CALIFORNIA INSTITUTE OF TECHNOLOGY
PHYSICS MATHEMATICS AND ASTRONOMY DIVISION

Sophomore Physics Laboratory (PH005/105)

Analog Electronics
Appendix

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Appendix A

Fourier Analysis

A.1 Discrete Spectrum

The Fourier analysis is a fundamental an extremely useful method to characterize a generic signal. It is based on the Fourier theorem which states[1] that any periodic function $v_T(t)$ can be represented as a series of sines with different amplitudes V_n , and frequency $\omega = n\omega_0$, i.e.

$$v_T(t) \quad = \quad \sum_{n=0}^{\infty} V_n \sin(\omega_0 n t)\,, \quad \omega_0 = \frac{2\pi}{T}\,.$$

The set of points (ω, V_n) is called discrete spectrum of the function and is given by the following integrals

$$V_n = \frac{2}{T}\int_0^T v_T(t)\sin(\omega_0 n t)\,dt.$$

A.1.1 Example: Square Wave

If we consider a square wave symmetric respect to the time axis

$$v_T(t) = \begin{cases} v_0, & 0 \le t < T/2 \\ -v_0, & T/2 \le t < T \end{cases}\,,$$

then the corresponding Fourier series is

$$v_T(t) = \sum_{i=0}^{\infty} \frac{4v_0}{\pi}\frac{1}{2n+1}\,\sin[(2n+1)\omega_0 t]\,.$$

[1]Here the Fourier series theorem is not enunciated in its most general form.

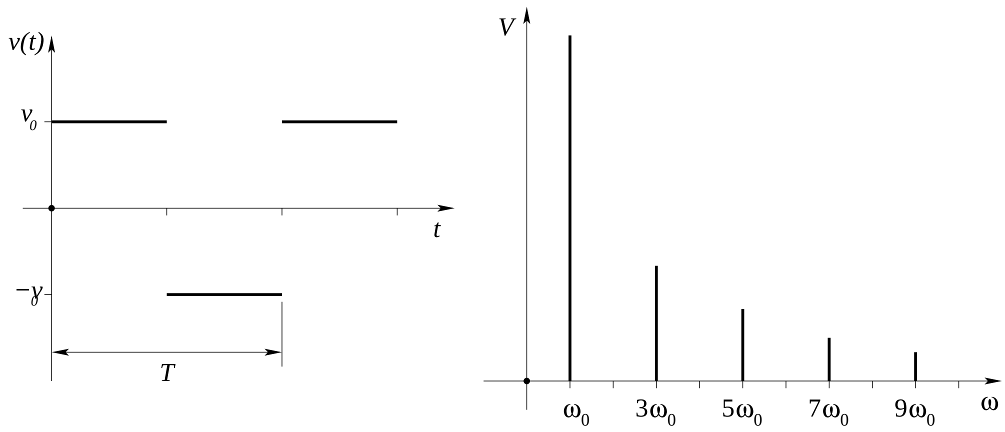


Figure A.1: Time domain representation (left) and frequency domain representation (right) of a square wave signal.

Figure A.1 shows the time domain representation of the signal and its frequency domain representation or frequency spectrum.

A.2 Continuous Spectrum

Using Fourier transform operator, the representation in the frequency domain can be extended to any type of signal $v(t)$

$$v(t) = \int_{-\infty}^{+\infty} V(\omega) e^{i\omega\tau} d\omega$$

and in this case we will have a continuum spectrum given by the Fourier integral

$$V(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} v(t) e^{-i\omega\tau} d\tau.$$

It is important to notice that $v(t)$ can be any type of signal even a random signal, in other words a noise. Real signals can be considered as the sum of deterministic and random signals. If we compute a spectrum of such a signal we expect to see the contribution of both, i.e. the noise spectrum which is present at all frequencies and the signal spectrum.

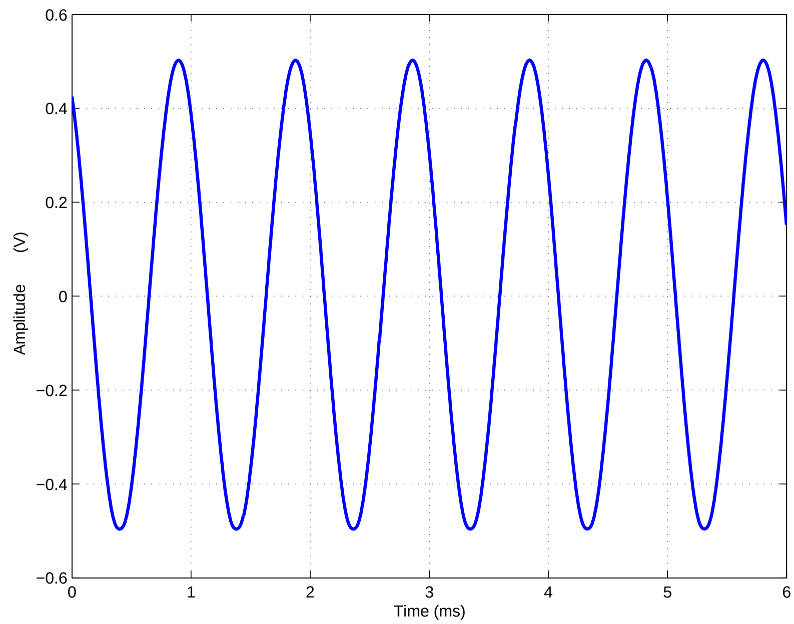


Figure A.2: Sinusoidal signal produced by a function generator (Tektroniχ GFG253) and acquired using a Spectrum analyzer (Stanford SR785).

A.2.1 Spectrum Estimation

For practical purposes, the spectrum is always estimated in a finite interval, (nobody can wait that long to compute a spectrum). The signal is acquired using an analog to digital converter and then numerically processed using the fast Fourier transform algorithm to estimate the spectrum. The results is a truncated discrete spectrum, which estimates the signal continuous spectrum.

A.2.2 Power Spectral Density and Units

The square of the Fourier coefficients $|V(\omega)|^2$, which are proportional to the signal power are calculated to estimate the signal spectrum. Normalizing those coefficients by the frequency step size (bin) of the discrete spectrum we obtain the so called *power spectral density* (PSD). This operation is mainly done to allow the amplitude comparison of spectra taken with different bins. In this case, the coefficients units are

$$\left[|V(\omega)|^2\right] = \frac{\text{[power arbitrary units]}}{\text{Hz}}.$$

Quite often, the square root of the PSD is considered, and unfortunately, it is quite common to find scientists who inaccurately call it PSD.

A.2.3 Example: Sinusoidal Function Generator

Figure A.2 shows a sinusoidal signal produced by a function generator and acquired with a digital instrument called spectrum analyzer. The frequency spectrum computed using the same instrument, is shown in figure A.3. The lower plot of figure A.3 shows the same spectrum between 500Hz and 10kHz with a horizontal linear scale to emphasize the harmonics content.

If we look at the time domain, it is quite difficult to see any harmonic distortion of the signal. On the contrary, the frequency domain representation clearly shows all the signal distortions.

The fundamental frequency is $\nu_0 = 1.02$kHz and the amplitude is $V_0 = 0.26$V/$\sqrt{\text{Hz}}$. The next harmonic, the second taller peak, has an amplitude $V_1 = 0.8$mV/$\sqrt{\text{Hz}}$, which implies that the fundamental frequency amplitude is at least more than 300 times larger than each high order harmonics.

Considering the time domain plot, the amplitude of the sinusoid is $V_0 = 0.5V_{pk}$, then the frequency bin amplitude must be $\Delta\nu = 3.7$Hz ($0.5/\sqrt{3.7} = 0.26$V/$\sqrt{\text{Hz}}$).

The spectrum also shows several peaks symmetric around the fundamental frequency ν_0 due to unwanted amplitude modulations of the fundamental frequency.

The other feature visible in the spectrum is the noise floor, i.e the noise level around the peaks base. This noise floor is reasonably flat above ν_0 with a magnitude $\delta V \simeq 0.2 - 0.3\mu$V/$\sqrt{\text{Hz}}$. Below ν_0 seems to have a negative slope and an average value of $\delta V \sim 0.7\mu$V/$\sqrt{\text{Hz}}$.

The so called power lines (60Hz and harmonics) are clearly visible in the spectrum.

In general, it is important to know and measure the resolution of the instrument to be certain that the noise level measurement is not dominated by the instrument noise. Moreover, the instrument resolution depends on the input dynamic range. The larger is the dynamic range the worst is the instrument resolution. Quite often, the dynamic range can be reduced removing the DC component of the signal to measure. In this particular case, a better resolution could be achieved reducing the dynamic range with a notch filter tuned at the frequency ν_0.

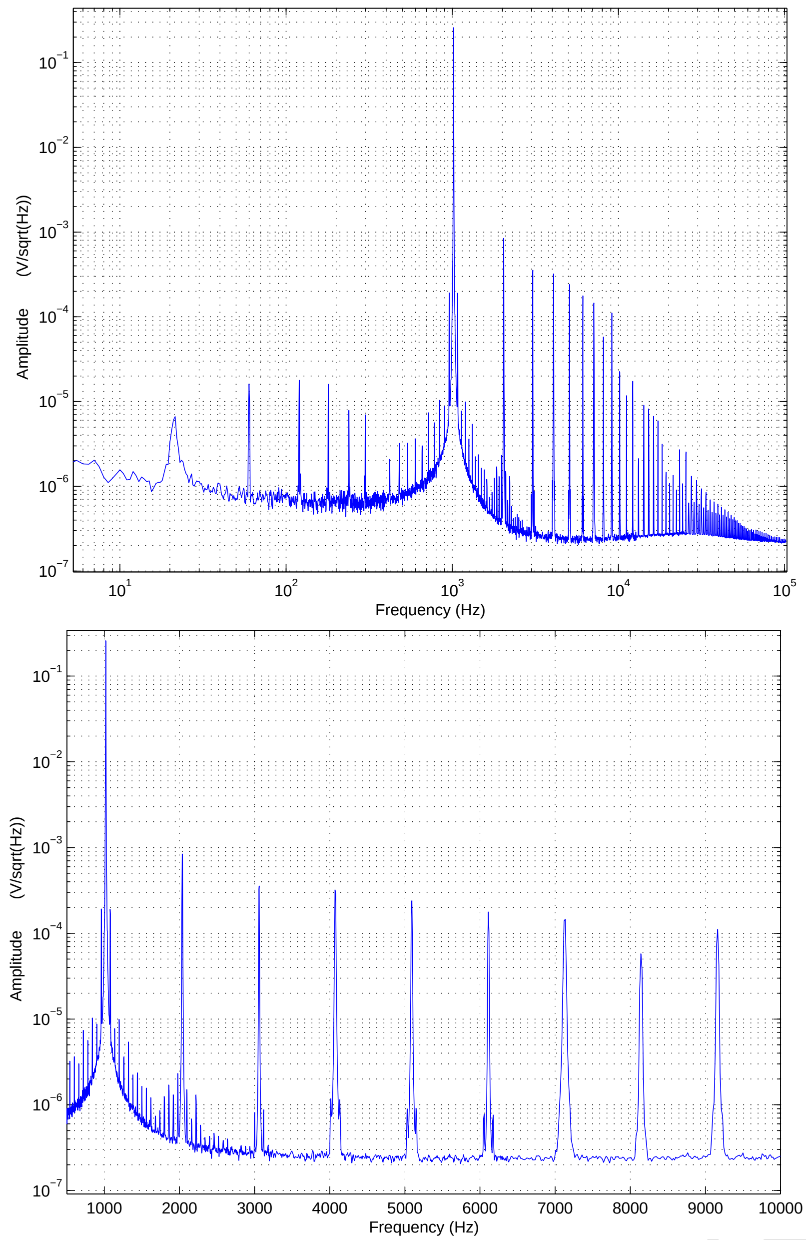


Figure A.3: Sinusoidal signal spectrum of figure A.2. Upper plot shows the spectrum with a logarithmic scale for the horizontal axis. The lower plot uses a linear scale between 500Hz to 10kHz.

Appendix B

Decibels

B.1 Definition of Decibel

The decibel is defined as 10 times the logarithm in base ten of a power P normalized to a reference power P_r , i.e.

$$X\ (\mathrm{dB}) = 10 \log_{10} \frac{P}{P_r}\,. \tag{B.1}$$

Considering that

$$P = \frac{V^2}{R} = RI^2,$$

and supposing that we use the same reference resistance R_r for P, P_r,we can rewrite equ. (B.1) as

$$X\ (\mathrm{dB}) = 20 \log_{10} \frac{V}{V_r} = 20 \log_{10} \frac{I}{I_r},$$

where V_r, and I_r are respectively the voltage and the current across the reference resistance R_r. In other words, we have to measure the voltage or the currents across equal impedaces, to get the decibels.

B.2 Generalization of the Use of Decibel

For practical purposes, the decibel is also used to report the ratio of homogeneus quantities such as the voltage output V_o over the voltage input V_i

of a two port network, or more in general the ratio of any kind of homogeneus quantities x_1, x_2

$$X\ (\mathrm{dB}) = 20 \log_{10} \frac{x_1}{x_2}\ .$$

In this case there is no normalization respect to a reference load R_r or power P_r.

B.3 Useful Table and Properties

The next table is quite useful to easily translate decibels into magnitude

(dB)	0	1	2	3	4	5	6	7	8	9	10
Magnitude	1	1.1	1.2	1.4	1.6	1.8	2	2.2	2.5	2.8	3.2

For conveniece, let´s rewrite some useful properties of the logarithm function

$$\begin{aligned} \log(x\, y) &= \log\ x + \log\ y, \\ \log(x/y) &= \log\ x - \log\ y, \\ \log x^n &= n \log\ x, \\ \log_a x &= \log_b x / \log_b a. \end{aligned}$$

B.4 Standard Power References

Decibels comes in many flavors (different reference powers) depending on the application, radio frequency, microwaves, optics, et cetera.

For example the following definition is quite often used

$$X\ (\mathrm{dBm})(R_r) = 10 \log_{10} \frac{V^2/R_r}{1\mathrm{mW}}$$

The value of R_r depends on the application field

	$R_r(\Omega)$
Radio Frequency	50
TV Frequencies	75
Audio Frequencies	600

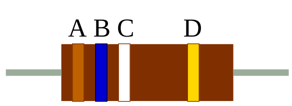
Appendix C

Resistor Color Code

Nominal values of resistances are coded using colors bands around the resistors (see figure below). The bands identify digits and the exponent in base ten for the resistance value and the tolerance as explained in the following table:

Band Number	1	2	3	4 (Tolerance band)	5
3 Bands	Digit	Digit	Exponent	Always 20%	
4 Bands	Digit	Digit	Exponent	Tolerance	
5 Bands	Digit	Digit	Exponent	Tolerance	Tolerance after 1000 hours

3 Band resistors have no band for the tolerance because it is assumed to be 20% of the nominal values.The fifth band is not an industry standard, but quite often it means the tolerance after 1000 hours of continuous use.



$$R = AB \cdot 10^C, \quad \Delta R = R \cdot D$$

The bands are counted from left to right. The following table reports the coding of the values using colors and a mnemonic sentence to remember the color code table.

Mnemonic Sentence	Color	Exponent	Tolerance (%)	Tolerance (%) 5th Band
Big	Black	0	20	
Bart	Brown	1	1	1%
Rides	Red	2	2	0.1%
Over	Orange	3		0.01%
Your	Yellow	4		0.001%
Grave	Green	5		
Blasting	Blue	6		
Violent	Violet	7		
Guns	Gray	8		
Wildly.	White	9		
Go	Gold	-1	5	
Shoot (him?)	Silver	-2	10	

For example, the nominal resistance of a 4 band resistor having the sequence brown, black, orange and gold is

$$\begin{array}{r} R_{nom.} = 10\text{k}\Omega \\ \Delta R_{nom.} = 5\%10\text{k}\Omega \end{array} \quad \Rightarrow \quad R_{nom.} \quad = (10.0 \pm 0.5)\text{k}\Omega$$

Resistor size (volume) is related to the power dissipation capability. Typical used values are 1/4W 1/2W, 1W.

Appendix D

The Cathode Ray Tube Oscilloscope

D.1 The Cathode Ray Tube Oscilloscope

The *cathode ray tube oscilloscope* is essentially an analog[1] instrument that is able to measure time varying electric signals. It is made of the following functional parts (see figure D.1):

- the cathode ray tube (CRT),
- the trigger,
- the horizontal input,
- the vertical input,
- time base generator.

Let's study in more detail each component of the oscilloscope.

D.1.1 The Cathode Ray Tube

The CRT is a vacuum envelope hosting a device called *an electron gun* , capable of producing an electron beam, whose transverse position can be modulated by two electric signals (see figures D.1 and D.7).

[1]Hybrid instruments combining the characteristics of digital and analog oscilloscopes, with a CRT, are also commercially available.

When the electron gun cathode is heated by wire resistance because of the Joule effect it emits electrons . The increasing voltage differences between a set of shaped anodes and the cathode accelerates electrons to a terminal velocity v_0 creating the so called electron beam.

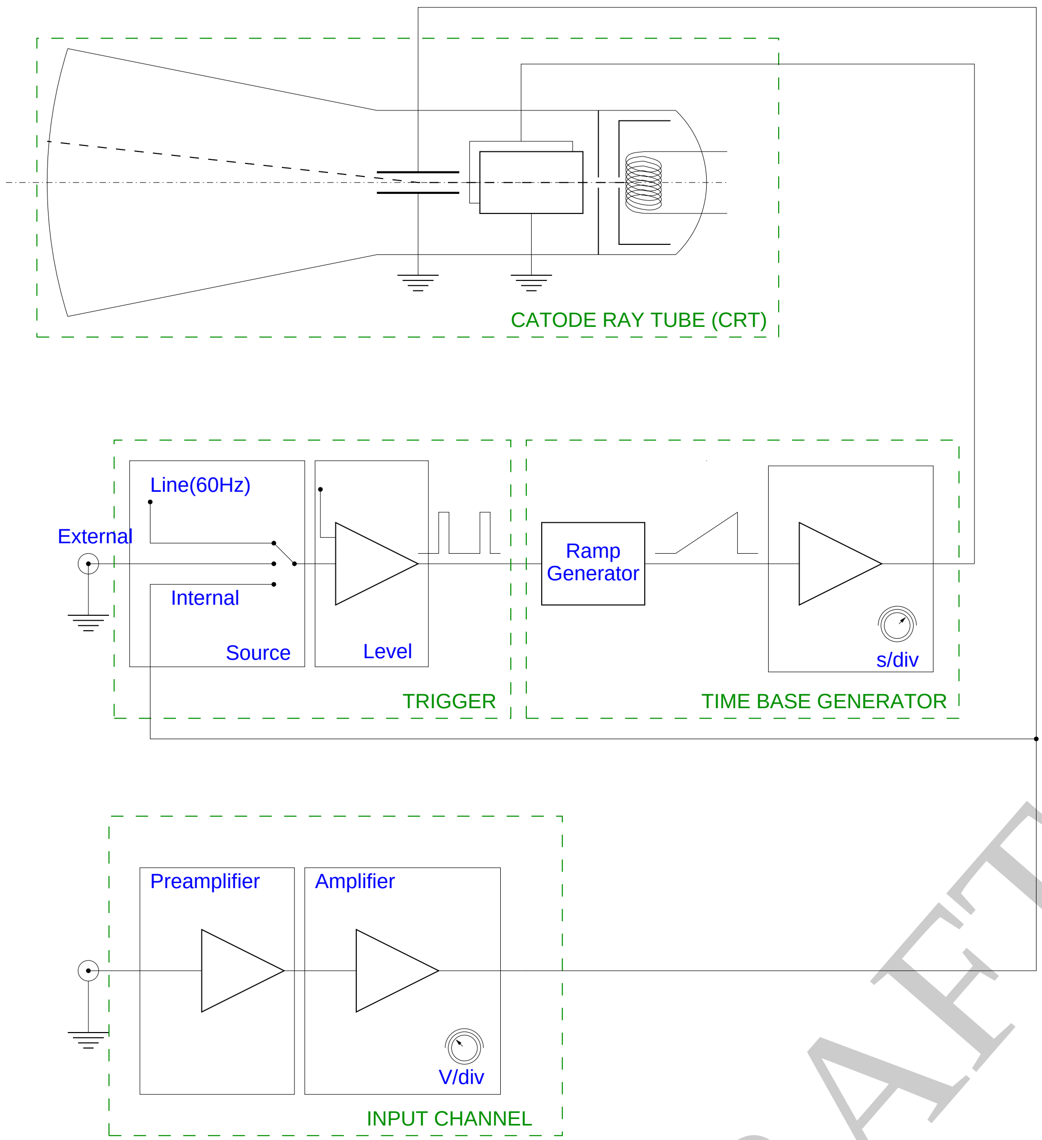


Figure D.1: Oscilloscope functional schematics

The beam then goes through two orthogonally mounted pairs of metallic plates. Applying voltage difference to those plates V_x and V_y, the beam is deflected along two orthogonal directions (x and y) perpendicular to its direction z. The deflected electrons will hit a plane screen perpendicular to the beam and coated with florescent layer. The electrons interaction with this layer generates photons, making the beam position visible on the screen.

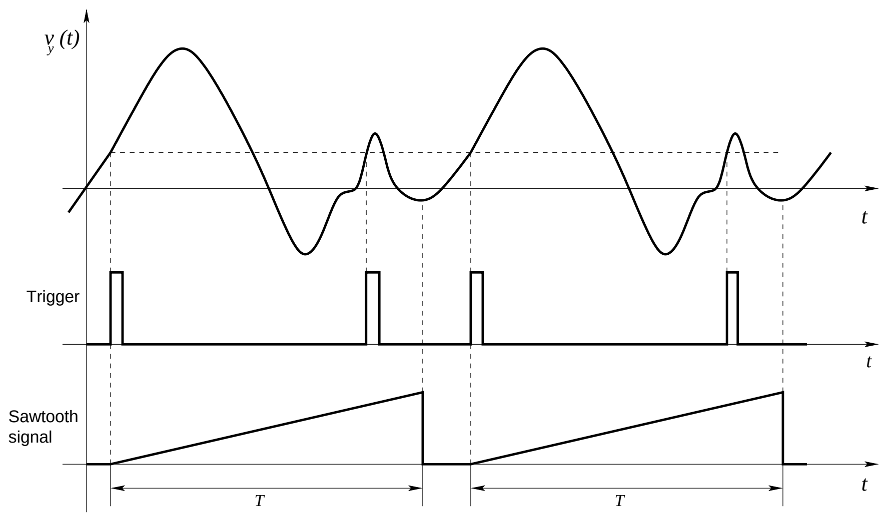


Figure D.2: Periodic Signal triggering.

D.1.2 The Horizontal and Vertical Inputs

The vertical and horizontal plates are independently driven by a variable gain amplifier to adapt the signals $v_x(t)$, and $v_y(t)$ to the screen range. A DC offset can be added to each input to position the signals on the screen. These two channels used to drive the signals to the plates signals are called horizontal and vertical inputs of the oscilloscope.

In this configuration the oscilloscope is an x-y plotter.

D.1.3 The Time base Generator

If we apply a sawtooth signal $V_x(t) = \alpha t$ to the horizontal input, the horizontal screen axis will be proportional to time t. In this case a signal $v_y(t)$

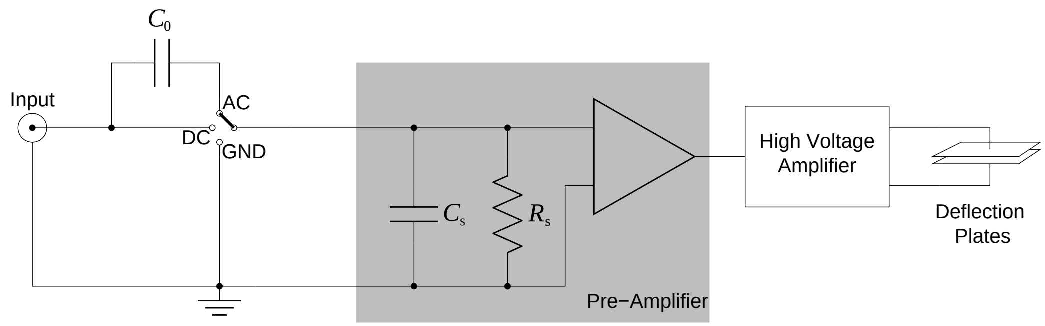


Figure D.3: Oscilloscope input impedance representation using ideal components (gray box). Input channel coupling is also shown.

applied to the vertical input, will depict on the oscilloscope screen the signal time evolution.

The internal ramp signal is generated by the instrument with an amplification stage that allows changes in the gain factor α and the interval of time shown on the screen. This amplification stage and the ramp generator are called the *time base generator.*

In this configuration, the horizontal input is used as a second independent vertical input, allowing the plot of the time evolution of two signals.

Visualization of signal time evolution is the most common use of an oscilloscope.

D.1.4 The Trigger

To study a periodic signal $v(t)$ with the oscilloscope, it is necessary to synchronize the horizontal ramp $V_x = \alpha t$ with the signal to obtain a steady plot of the periodic signal. The trigger is the electronic circuit which provides this function. Let's qualitatively explain its behavior.

The trigger circuit compares $v(t)$ with a constant value and produces a pulse every time the two values are equal and the signal has a given slope. The first pulse triggers the start of the sawtooth signal of period[2] T , which will linearly increase until it reaches the value $V = \alpha T$, and then is reset to zero. During this time, the pulses are ignored and the signal $v(t)$ is plotted for a duration time T. After this time, the next pulse that triggers the sawtooth signal will happen for the same previous value and

[2]In general, the sawtooth signal period T and the period of $v(t)$ are not equal.

slope sign of $v(t)$, and the same portion of the signal will be re-plotted on the screen.

D.2 Oscilloscope Input Impedance

A good approximation of the input impedance of the oscilloscope is shown in the circuit of figure D.3. The different input coupling modes (DC AC GND) are also represented in the circuit.

The amplifying stage is modeled using an ideal amplifier (infinite input impedance) with a resistor and a capacitor in parallel to the amplifier input.

The switch allows to ground the amplifier input and indeed to vertically set the origin of the input signal (GND position), to directly couple the input signal (DC position), or to mainly remove the DC component of the input signal (AC position).

D.3 Oscilloscope Probe

An oscilloscope probe is a device specifically designed to minimize the capacitive and resistive load added when the instrument is connected to the circuit. The price to pay is an attenuation of the signal that reaches the oscilloscope input[3].

Let's analyze the behavior of a passive probe. Figure D.4 shows the schematics of the equivalent circuit of a passive probe and of the input stage of an oscilloscope. The capacitance of the probe cable can be considered included in C_s

Considering the voltage divider equation, we have

$$H(j\omega) = \frac{V_s}{V_i} = \frac{Z_s}{Z_p + Z_s}, \tag{D.1}$$

where

$$\frac{1}{Z_s} = j\omega C_s + \frac{1}{R_s}, \qquad \frac{1}{Z_p} = j\omega C_p + \frac{1}{R_p},$$

[3]Active probes can partially avoid this problems by amplifying the signal.

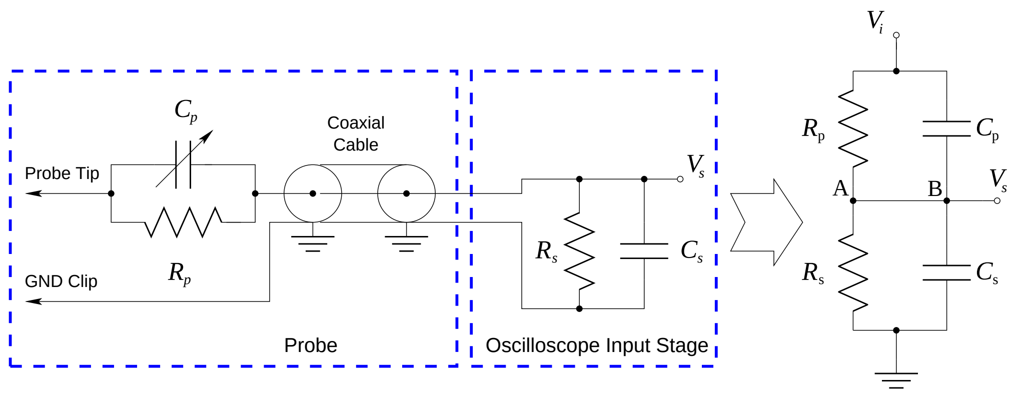


Figure D.4: Oscilloscope input stage and passive probe schematics.The equivalent circuit made of ideal components for the probe shielded cable is not shown.

and then

$$Z_s = \frac{R_s}{j\omega\tau_s + 1}, \qquad Z_p = \frac{R_p}{j\omega\tau_p + 1}.$$

Defining the following parameters

$$\tau_p = C_p R_p, \quad \alpha = \frac{R_s}{R_s + R_p}, \qquad \beta = \frac{C_p}{C_s + C_p},$$

and after some tedious algebra, equation (D.1) becomes

$$H(j\omega) = \alpha \frac{1 + j\omega\tau_p}{1 + j\omega\frac{\alpha}{\beta}\tau_p},$$

which is the transfer function from the probe input to the oscilloscope input before the ideal amplification stage.

The DC and high frequency gain of the transfer function $H(j\omega)$ are respectively

$$H(0) = \alpha, \qquad H(\infty) = \beta.$$

The numerator and denominator of $H(j\omega)$ are respectively equal to zero, (the zeros and poles of H) when

$$\omega = \omega_z = j\frac{1}{\tau_p}, \qquad \omega = \omega_p = j\frac{\beta}{\alpha}\frac{1}{\tau_p}.$$

Figure D.5 shows the qualitative behavior of H for $\frac{\alpha}{\beta} > 1$.

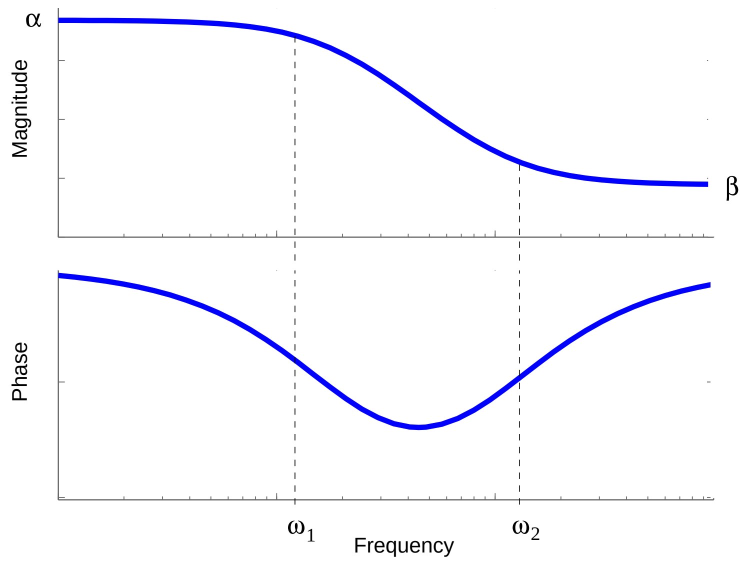


Figure D.5: Qualitative transfer function from the under compensated probe input to the oscilloscope input before the ideal amplification stage. As usual, the oscilloscope input is described having an impedance $R_s||C_s$.

D.3.1 Probe Frequency Compensation

By tuning the variable capacitor C_p of the probe, we can have three possible cases

$$\frac{\alpha}{\beta} < 1 \quad \Rightarrow \quad \text{over-compensation}$$
$$\frac{\alpha}{\beta} = 1 \quad \Rightarrow \quad \text{compensation}$$
$$\frac{\alpha}{\beta} > 1 \quad \Rightarrow \quad \text{under-compensation}$$

if $\alpha < \beta$ the transfer function attenuates more at frequencies above ω_z, and the input signal V_i is distorted.

if $\alpha = \beta$ the transfer function is constant and the input signal $V_i$will be undistorted, and attenuated by a factor α.

if $\alpha > \beta$ the transfer function attenuates more at frequencies below ω_p and the input signal $V_i$is distorted.

The ideal case is indeed the compensated case, because we will have increased the input impedance by a factor α without distorting the signal.

The probe compensation can be tuned using a signal, which shows a clear distortion when it is filtered. A square wave signal is very useful in this case because, it shows a different distortion if the probe is under or over compensated. Figure D.6 sketches the expected square wave distortion for the two un-compensated cases.

It is worthwhile to notices that

$$\frac{\alpha}{\beta} = 1, \qquad \Rightarrow \frac{R_s}{R_p} = \frac{C_p}{C_s}.$$

This condition implies that:

- the voltage difference V_1 across R_s is equal the voltage difference V_2 across C_s, i.e $V_1 = V_2$
- the voltage difference V_3 across R_p is equal the voltage difference V_4 across C_p , i.e. $V_3 = V_4$
- and indeed $V_1 + V_2 = V_3 + V_4$.

This means that no current is flowing through the branch AB, and we can consider just the resistive branch of the circuit to calculate V_s. Applying the voltage divider equation, we finally get

$$V_s = \frac{R_s}{R_s + R} V_i$$

The capacitance of the oscilloscope does not affect the oscilloscope input anymore, and the oscilloscope+probe input impedance R_i becomes greater, i.e.

$$R_i = R_s + R_p.$$

D.4 Beam Trajectory

Let's consider the electron motion through one pair of plates.

The electron terminal velocity v_0 coming out from the gun can be easily calculated considering that its initial potential energy is entirely converted into kinetic energy, i.e

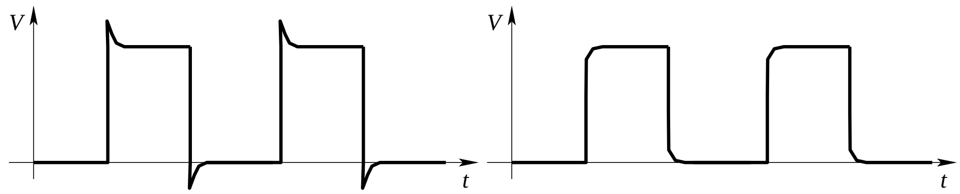


Figure D.6: Compensation of a passive probe using a square wave. Left figure shows an over compensated probe, where the low frequency content of the signal is attenuated. Right figure shows the under compensated case, where the high frequency content is attenuated.

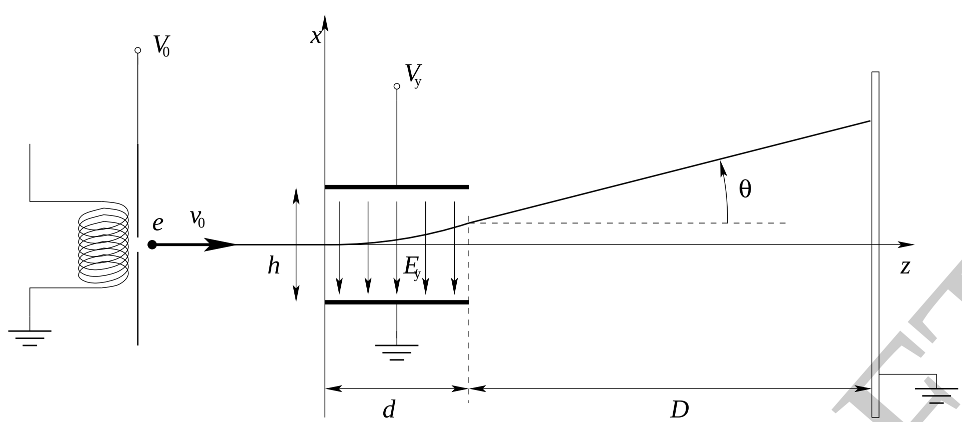


Figure D.7: CRT tube schematics. The electron enters into the electric field and makes a parabolic trajectory. After passing the electric field region it will have a vertical offset and deflection angle θ.

$$\frac{1}{2}\mu v_0^2 = eV_0, \quad \Rightarrow \quad v_0 = \sqrt{2\frac{eV_0}{\mu}},$$

where μ is the electron mass, e the electron charge, and V_0 the voltage applied to the last anode.

If we apply a voltage V_y to the plates whose distance is h, the electrons will feel a force $F_y = eE_y$ due to an electric field

$$|E_y| = \frac{V_y}{h}.$$

The equation of dynamics of the electron inside the plates is

$$\begin{aligned} \mu\ddot{z} &= 0, \quad \Rightarrow \quad \dot{z} = v_0, \\ \mu\ddot{y} &= e|E_y|. \end{aligned}$$

Supposing that V_y is constant, the solution of the equation of motion will be

$$\begin{aligned} z(t) &= \sqrt{2\frac{eV_0}{\mu}}t, \\ y(t) &= \frac{1}{2}\frac{eV_y}{\mu h}t^2. \end{aligned}$$

Removing the dependency on the time t, we will obtain the electron beam trajectory , i.e.

$$y = \frac{1}{4h}\frac{V_y}{V_0}z^2,$$

which is a parabolic trajectory.

Considering that the electron is transversely accelerated until $z = d$, the total angular deflection θ will be

$$\tan\theta = \left(\frac{\partial y}{\partial z}\right)_{z=d} = \frac{1}{2}\frac{d}{h}\frac{V_y}{V_0}.$$

and displacement Y on the screen is

$$Y(V_y) = y(z=d) + \tan\theta D,$$

i.e.,

$$Y(V_y) = \frac{1}{2}\frac{d}{h}\frac{1}{V_0}(\frac{d}{2} + D)V_y.$$

Y is indeed proportional to the voltage applied to the plates through a rather complicated proportional factor.

The geometrical and electrical parameters of this proportional factor play a fundamental role in the resolution of the instrument. In fact, the smaller the distance h between the plates, or the smaller the gun voltage drop V_0, the larger is the displacement Y. Moreover, Y increases quadratically with the electron beam distance d.

D.4.1 CRT Frequency Limit

The electron transit time through the plates determine the maximum frequency that a CRT can plot. In fact, if the transit time τ is much smaller than the period T of the wave form $V(t)$, we have

$$V(t) \simeq \text{constant}, \qquad \text{if} \quad \tau \ll T,$$

and the signal is not distorted.

The transit time is

$$\tau = \frac{d}{v_0} = d\sqrt{\frac{\mu}{2eV_0}}.$$

Supposing that

$$\begin{cases} V_0 = 1\text{kV} \\ d = 20\text{mm} \\ \mu c^2 \simeq 0.5\text{MeV} \\ e = 1\text{eV} \end{cases} \qquad \Rightarrow \qquad \tau \simeq 1\text{ns}$$

Appendix E

Electromagnetic Field Noise

E.1 Introduction

Human and natural activities fill the surrounding space with electromagnetic fields (radiation) creating a very complex and unpredictable frequency spectrum of radiation. For example, domestic appliances, bulbs, fluorescent lights, and power line grids mainly irradiate at 60Hz and harmonics of 60Hz. Radios, televisions, wireless internet connections, and cellular phones networks are other typical sources, which fill the radiation spectrum from the kilohertz to the gigahertz region. Light mainly produced by the sun pervades the spectrum in the optical region. Radioactivity, gamma ray burst (GRB) emitted by astrophysical sources are for eaxmple responsible for filling the high and very high region of the spectrum.

Portion of this so complex spectrum can be attenuated by the so called electromagnetic shields but some others portions because of the energy involved cannot be effectively even attenuated.

The so called radio frequency noise can be easily attenuated (shielded) using a quite simple device known as the *Faraday cage*.

E.2 The Faraday Cage

Gauss's law states that a closed surface will prevent extern electrostatic fields from reaching the space enclosed by the surface. If the electric field is slowly varying i.e., its wavelength λ is large compared to the typical size d of the enclosure), then the field on the surface can be considered static and Gauss's law is then applicable. This enclosure is commonly called *Faraday cage.*

Using this crude approximation we can state that all frequencies much smaller than the following

$$\nu^* \sim \frac{c}{d}$$

where c is the speed of light, will be effectively attenuated. For example if $d = 1$m then the Faraday cage will attenuate the external electromagnetic fields with frequencies much smaller than $\nu^* \sim 300$MHz.

E.3 Practical Considerations

Normally, when we perform a measurement we cannot easily fit the lab in a small Faraday cage. Anyway, most of the time it is sufficient to enclose the physical system under measurement inside the cage . Then to perform the measurement we will have to connect the instrument sitting outside the cage to the system. The instruments leads acting like an antenna will still pick-up some of the ambient electromagnetic radiation. This effect can be amplified if we touch one of the leads increasing the antenna effect. A way to minimize this effect is to connect Faraday cages together. Reasonably good instruments have a built in Faraday cage connected to ground. Connecting the cages to ground will create a more or less single effective cage which will attenuate the electromagnetic noise pick-up.

Appendix F

Common Emitter BJT Amplifier

The common emitter BJT amplifier is one of the most simple design that allows to set the voltage amplification A_v quite independently from the BJT characteristics.

To properly set the BJT working point we have to forward bias the emitter base junction and reverse bias the collector base junction. But this is not enough if we want to build an amplifier. The other requirement is to set the voltage V_{CE} where the VCE characteristic is flat and wide enough to accommodate the output signal excursion. In other words, we don´t want the output to swing into the saturation region or into the break down region.

The design parameters we have to fix are are I_C,V_{CE},V_{CC}, and essentially, the VCE characteristics contains all the information we need to properly bias the BJT. As last remark, voltage gain and bias point are "intimately" related and cannot be completely independent.

F.1 BJT Bias

The analysis of the circuit becomes quite easy if we observe from the V_{CE} characteristic that

$$I_C \gg I_B \,. \qquad \text{(F.1)}$$

In fact, in this case we have that $I_B$is negligible and the resistors R_B and R_b act as a simple voltage divider, i.e.

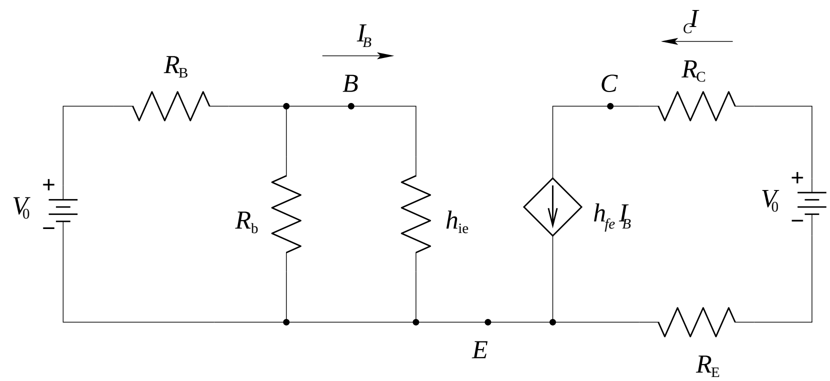


Figure F.1: Common emitter equivalent circuit which simplifies the BJT biasing

$$V_{BE} = \frac{R_b}{R_b + R_B} V_{CC} \,. \tag{F.2}$$

The voltage difference V_{BE} must be the voltage drop of a forward polarized diode junction typically 0.7V. This is one of the conditions we have to fulfill to properly bias the BJT, and therefore equation F.2 sets the value of one resistor as a function of the other resistor and the voltages.

The other condition to fulfill is on V_{CE}. Applying the KVL to the output mesh we will have

$$V_{CE} = (R_C + R_E)\, I_C + V_{CC} \tag{F.3}$$

Equation F.2, F.3 must be satisfied, but they are not enough to set all the resistor values. The voltage gain will provide another constraint to set all the resistor values.

F.2 BJT Gain

Using the equivalent small signal circuit model for the BJT and considering the impedance of the ideal voltage and current sources we can construct the circuit show in figure F.2. Then from that figure it is finally easy to compute the voltage gain A_v and the input and output impedance R_i and R_o of the circuit.

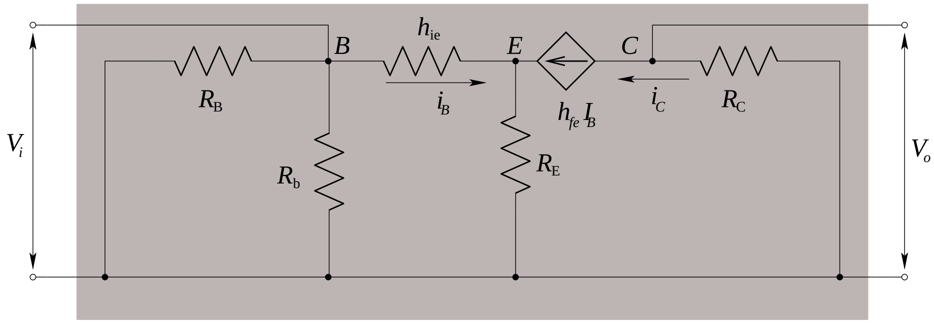


Figure F.2: Small signal circuit model for the common emitter BJT amplifier

In fact, the input and the output voltage are simply

$$\begin{cases} V_i = R_E I_E \simeq R_E I_C, \qquad (I_B \ll I_C) \\ V_o = R_C I_C \end{cases}, \qquad \Rightarrow \qquad A_v \simeq \frac{R_C}{R_E}$$

F.3 Input and Output Impedance

The input impedance is the impedance seen from the inputs lead , and can be easily computed considering that the ideal current source is an open circuit, i.e.

$$R_i = R_b || R_B || (h_{ie} + R_e)$$

The output impedance is then

$$R_o = R_C$$

F.4 Resume

Summarizing the results we have

$$\begin{aligned} V_{BE} &= \frac{R_b}{R_b + R_B} V_{CC} \\ V_{CC} &= (R_C + R_E)\, I_C + V_{CE} \\ A_v &= \frac{R_C}{R_E} \end{aligned}$$

and resolving the equation respect the unknown parameters

$$\begin{aligned} R_B &= R_b\left(\frac{V_{CC}}{V_{BE}} - 1\right) \\ R_E &= \frac{V_{CC} - V_{CE}}{I_C(1 + A_v)} \\ R_C &= A_v R_E \end{aligned}$$

F.5 Example

Let´s set the following design values

$$\begin{cases} A_v &= 10 \\ V_{CC} &= 20\text{V} \\ I_C &= 10\text{mA} \end{cases}$$

Picking up a value for R_b and considering that to have a maximum dynamic

$$V_{CE} \simeq 10.3\text{V}$$

we finally get

$$\begin{cases} R_b &\simeq 1.0\text{k}\Omega \\ R_B &\simeq 27.5\text{k}\Omega \\ R_C &\simeq 850\Omega \\ R_E &\simeq 85\Omega \end{cases}$$