10} X$$

For example  $20\text{dB} = 10$ ,  $40\text{dB}=100$ , etc... Practically, plotting a quantity in dB (which is not a units symbol such as m,s,kg,  $\Omega$  ) corresponds to plot the quantity on a logarithmic scale.

The phase can be expressed in radians (rad) or in degrees (deg).

Logarithmic scales have the advantage of emphasizing asymptotic trends and the disadvantage of flattening small variations. In other words, variations much smaller than the range of the plotted values become quite often indistinguishable in a logarithmic scale. To find out such kind of behavior, it is a good practice to look at magnitudes in both linear and logarithmic plots.

The Asymptotic Bode diagram [1], a simplification of the frequency response of a system is a convenient approximation of the characteristics of  $H(\omega)$ .

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<sup>2</sup>A thorough understanding of the transfer function of a circuit requires the concept of the Fourier transform and the Laplace transform and the convolution theorem. See [2] appendix C

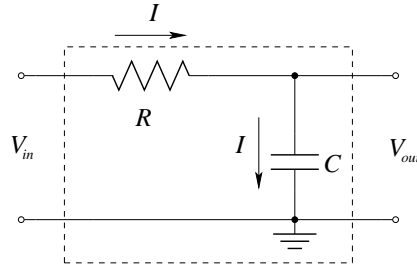


Figure 1.5: RC low-pass filter circuit.

### 1.6.2 The RC Low-Pass Filter

Figure 1.5 shows the *RC low-pass filter circuit*. The input and output voltage differences are respectively<sup>3</sup>

$$V_{in} = Z_{in}I = \left( R + \frac{1}{j\omega C} \right) I,$$

$$V_{out} = Z_{out}I = \frac{1}{j\omega C} I,$$

and the transfer function is indeed

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\tau\omega}, \quad \tau = RC.$$

or

$$H(\omega) = \frac{1}{1 + j\omega/\omega_0}, \quad \omega_0 = \frac{1}{RC}.$$

Computing the magnitude and phase of  $H(\omega)$ , we obtain

$$|H(\omega)| = \frac{1}{\sqrt{1 + \tau^2\omega^2}}$$

$$\arg(H(\omega)) = -\arctan\left(\frac{\omega}{\omega_0}\right)$$

Figure 1.6 shows the magnitude and phase of  $H(\omega)$ . The parameter  $\tau$  and  $\omega_0$  are called respectively the *time constant* and *angular cut-off frequency*

<sup>3</sup> $V_{out}$  as function of  $V_{in}$  can be directly calculated using the voltage divider equation.

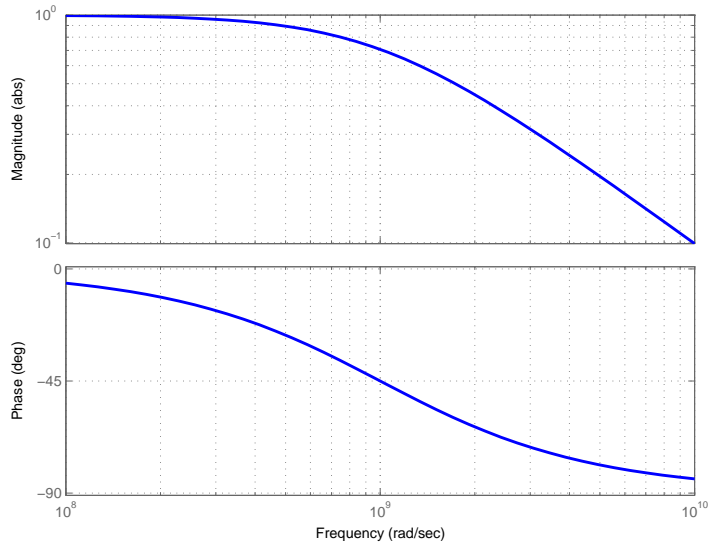


Figure 1.6: RC low-pass filter circuit transfer function.

of the circuit. The cut-off frequency is the frequency where the output  $V_{out}$  is attenuated by a factor  $1/\sqrt{2}$ .

It is worthwhile to analyze the qualitative behavior of the capacitor voltage difference  $V_{out}$  at very low frequency and at very high frequency.

For very low frequency the capacitor is an open circuit and  $V_{out}$  is essentially equal to  $V_{in}$ . For high frequency the capacitor acts like a short circuit and  $V_{out}$  goes to zero.

The capacitor produces also a delay as shown in the phase plot. At very low frequency the  $V_{out}$  follows  $V_{in}$  (they have the same phase). The output  $V_{out}$  loses phase ( $\omega t = \varphi \Rightarrow t = \varphi/\omega$ ) when the frequency increases. The output  $V_{out}$  starts lagging due to the negative phase  $\varphi$ , and then approaches a maximum delay at a phase shift of  $-\pi/2$ .

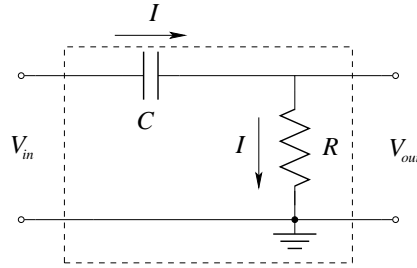


Figure 1.7: RC high-pass filter circuit

### 1.6.3 The RC High-Pass Filter

Figure 1.7 shows the *RC high-pass filter circuit*. The input and the output voltage differences are respectively

$$\begin{aligned} V_{in} &= Z_{in}I = \left( R + \frac{1}{j\omega C} \right) I, \\ V_{out} &= Z_{out}I = RI, \end{aligned}$$

and indeed the transfer function is

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega\tau}{1 + j\tau\omega}, \quad \tau = RC.$$

or

$$H(\omega) = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}, \quad \omega_0 = \frac{1}{RC}.$$

Computing the magnitude and phase of  $H(\omega)$ , we obtain

$$\begin{aligned} |H(\omega)| &= \frac{\tau\omega}{\sqrt{1 + \tau^2\omega^2}}, \\ \arg(H(\omega)) &= \arctan\left(\frac{\omega_0}{\omega}\right) \end{aligned}$$

Figure 1.8 shows the magnitude and phase of  $H(\omega)$ . The definitions in the previous subsection, for  $\tau$  and  $\omega_0$ , hold for the RC high-pass filter.

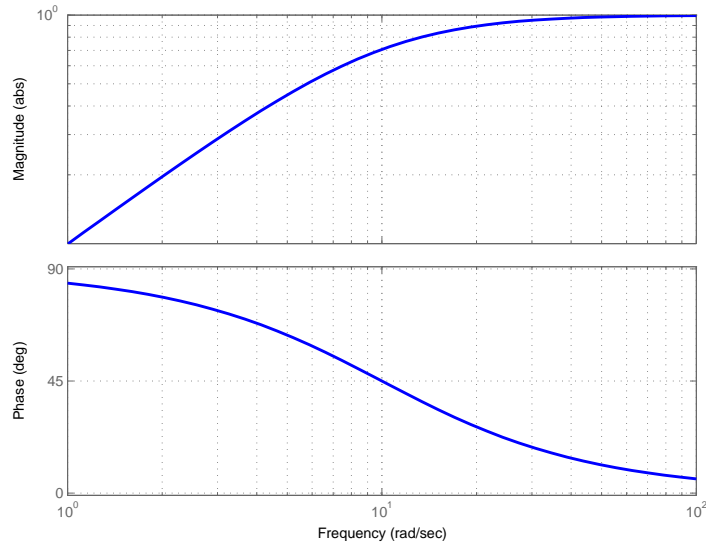


Figure 1.8: RC high-pass filter circuit transfer function.

## 1.7 Ideal Sources

### 1.7.1 Ideal Voltage Source

An ideal voltage source is a source able to deliver a given voltage difference  $V_s$  between its leads independent of the load  $R$  attached to it (see figure 1.9). It follows from Ohm's law that a voltage source is able to produce the current  $I$  necessary to keep constant the voltage difference  $V_s$  across the load  $R$ . The symbol for the ideal voltage source is shown in figure 1.9.

Quite often, a real voltage source exhibits a linear dependence on the resistive load  $R$ . It can be represented using an ideal voltage source  $V_s$  in series with a resistor  $R_s$  called input resistance of the source. Applying Ohm's law, it can be easily shown that the voltage and current through the load  $R$  are

$$V = \frac{R}{R + R_s} V_s, \quad I = \frac{V_s}{R + R_s}.$$

If we assume

$$R \gg R_s, \Rightarrow V \simeq V_s, \quad I \simeq \frac{V_s}{R}.$$

Under the previous condition, the real voltage source approximates the ideal case.

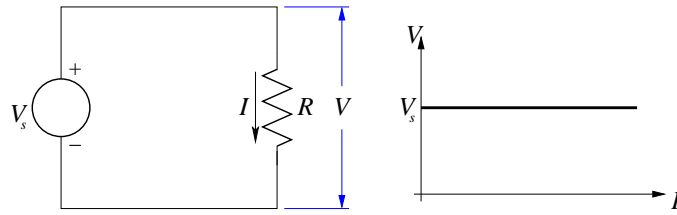


Figure 1.9: Ideal voltage source.

### 1.7.2 Ideal Current Source

An ideal current source is a source able to deliver a given current  $I_s$  that does not depend on the load  $R$  attached to it (see figure 1.10). It follows from Ohm's law that an ideal current source is able to produce the voltage difference  $V$  across the load  $R$  needed to keep  $I_s$  constant. The symbol for the ideal current source is shown in figure 1.10.

A real current source exhibits a dependence on the resistive load  $R$ , which can be represented using an ideal current source  $I_s$  in parallel with a resistor  $R_s$ . Applying Ohm's law and the KCL, it can be easily shown that the voltage and current through the load  $R$  are

$$I = \frac{R_s}{R + R_s} I_s, \quad V = \frac{R_s R}{R_s + R} I_s.$$

If we suppose

$$R_s \gg R, \Rightarrow I \simeq I_s, \quad V \simeq R I_s \gg 0$$

Under the previous condition, the real current source approximates the ideal case.

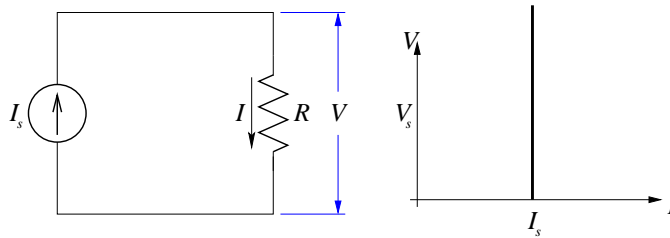


Figure 1.10: Ideal current source.

## 1.8 Equivalent Networks

Quite often, the analysis of a network becomes easier by replacing part of it with an equivalent and simpler network or dividing it into simpler sub-networks.

For example, the voltage divider is an equation easy to remember that allows to divide a complex circuit in two parts simplifying the search of the solution.

Thévenin and Norton theorems give us two methods to calculate equivalent circuits which behave like the original circuit, as seen from two points of it. The techniques briefly explained in this section, will be then extensively used in the next chapters .

### 1.8.1 Voltage Divider

The voltage divider equation is applicable every time we have a circuit which can be re-conducted to a series of two simple or complex components. Considering the circuit branch of figure 1.11 and applying Ohm's law, we have

$$\begin{aligned} V_{Tot} &= (Z_1 + Z_2)I \\ V_2 &= Z_2I \end{aligned}$$

and indeed

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_{Tot},$$

which is the voltage divider equation.

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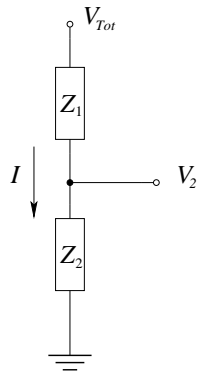


Figure 1.11: Voltage divider circuit.

## 1.8.2 Thévenin Theorem

Thévenin theorem allows us to find an equivalent circuit for a network seen from two points **A** and **B** using an ideal voltage source  $V_{Th}$  in series with an impedance  $Z_{Th}$ .

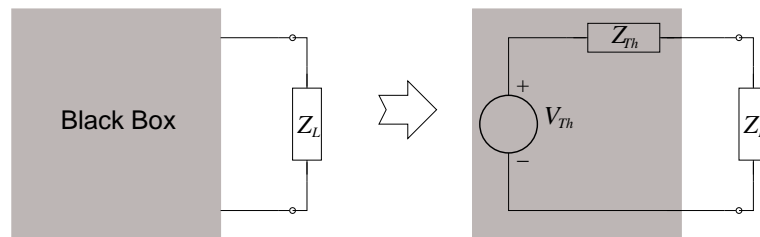


Figure 1.12: Thévenin equivalent circuit illustration.

The equivalence means that if we place a load  $Z_L$  between **A** and **B** in the original circuit (see figure 1.12) and measure the voltage  $V_L$  and the current  $I_L$  across the load, we will obtain exactly the same  $V_L$  and  $I_L$  if  $Z_L$  is placed in the equivalent circuit. This must be true for any load we connect between the points **A** and **B**.

The previous statement and the linearity of the circuit can be used to find  $V_{Th}$  and  $Z_{Th}$ . In fact, if we consider  $Z_L = \infty$  (open circuit, OC), we will have

$$V_{Th} = V_{OC}.$$

The Voltage  $V_{Th}$  is just the voltage difference between the two leads **A** and **B**.

For  $Z_L = 0$  (short circuit, SC) we must have

$$I_{SC} = \frac{V_{Th}}{Z_{Th}} = \frac{V_{OC}}{Z_{Th}}.$$

and therefore

$$Z_{Th} = \frac{V_{OC}}{I_{SC}}.$$

The last expression says that the Thévenin impedance is the impedance seen from the points **A** and **B** of the original circuit.

If the circuit is known, the Thévenin parameter can be calculated in the case for the terminals **A** and **B** open. In fact,  $V_{Th}$  is just the voltage across **A** and **B** of a known circuit. Replacing the ideal voltage sources with short circuits (their resistance is zero) and ideal current sources with open circuits ( their resistance is infinite), we can calculate the impedance  $Z_{Th}$  seen from terminals **A** and **B**.

Considering the previous results, we can finally state Thévenin theorem as follows:

*Any circuit seen from two points can be replaced by an ideal voltage source of voltage  $V_{Th}$  in series with impedance  $Z_{Th}$ .  $V_{Th}$  is the voltage difference between the two point of the original circuit.  $Z_{Th}$  is the impedance seen from these two points, short-circuiting all the ideal voltage generators and open-circuiting all the ideal current generators.*

### Example:

We want to find the Thévenin circuit of the network enclosed in the gray rectangle of figure 1.13. To find  $R_{Th}$ , and  $V_{Th}$  we have to disconnect the circuit in the points **A** and **B**. In this case, the voltage difference between these two points, thanks to the voltage divider equation, is

$$V_{Th} = \frac{R_1}{R_1 + R_2} V_s.$$

Short circuiting  $V_s$  we will have  $R_1$  in parallel with  $R_2$ . The Thévenin resistance  $R_{Th}$  will be indeed

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}.$$

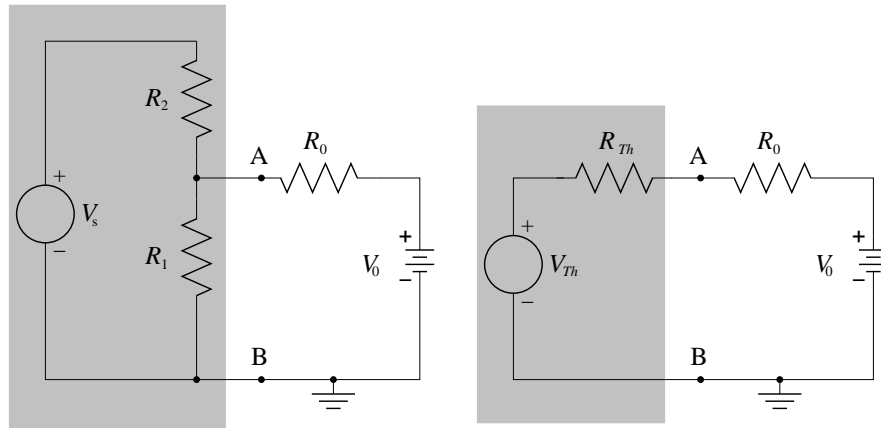


Figure 1.13: Thévenin equivalent circuit example

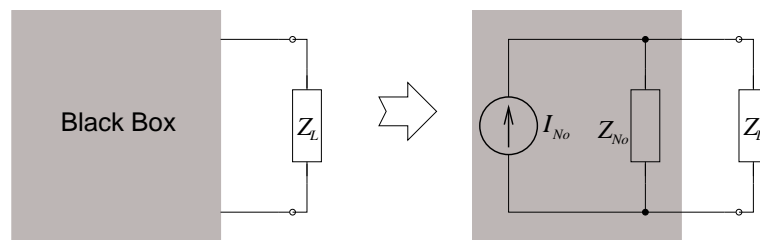


Figure 1.14: Norton equivalent circuit illustration.

### 1.8.3 Norton Theorem

Any kind of active network seen from two points **A** and **B** can be replaced by an ideal current generator  $I_{No}$  in parallel with a impedance  $Z_{No}$ . The current  $I_{No}$  corresponds to the short-circuit current of the two points **A** and **B**. The Resistance  $Z_{No}$  is the Thévenin resistance  $Z_{No} = Z_{Th}$ .

The proof of this theorem is left as exercise.

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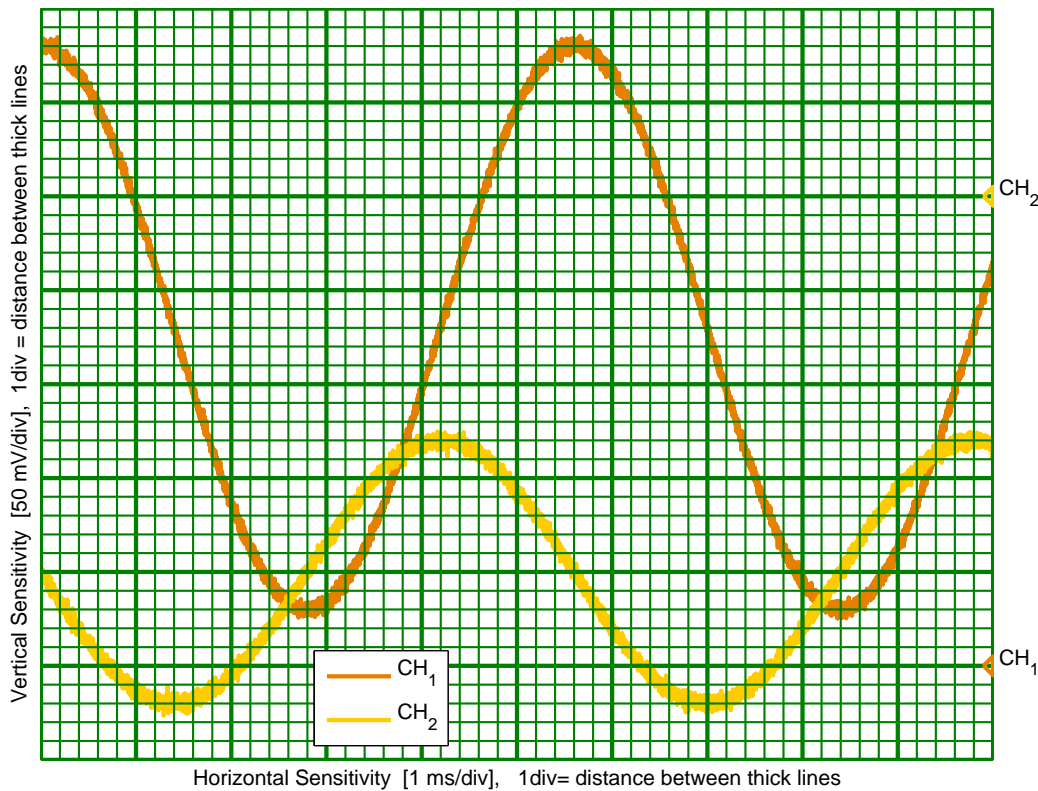
## 1.9 First Laboratory Week

This first laboratory class is essentially intended to help the student to become acquainted with the laboratory instrumentation (function generator, digital multimeter, circuit bread board, connectors), and with the use of passive components and their mathematical descriptions.

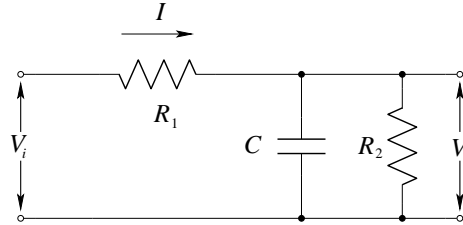
### 1.9.1 Pre-laboratory Problems

To complete the preparatory problems, it is recommended to read sections 1.1 to 1.6, and appendix C for the laboratory procedure.

1. Considering the figure below (a “snapshot” of an oscilloscope display), determine the peak to peak amplitude, the DC offset, the frequency of the two sinusoidal curves, and the phase shift between the two curves (channels horizontal axis position is indicated by an arrow and the channel name on the right of the figure).

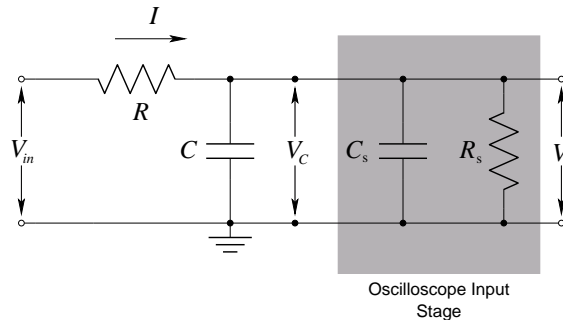


1. Sketch in a graph the magnitude and phase of the RC series circuit input impedance  $Z_i$ . Determine the two asymptotic behavior of  $|Z_i|$ , i.e. where the resistive and the capacitive behavior dominate.
2. Assuming that  $R_2 \gg R_1$ , repeat the previous problem for the following circuit



Hint: in this case there are three dominant behaviors, two resistive and one capacitive, and two frequencies which separate the three dominant behaviors.

3. The circuit below includes the impedance of the input channel of the CRT oscilloscope, and  $V_s$  is indeed the real voltage measured by the instrument.



Find the voltage  $V_s(\omega)$ , and the angular cut-off frequency  $\omega_0$  of the transfer function  $V_s/V_{in}$  (i.e. the value of  $\omega$  for which  $|V_s/V_{in}| = 1/\sqrt{2}$ ).

Show that for  $\omega = 0$  the  $V_s(\omega)$  formula simplifies and becomes the resistive voltage divider equation.

Demonstrate that the conditions to neglect the input impedance of the oscilloscope are the following :

$$C \gg C_s, \quad R \ll R_s$$

(Hint: use the voltage divider equation to write  $V_C$ .)

4. Considering the previous circuit, calculate the value of  $R$  needed to obtain  $V_{in} \simeq V_s$  with a fractional systematic error of 1%, if  $\omega = 0$  rad/s and  $R_s = 1\text{M}\Omega$ .
5. Determine the values of  $R$ , and  $C$  needed to get a RC series circuit cut-off frequency of 20kHz. Choose values which make the oscilloscope impedance negligible ( $R_s = 1\text{M}\Omega$ ,  $C_s = 25\text{pF}$ ).

### 1.9.2 Procedure

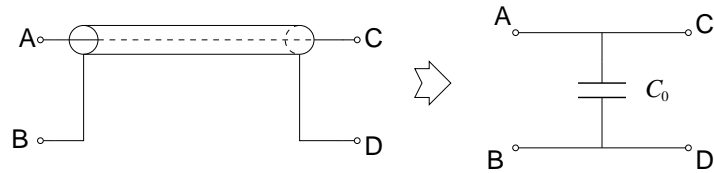
Whenever you work with electronic circuits as a beginner (all students are considered beginners; it doesn't matter which personal skills they already have), some extra precautions must be taken to avoid injuries. These are the main ones:

- NEVER CONNECT INSTRUMENT PROBES OR LEADS TO AN OUTLET OR MORE IN GENERAL TO THE POWER LINE.
- DO NOT TRY TO FIX/IMPROVE AN INSTRUMENT BY YOURSELF.
- DO NOT POWER AN INSTRUMENT WHICH IS NOT WORKING OR DISASSEMBLED.
- DO NOT TOUCH A DISASSEMBLED OR PARTIALLY DISASSEMBLED INSTRUMENT, EVEN IF IT IS NOT POWERED.
- WEAR PROTECTIVE GOGGLES EVERY TIME YOU USE A SOLDERING IRON.
- TO AVOID EXPLOSIONS, NEVER USE A SOLDERING IRON ON A POWERED CIRCUIT AND BATTERIES.
- PLACE A FAN TO DISPERSE SOLDER VAPORS DURING SOLDERING WORK.

Read carefully the text before starting the laboratory measurements.

BNC cables and wires terminated with banana connectors are available to connect the circuit under measurement to the instruments.

BNC<sup>4</sup> cables, a diffused type of radio frequency (RF) coaxial shielded cable, have an intrinsic and quite well defined capacitance due to their geometry as shown in the figure below



They have typical linear density capacitance  $\Delta C / \Delta l \simeq 98 \text{ pF/m}$ . Wires terminated with banana connectors have smaller capacitance than BNC cables but not as constant as BNC cables (Why?).

1. Build a RC low-pass or a RC high-pass filter with a cut-off frequency between 10kHz to 100kHz. Choose components values which make the perturbation of the input impedance of the oscilloscope negligible. Then, do the following
  - (a) Verify the circuit transfer function, and in particular for frequencies  $\nu \ll \nu_0$  and  $\nu \gg \nu_0$
  - (b) Find the cut-off frequency  $\nu_0$  knowing the expected magnitude and phase values, and compare with the theoretical values
  - (c) Change the capacitor value to be comparable to the oscilloscope input capacitance or the BNC cable capacitance and experimentally find the new cut-off frequency  $\nu_1$ . Compare with the theoretical value.
  - (d) Drive the circuit input with a square wave and verify the transient response of the circuit.
  
2. Measure the output impedance of the function generator for a given fixed sinusoidal frequency. Place at least three different loads to perform the measurement.

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<sup>4</sup>“BNC” seems to stand for Bayonet Neill Concelman (named after Amphenol engineer Carl Concelman). Other sources claims that the acronym means British Navy Connector. What is certain is that the BNC connector was developed in the late 1940’s as a miniature version of the type C connector (what does the “C” stand for ?)

# Bibliography

[1] Ref for Maxwell equations versus Kirchhoff's laws.

[2] Microelectronics, Jacob Millman, and Arvin Grabel , Mac-Graw Hill

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