Laser Interferometer Space Antenna (LISA)

Time-Delay Interferometry for LISA: The Next Generation!

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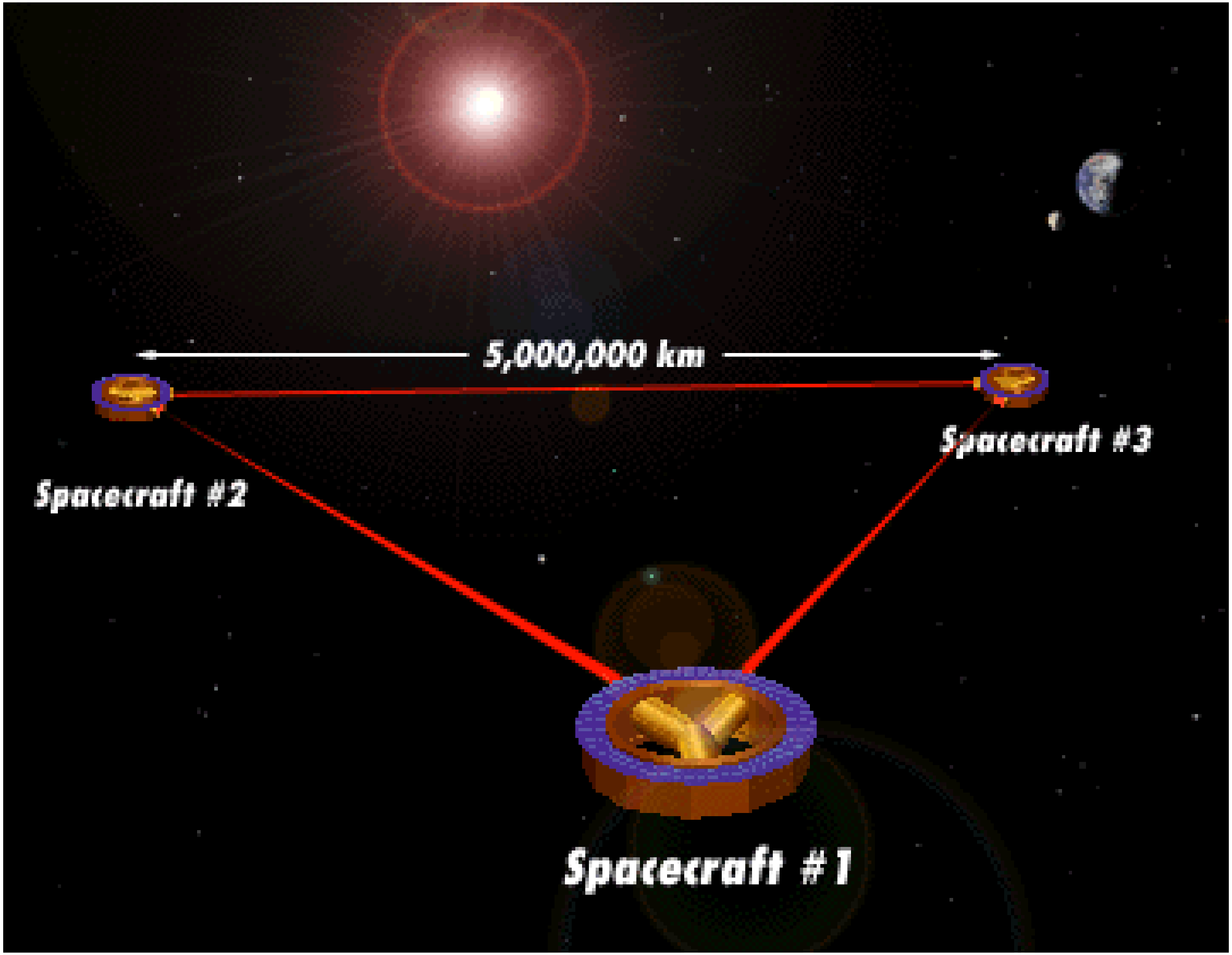
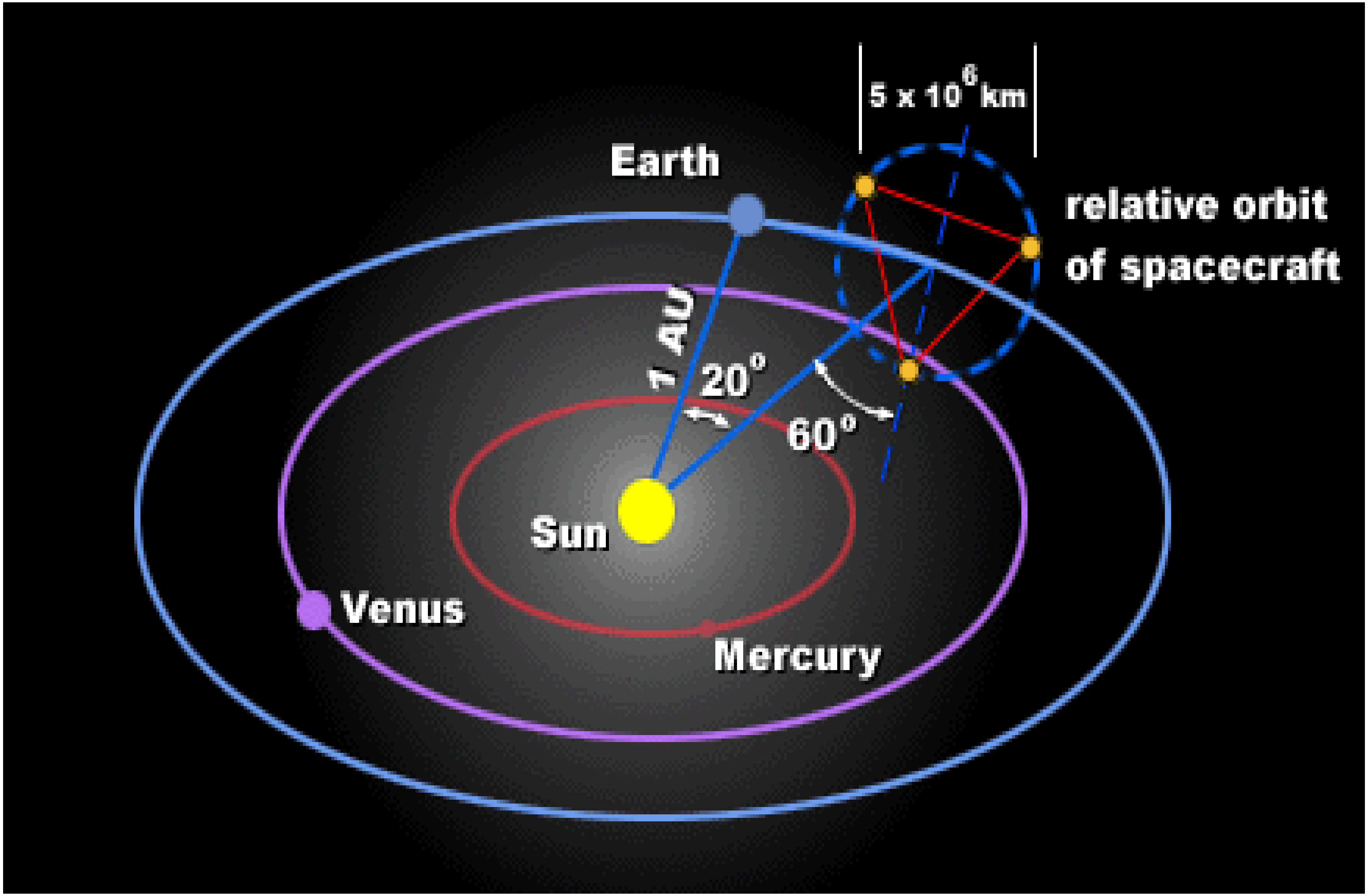
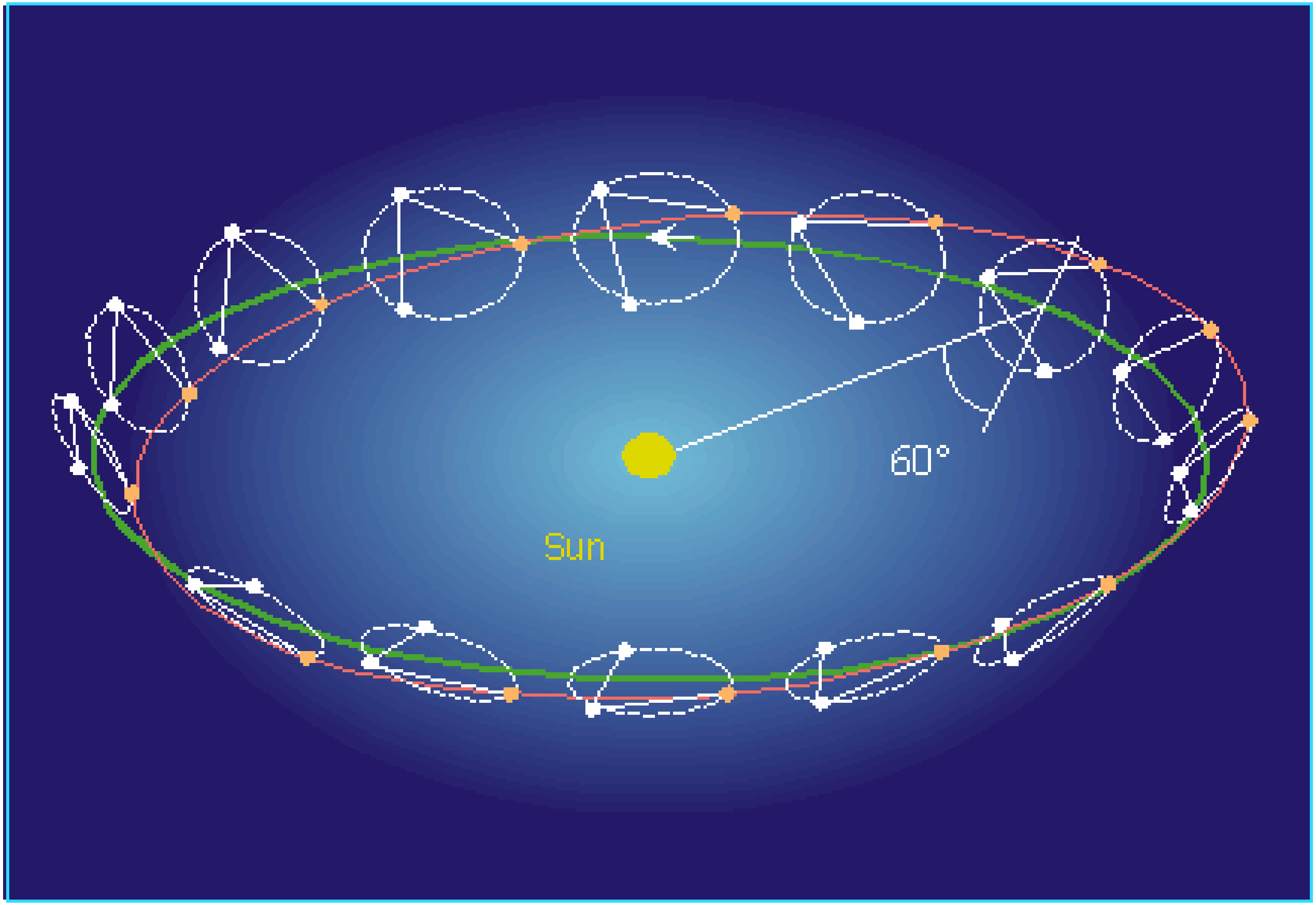
esa


5,000,000 km
Spacecraft #2
Spacecraft #3
Spacecraft #1

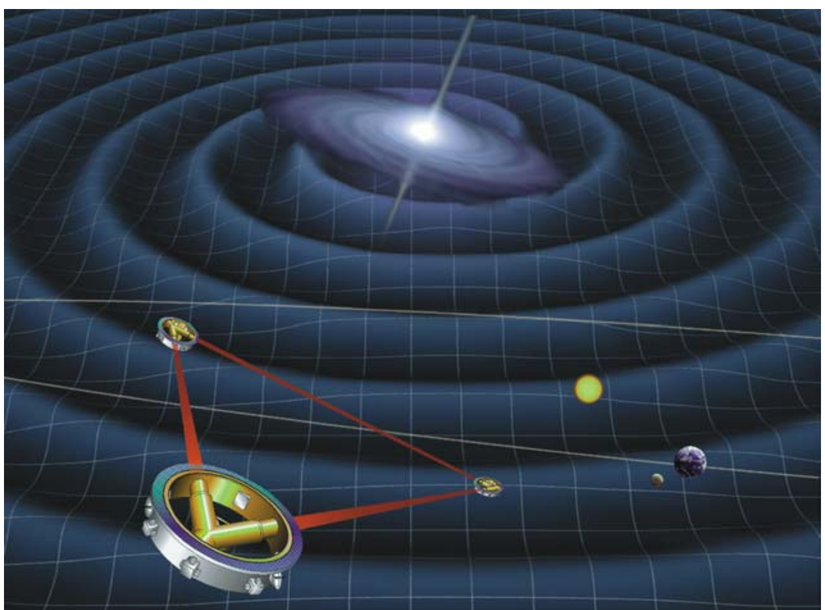

5 x 10^6 km
Earth
relative orbit
of spacecraft
1 AU
20°
60°
Sun
Venus
Mercury



W.M. Folkner *et al.*, C.Q.G., **14**, 1543, (1997)

Earth vs. Space-based Interferometers

- Earth-based interferometers have arm lengths essentially equal. This is in order to directly remove laser frequency fluctuations at the photodetector, where the two beams interfere.
- They operate in the long-wavelength limit ($L \ll \lambda$).



- By contrast, LISA will have arm lengths significantly different ($\Delta L/L \sim 10^{-2}$), with $L = 5 \times 10^6$ km.
- Over much of its sensitivity frequency-band, it will **<u>not</u>** operate in the long-wavelength regime.
- Time-of-flight delays in the response to the wave, and travel times along the beams in the detector must be allowed for, in order to derive a correct theory of the detector response.

The Gravitational Wave Signal

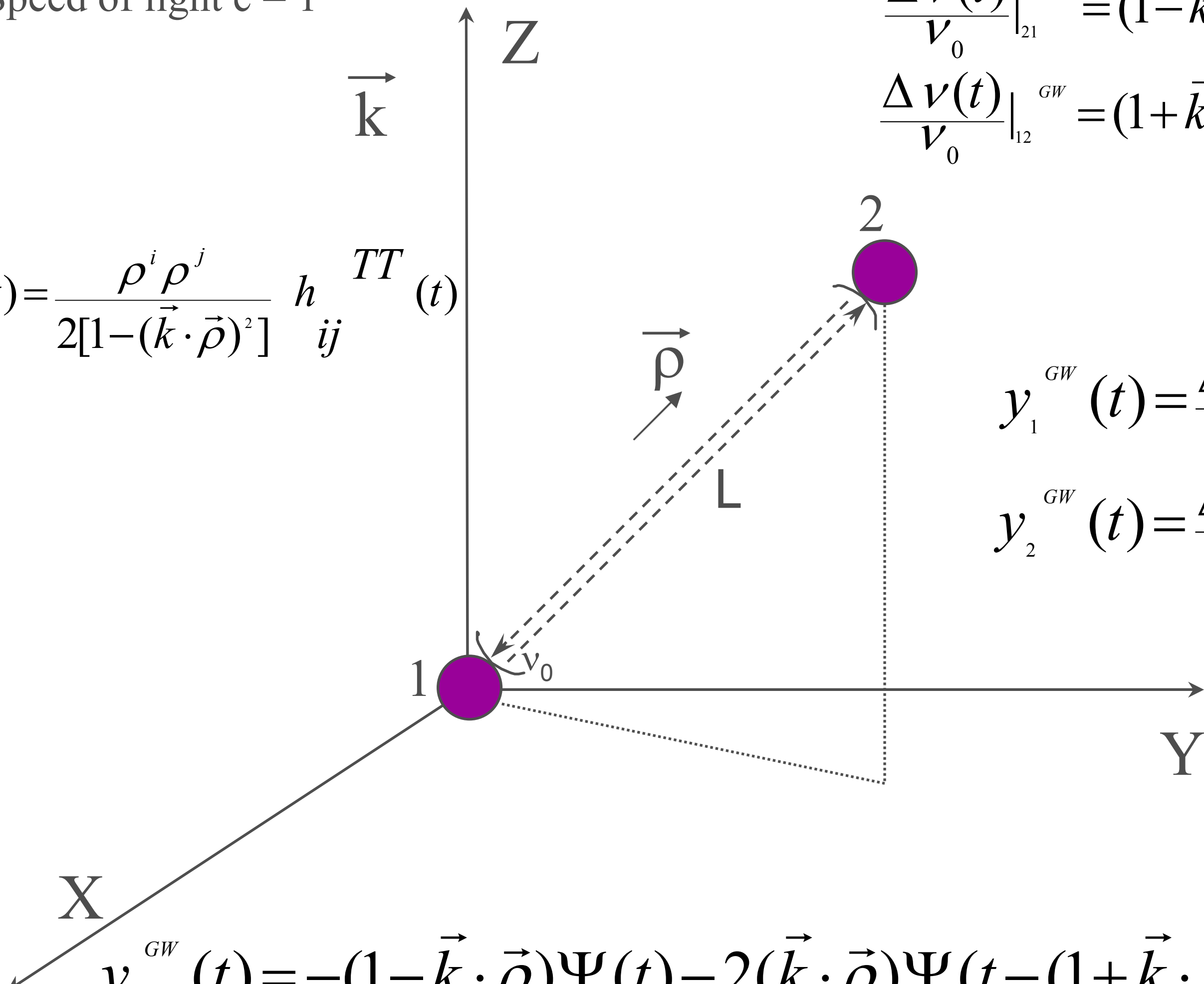
Estabrook, F.B., & Wahlquist, H.D., *Gen.Relativ.Gravit.* **6**,439 (1975)

Speed of light c = 1

$$\frac{\Delta\nu(t)}{\nu_0}\Big|_{21}^{GW} = (1-\vec{k}\cdot\vec{\rho})[\Psi(t-(1+\vec{k}\cdot\vec{\rho})L)-\Psi(t)]$$

$$\frac{\Delta\nu(t)}{\nu_0}\Big|_{12}^{GW} = (1+\vec{k}\cdot\vec{\rho})[\Psi(t-L)-\Psi(t-\vec{k}\cdot\vec{\rho}L)]$$

$$\Psi(t)=\frac{\rho^i\rho^j}{2[1-(\vec{k}\cdot\vec{\rho})^2]}\, h_{ij}^{TT}(t)$$



$$y_1^{GW}(t)=\frac{\Delta\nu(t)}{\nu_0}\Big|_{21}^{GW}+\frac{\Delta\nu(t-L)}{\nu_0}\Big|_{12}^{GW}$$

$$y_2^{GW}(t)=\frac{\Delta\nu(t)}{\nu_0}\Big|_{12}^{GW}+\frac{\Delta\nu(t-L)}{\nu_0}\Big|_{21}^{GW}$$

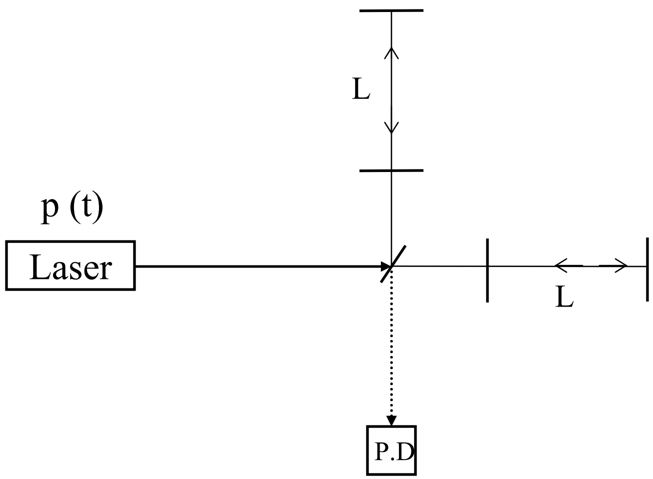
$$y_1^{GW}(t)=-(1-\vec{k}\cdot\vec{\rho})\Psi(t)-2(\vec{k}\cdot\vec{\rho})\Psi(t-(1+\vec{k}\cdot\vec{\rho})L)+(1+\vec{k}\cdot\vec{\rho})\Psi(t-2L)$$

$$y_2^{GW}(t)=-(1+\vec{k}\cdot\vec{\rho})\Psi(t-\vec{k}\cdot\vec{\rho}L)+2(\vec{k}\cdot\vec{\rho})\Psi(t-L)+(1-\vec{k}\cdot\vec{\rho})\Psi(t-2L-\vec{k}\cdot\vec{\rho}L)$$

Statement of The Problem

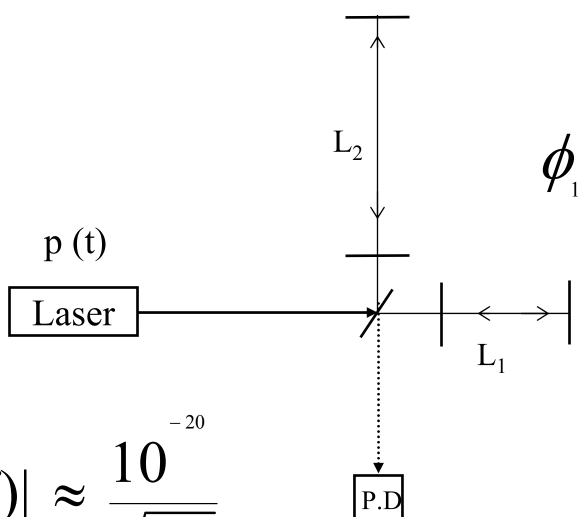
p (t) = Laser phase fluctuations

$$\frac{1}{2\pi\nu_0}\frac{dp(t)}{dt} \equiv \left[\frac{\Delta\nu(t)}{\nu_0}\right]_{Laser} = C(t)$$



$$\phi_1(t) = h_1(t) + p(t - 2L_1) + n_1(t)$$

$$\phi_2(t) = h_2(t) + p(t - 2L_2) + n_2(t)$$

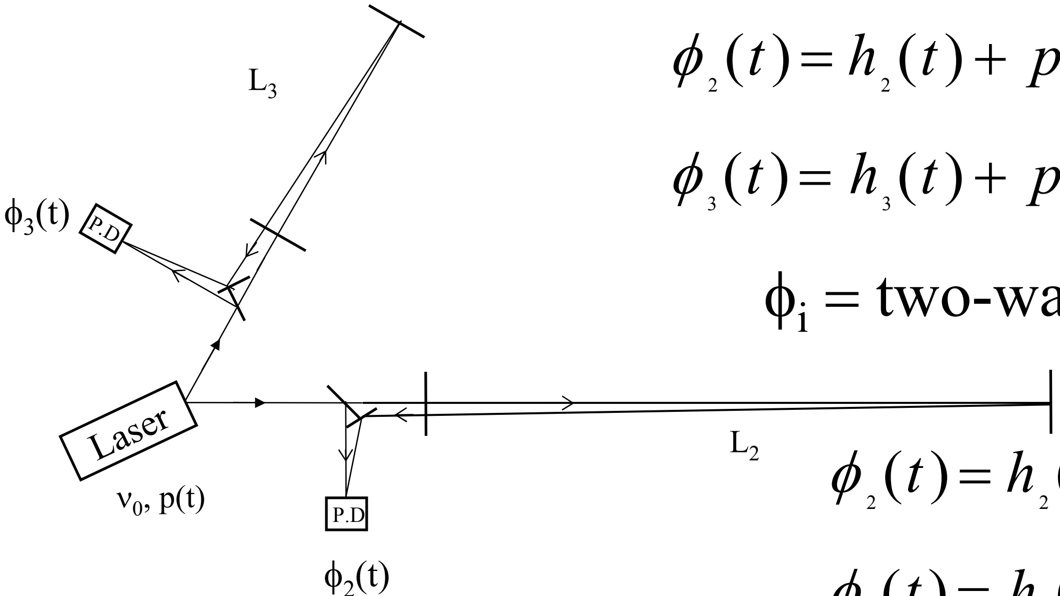


$$\phi_1(t) - \phi_2(t) \Rightarrow p(t-2L_1) - p(t-2L_2) \cong 2\frac{dp}{dt}\varepsilon L_1$$

$$|\tilde{h}(f)| \approx \frac{10^{-20}}{\sqrt{Hz}}$$

$$|\tilde{C}(f)| \approx \frac{10^{-13}}{\sqrt{Hz}}\ ,\ \varepsilon \cong 3\times 10^{-2} \Rightarrow \frac{5\times 10^{-16}}{\sqrt{Hz}}$$

Unequal-arm Interferometers



$$\phi_2(t) = h_2(t) + p(t - 2L_2) - p(t) + n_2(t)$$

$$\phi_3(t) = h_3(t) + p(t - 2L_3) - p(t) + n_3(t)$$

ϕ_i = two-way phase measurements

$$\phi_2(t) = h_2(t) + [D_2 D_2 - I]\, p(t) + n_2(t)$$

$$\phi_3(t) = h_3(t) + [D_3 D_3 - I]\, p(t) + n_3(t)$$

$$where \quad : D_i \Psi(t) \equiv \Psi(t - L_i)$$

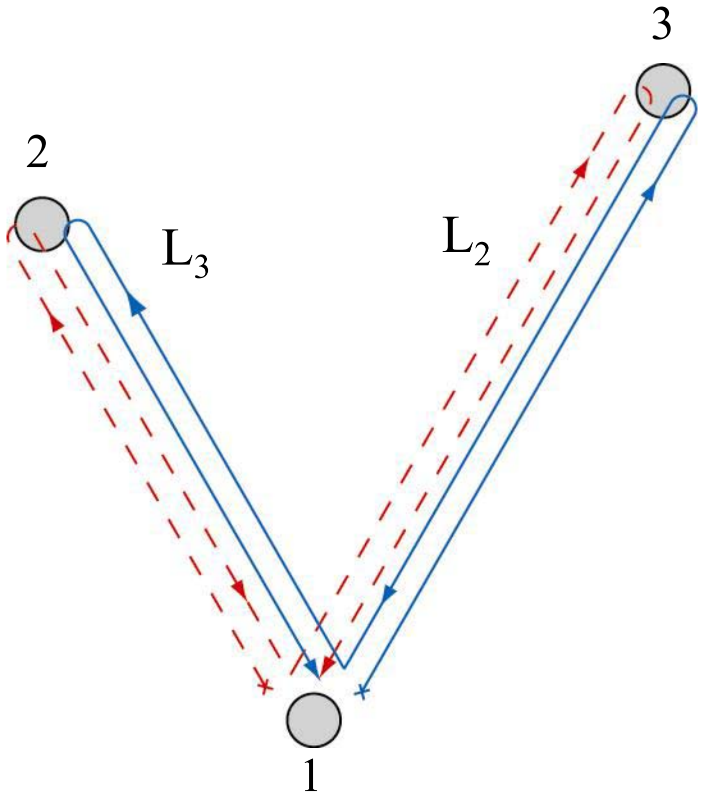
S.V. Dhurandhar, K.R. Nayak, and J-Y. Vinet, *Phys. Rev. D*, **65**, 102002 (2002).

$$\begin{aligned} X(t) &\equiv [D_3 D_3 - I]\phi_2(t) - [D_2 D_2 - I]\phi_3(t) \\ &= [D_3 D_3 - I][h_2(t) + n_2(t)] - [D_2 D_2 - I][h_3(t) + n_3(t)] \\ &\quad + [(D_3 D_3 - I)(D_2 D_2 - I) - (D_2 D_2 - I)(D_3 D_3 - I)]\, p(t) \end{aligned}$$

M. Tinto, & J.W. Armstrong, *Phys. Rev. D*, **59**, 102003 (1999).

Unequal-arm Interferometers (Cont.)

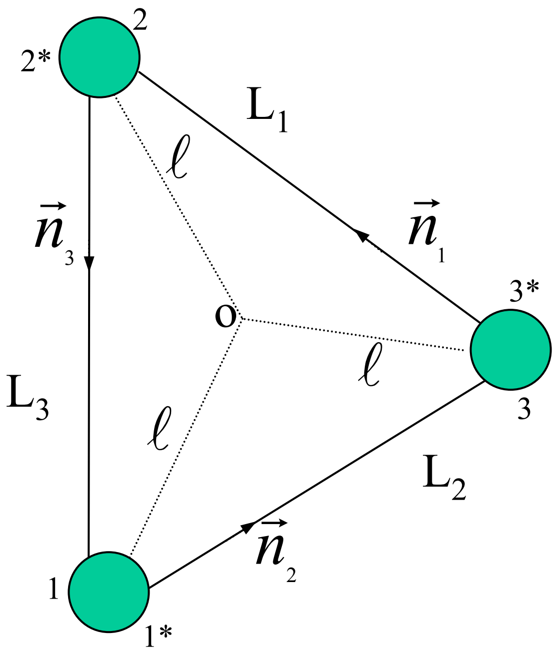
$$X(t) = [\phi_3(t) + D_3 D_3 \phi_2(t)] - [\phi_2(t) + D_2 D_2 \phi_3(t)]$$



- One can actually regard X as given by the interference of two beams that propagate within the two arms of LISA, each experiencing a delay equal to $(2L_1 + 2L_2)$.
- X is actually a zero-area Sagnac Interferometer, synthesized by properly combining measurements from each arm.

D.A. Shaddock, M. Tinto, F.B. Estabrook & J.W. Armstrong, *Phys. Rev. D*, **68**, 061303 (R) (2003).

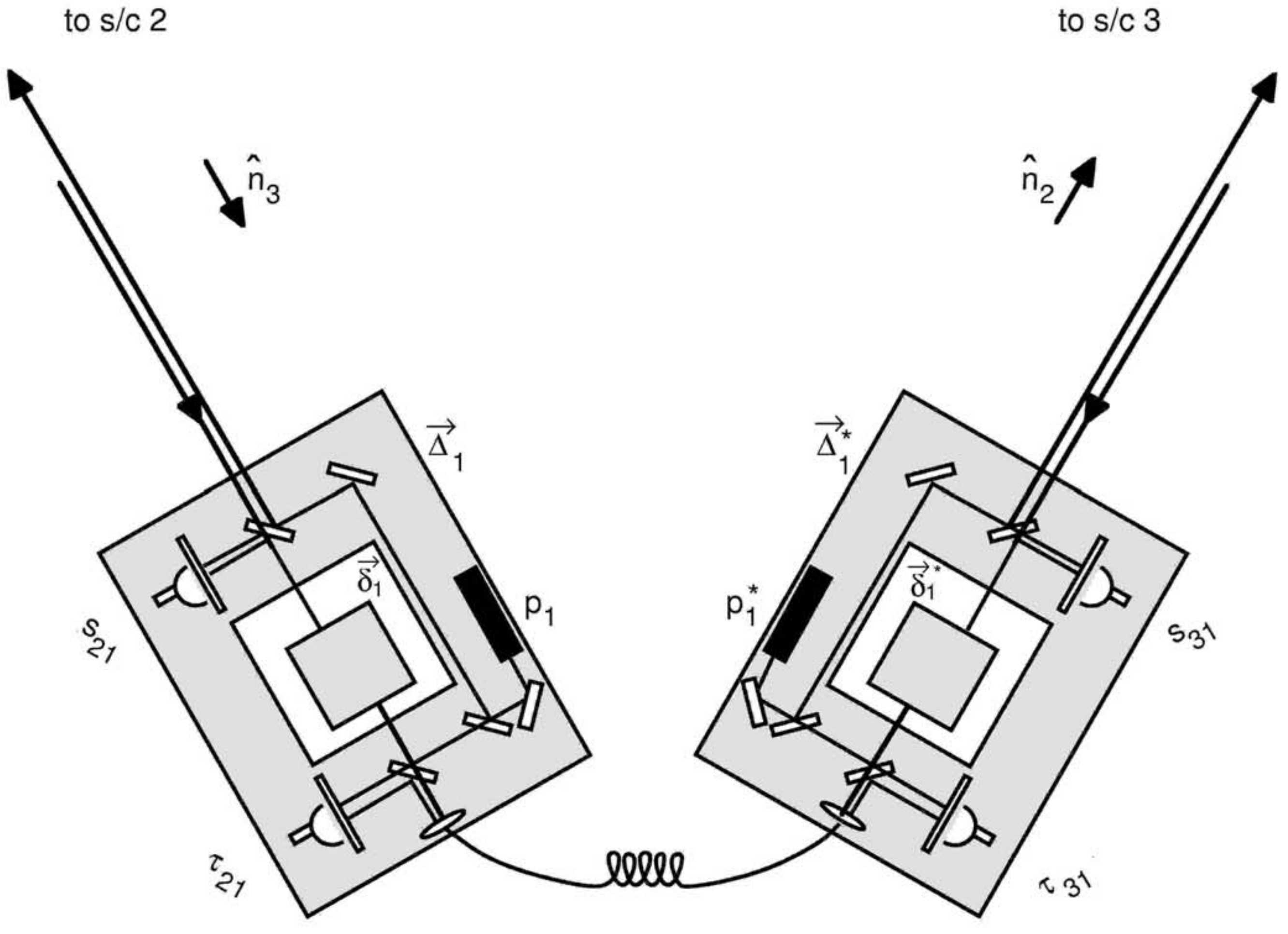
Time-delay Interferometry



- It is best to think of LISA as a closed array of six one-way delay lines between the test masses.
- This approach allows us to reconstruct the unequal-arm Michelson interferometer, as well as new interferometric combinations, which offer advantages in hardware design, in robustness to failures of single links, and in redundancy of data.

M. Tinto: *Phys. Rev. D,* **53**, 5354 (1996); *Phys. Rev. D,* **58**, 102001 (1998)
J.W. Armstrong, F.B. Estabrook, and M. Tinto: *Ap. J.,* **527**, 814 (1999).
M. Tinto, D.A. Shaddock, J. Sylvestre, & J.W. Armstrong: Phys. Rev. D **67**, 122003 (2003)

Time-delay Interferometry & The Drag-free Configuration



F.B. Estabrook, M. Tinto, & J.W. Armstrong, *Phys. Rev. D,* **62**, 042002 (2000)

One-way Measurements

$$D_i \Psi(t) = \Psi(t - L_i) \equiv \Psi_{,i}$$

$$s_{31} = (p_3 - \nu_0 \vec{n}_2 \cdot \vec{\Delta}_3)_{,2} - (p^*_1 + \nu_0 \vec{n}_2 \cdot \vec{\Delta}^*_1) + 2\nu_0 \vec{n}_2 \cdot \vec{\delta}^*_1 + s_{31}^{GW} + s_{31}^{opt.\,path}$$

$$\tau_{31} = p_1 - p^*_1 + 2\nu_0 \vec{n}_3 \cdot [\vec{\delta}_1 - \vec{\Delta}_1] + \mu_1$$

$$s_{21} = (p^*_2 + \nu_0 \vec{n}_3 \cdot \vec{\Delta}^*_2)_{,3} - (p_1 - \nu_0 \vec{n}_3 \cdot \vec{\Delta}_1) - 2\nu_0 \vec{n}_3 \cdot \vec{\delta}_1 + s_{21}^{GW} + s_{21}^{opt.\,path}$$

$$\tau_{21} = p^*_1 - p_1 - 2\nu_0 \vec{n}_2 \cdot [\vec{\delta}^*_1 - \vec{\Delta}^*_1] + \mu_1$$

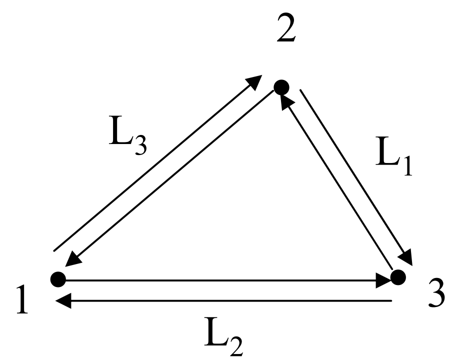
- Eight other relations are obtained by cyclic permutation of the indices in the equations above.

Note that: $$\frac{1}{2}[\tau_{21} - \tau_{31}] = [p^*_1 + \nu_0 \vec{n}_2 \bullet \vec{\Delta}^*_1] - [p_1 - \nu_0 \vec{n}_3 \bullet \vec{\Delta}_1] - \nu_0 \vec{n}_3 \bullet [\vec{\delta}^*_1 + \vec{\delta}_1]$$

$$\eta_{31} \equiv s_{31} + \frac{1}{2}[\tau_{21} - \tau_{31}] \; ; \; \eta_{21} \equiv s_{21} - \frac{1}{2}[\tau_{21} - \tau_{31}]_{,3}$$ →

Equivalent problem: remove 3 random processes from 6 data combinations

Six-Pulse Data Combinations



$\alpha, \beta, \gamma, \zeta$

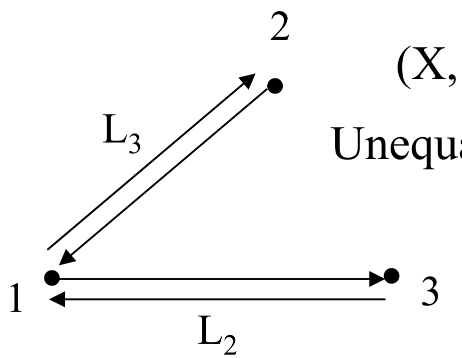
$$\alpha = (\eta_{31} + \eta_{23,2} + \eta_{12,12}) - (\eta_{21} + \eta_{32,3} + \eta_{13,13})$$

? ?

$$\zeta = (\eta_{31,1} - \eta_{32,2} + \eta_{12,2}) - (\eta_{13,3} - \eta_{23,3} + \eta_{21,1})$$

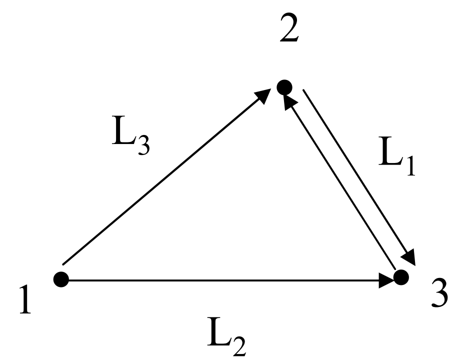
$$= (p_{1,23} - p_{1,1}) - (p_{1,23} - p_{1,1}) + (GW + Secondary\ noises)$$

Eight-Pulse Data Combinations



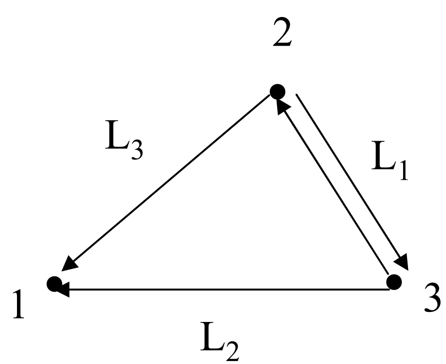
(X, Y, Z)

Unequal-arm Michelson



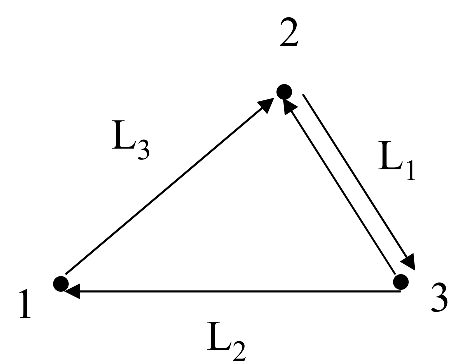
(P, Q, R)

Beacon



(E, F, G)

Monitor



(U, V, W)

Relay

Data Combinations (cont)

- In summary: there are 6 optical benches, 6 lasers, and a total of 12 Doppler time series observed.
- The 6 beams exchanged between distant spacecraft contain the information about the GW signal (s_{ij}); the other 6 signals (τ_{ij}) are for comparison of the lasers and relative optical bench motions within the spacecraft.
- The functional space of interferometric combinations is 3-dimensional.

$$\zeta - \zeta_{,123} = \alpha_{,1} - \alpha_{,23} + \beta_{,2} - \beta_{,31} + \gamma_{,3} - \gamma_{,12}$$

$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \zeta$$

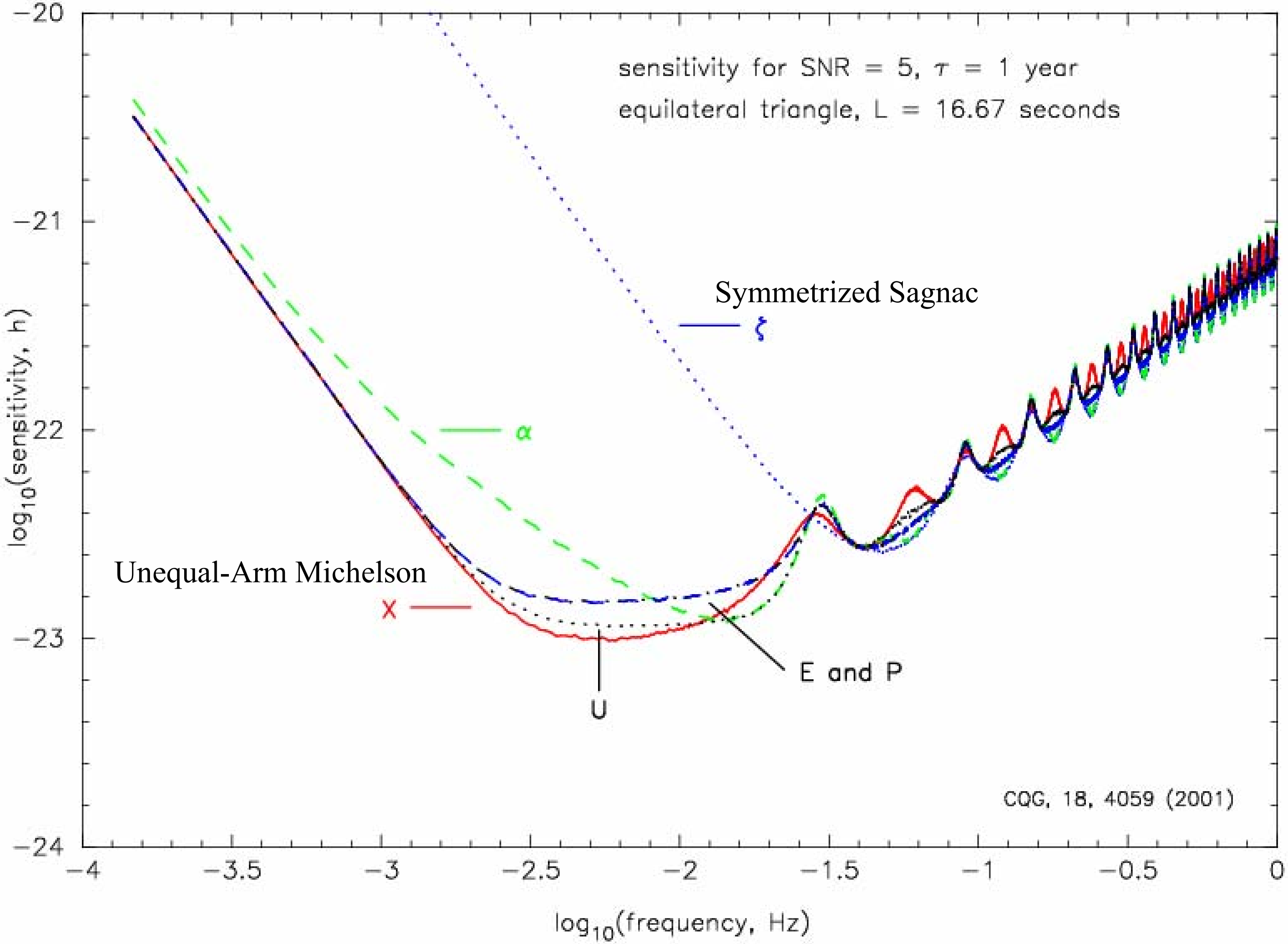
$$Y_{,2} = \beta_{,13} - \gamma_{,3} - \alpha_{,1} + \zeta$$

$$Z_{,3} = \gamma_{,21} - \alpha_{,1} - \beta_{,2} + \zeta$$

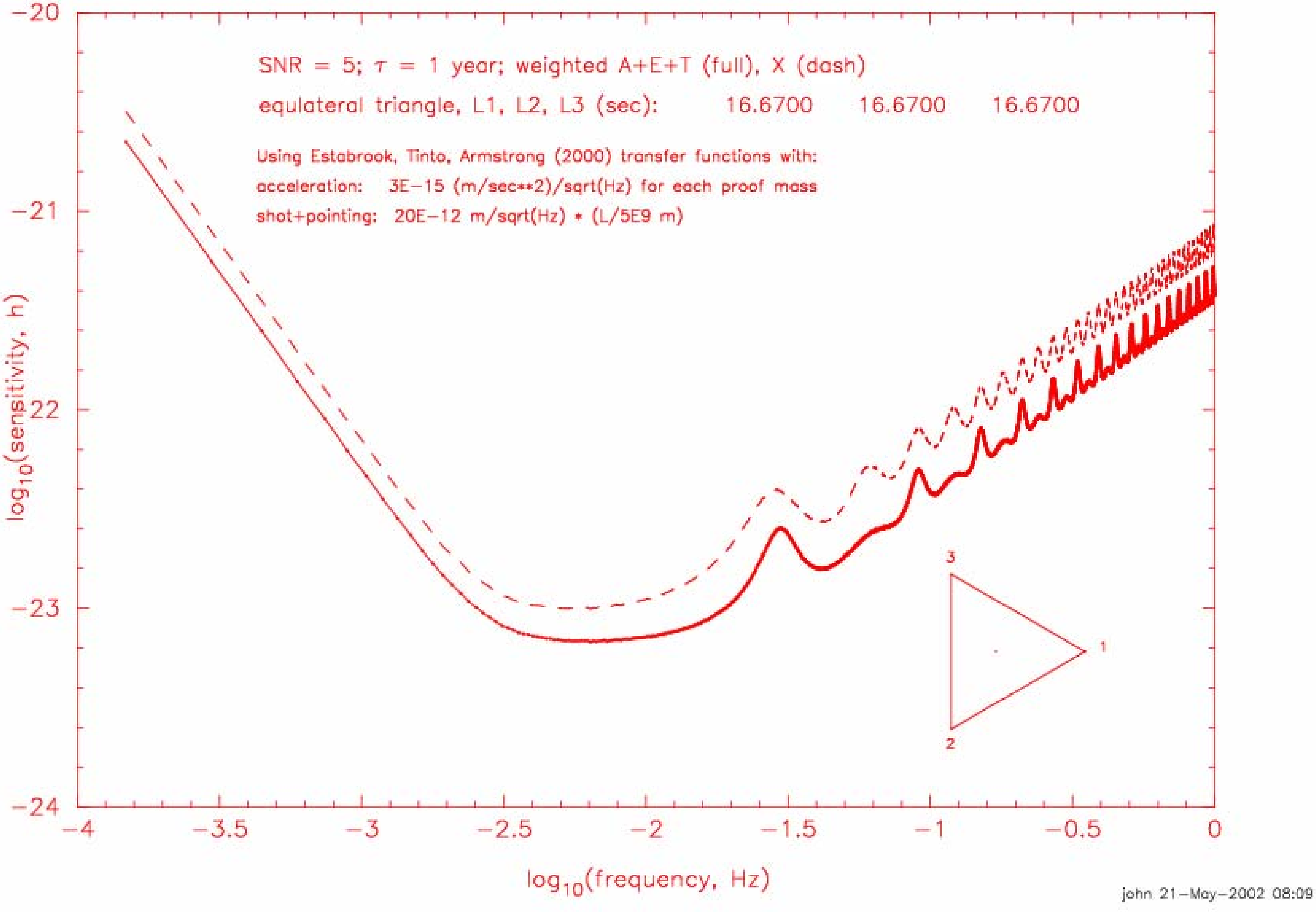
J.W. Armstrong, F.B. Estabrook, and M. Tinto, *Ap. J.,* **527**, 814 (1999).

S.V. Dhurandhar, K.R. Nayak, and J-Y. Vinet, *Phys. Rev. D*, **65**, 102002 (2002).

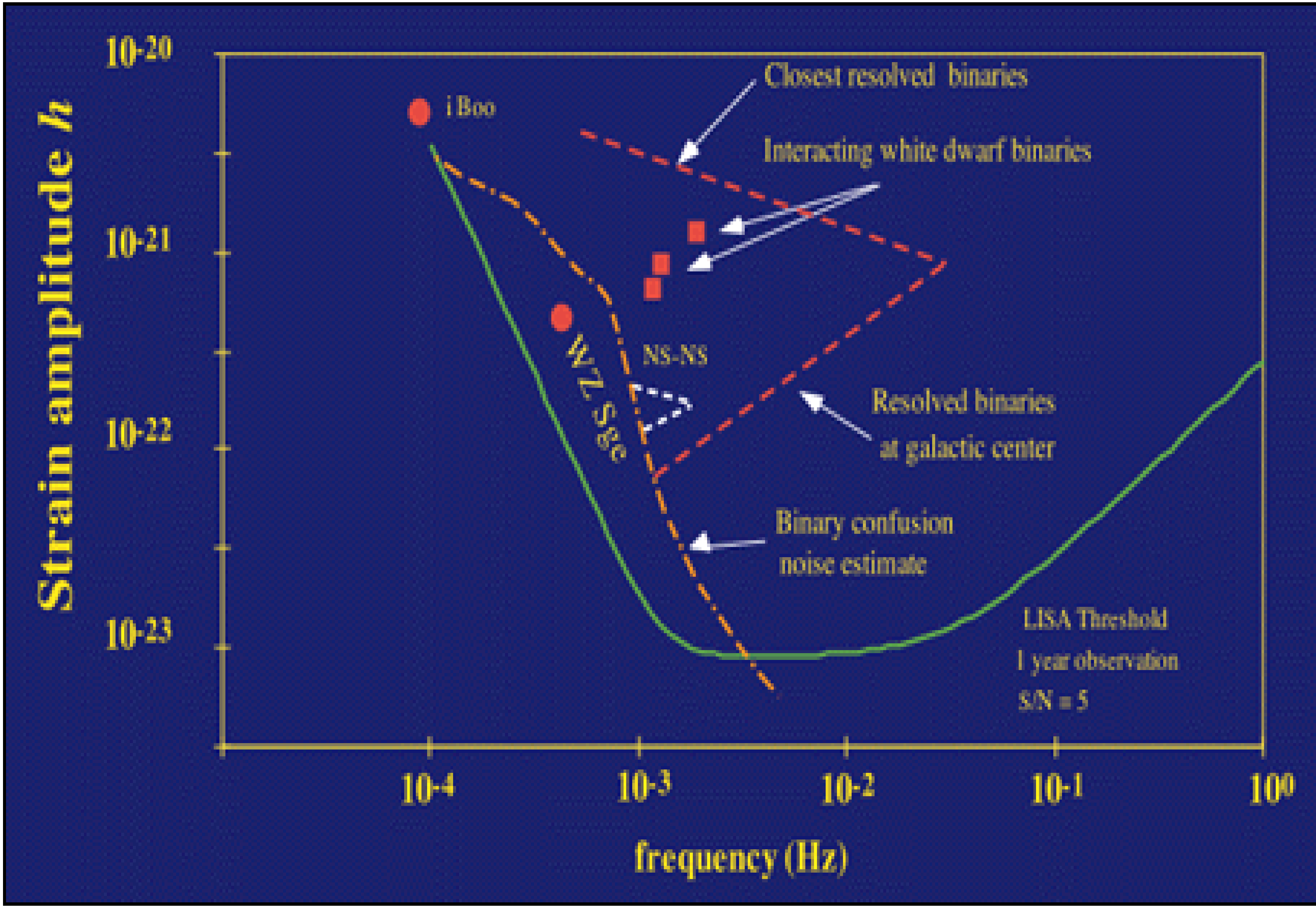
Sensitivities



Optimal Data Combinations



T.A. Prince, M. Tinto, S.L. Larson, and J.W Armstrong, *Phys. Rev. D*, **66**, 122002 (2002).



N.J. Cornish & S.L. Larson, *Phys. Rev. D* **67**, 103001 (2003) **A. Krolak and M. Tinto**, *gr-qc/0302013*

NOISE CALIBRATION and DETECTION OF A STOCHASTIC BACKGROUND

- The gravitational wave background will be below the anticipated sensitivity curve of ζ by several orders of magnitude.

- The Sagnac combination provides a way for estimating the instrumental noise sources: Sagnac greatly attenuates the gravitational wave signal, but instrumental noise persists.

- This allows us to infer the actual on-orbit LISA instrumental noise in the Michelson interferometer mode X, and in turn to detect the stochastic background.

- The ζ combination can of course be used also as a discriminator for sinusoidal signals and bursts.

M. Tinto, J.W. Armstrong, & F.B. Estabrook, *Phys. Rev. D,* **63**, 021101(R) (2001).
J. Sylvestre, & M. Tinto, *Phys. Rev. D: to appear;* gr-qc/0308085

The Laser Frequencies Are Different, and the Spacecraft Are Not Stationary

- Both frequency offsets between lasers, and Doppler drifts, now bring in noise from the onboard oscillators (USO=Ultra Stable Oscillators) used in the down-conversion of phototube fringe rates.
- A state-of-the-art USO with a frequency stability of $\sim 10^{-13}$ in the millihertz band will introduce relative frequency fluctuations in the interferometric data equal to 3.0×10^{-20} (1 year integration, 5 σ level) in the X-combination.

R. Hellings, G. Giampieri, L. Maleki, M. Tinto, K. Danzmann, J. Hough, and D. Robertson, *Optics Communications*, **124**, 313, (1996)

M. Tinto, F.B. Estabrook, & J.W. Armstrong, *Phys. Rev. D*, 65, 082003, 2002

Moving spacecraft Arrays and Clocks Synchronization

- All the analyses above assumed the clocks onboard the LISA S/Cs to be synchronized to each other in the frame attached to the LISA array.
- In a rotating reference frame, the Sagnac effect prevents the implementation of the *Einstein's Synchronization Procedure,* i.e. synchronization by transmission of electromagnetic signals (GPS is a good example of this problem!)
- To account for the Sagnac effect, one introduces an <u>**hypothetical**</u> inertial reference frame, and time in this frame is the one adopted by the spacecraft clocks!
- In other words, the onboard receivers have to convert time information received from Earth to time in this inertial reference frame (SSB).

N. Asbby, "The Sagnac effect in the GPS System", http://digilander.libero.it/solciclos/

M. Tinto, F.B. Estabrook, & J.W. Armstrong, gr-qc/0310017, October 6, 2003

Moving spacecraft Arrays and Clocks Synchronization (Cont.)

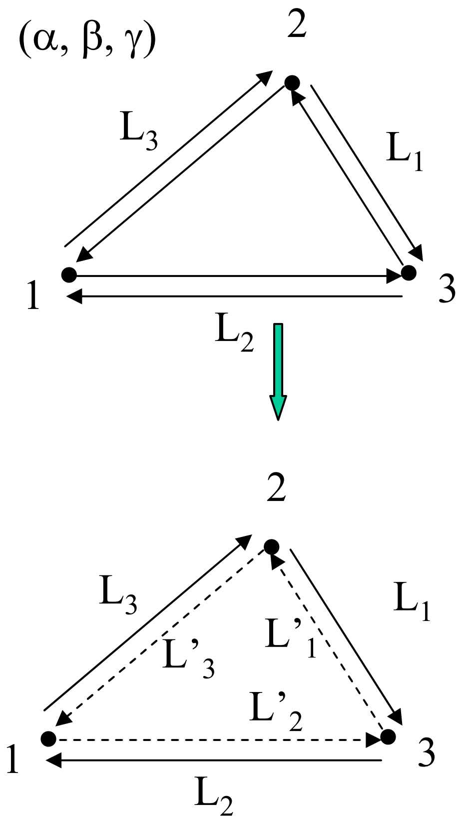
- In the SSB frame, the differences between back-forth delay times are very much larger than has been previously recognized.
- The reason is in the aberration due to motion and changes of orientation in the SSB frame.
- With a velocity V=30 km/s, the light-transit times of light signals in opposing directions (L_i, and L'_i) will differ by as much as 2VL (a few thousands km)
- They will also change in time due to rotation (0.1 m/s); this however is significantly smaller than the spacecraft relative velocity (10 m/s).

D.A. Shaddock, Phys. Rev. D: to appear; gr-qc/0306125 **Cornish & Hellings**, gr-qc/0306096

TDI with Moving spacecraft Arrays

- The TDI expressions above do not account for:
  - The Sagnac Effect
  - Time-dependence (velocity) of the arm lengths in the TDI expressions (the "Flex-effect")
- Both effects prevent the perfect cancellation of the laser frequency fluctuations in the presently existing TDI combinations.
- With a laser frequency stability of 30 $Hz/Hz^{1/2}$ the remaining laser frequency fluctuations could be as much as **30 times** larger than the secondary noise sources.

The Sagnac Effect and the Sagnac Combinations



- In presence of rotation, the amount of time spent by a beam to propagate clockwise is different by the time it spends to propagate counterclockwise along the same arm => $(L_1, L_2, L_3, L'_1, L'_2, L'_3)$.
- The Sagnac effect prevents the perfect cancellation of the laser frequency fluctuations in the existing expressions of the Sagnac combinations $(\alpha, \beta, \gamma, \zeta)$.
- @ 10^{-3} Hz the laser frequency fluctuations remaining in $(\alpha, \beta, \gamma, \zeta)$ would be about **30 times** larger than the secondary noise sources.

$$|(L_1 + L_2 + L_3) - (L'_1 + L'_2 + L'_3)| = 4\,|\vec{\Omega}\cdot\vec{A}| \cong 14 \;\; km$$

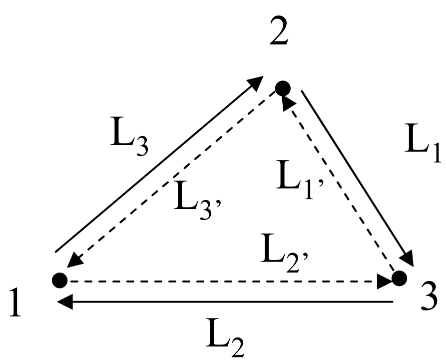
$\alpha, \beta, \gamma \longrightarrow \alpha_1, \alpha_2, \alpha_3$

D.A. Shaddock, Phys. Rev. D: to appear; gr-qc/0306125 **Cornish & Hellings**, gr-qc/0306096

Solution to the Sagnac Effect

- The solution to this problem (for a rigid rotation!) is rather simple:

  Make each light-beam go around clockwise and counterclockwise once. They both experience a total delay equal to ($L_1 + L_2 + L_3 + L'_1 + L'_2 + L'_3$) and the laser noise cancels out exactly when they interfere.



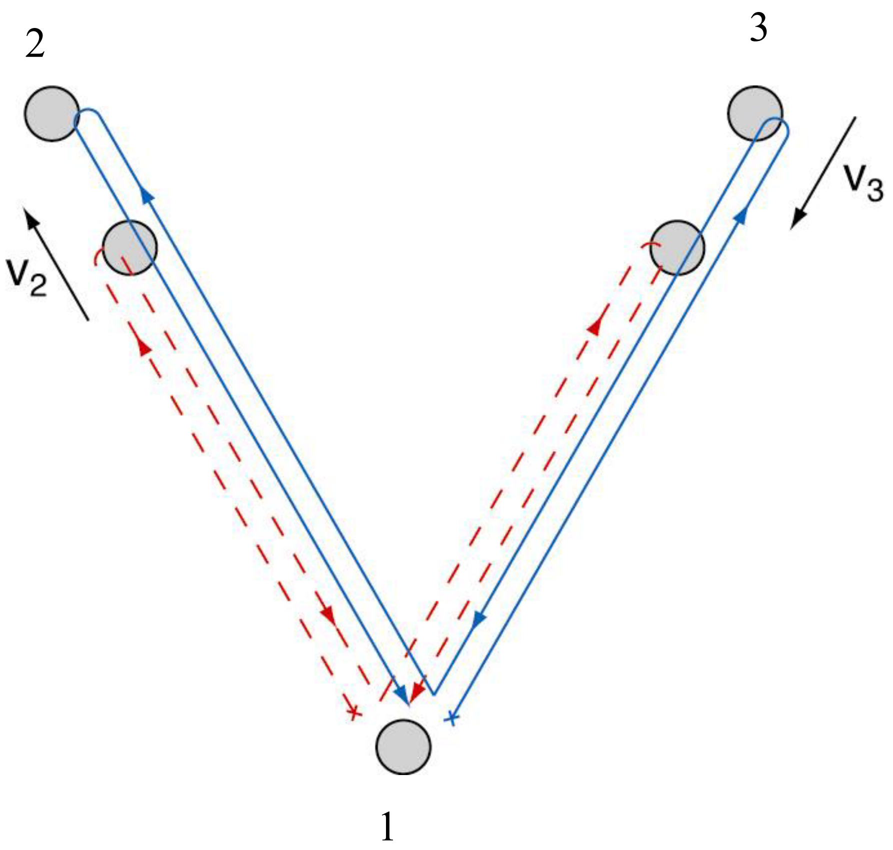
α, β, γ → $\alpha_1, \alpha_2, \alpha_3$

$$\lambda \equiv \eta_{31} + \eta_{23,2} + \eta_{12,12} \qquad \mu \equiv \eta_{21} + \eta_{32,3'} + \eta_{13,1'3'}$$

$$\alpha_1(t) \equiv [\lambda + \mu_{,123}] - [\mu + \lambda_{,1'2'3'}]$$

$$\textit{where} \quad \eta_{ij,kl} = \eta_{ij}(t - L_k - L_l)$$

"Flexy"



D. Summers, ESA Presentation (February 2003): Unpublished

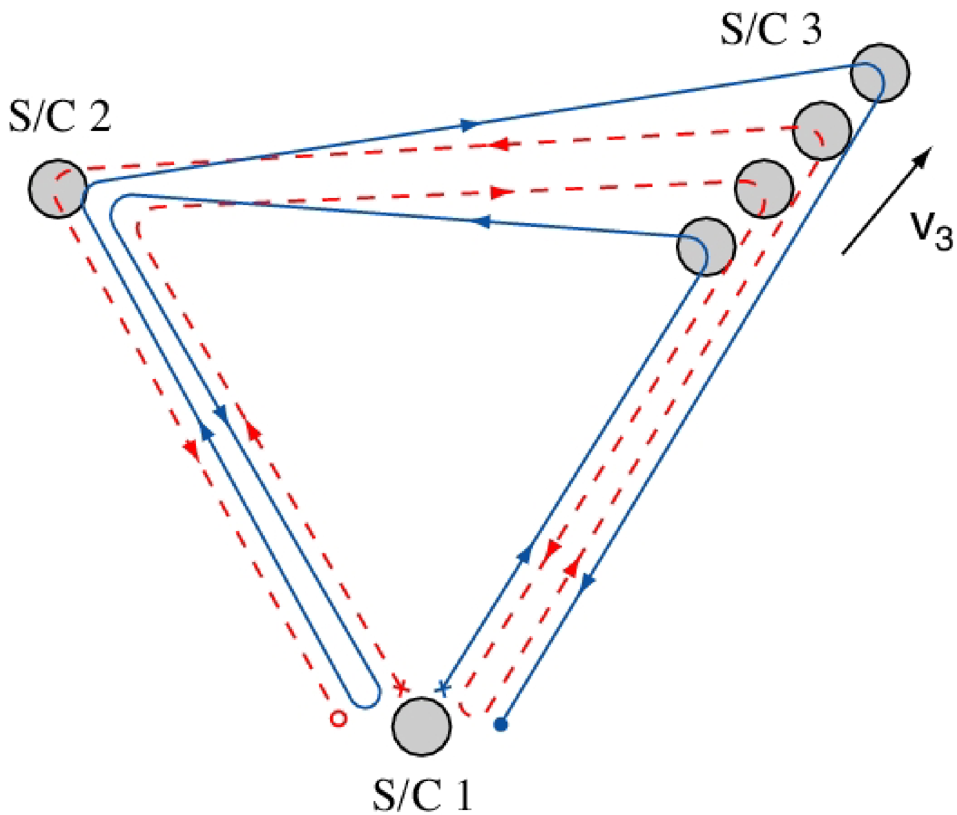
Cornish & Hellings, gr-qc/0306096

D.A. Shaddock, M. Tinto, F.B. Estabrook & J.W. Armstrong, *Phys. Rev. D*, **68**, 061303 (R) (2003).

"Flexy" (Cont.)

- This effect prevents the perfect cancellation of the laser noise in all the "first generation" TDI combinations.
- The remaining laser noise depends on the difference of the velocities of the two arms.
- For X, @ 10^{-4} Hz, residual laser frequency fluctuations would be about 5 times larger than the secondary noise sources; @ 10-3 Hz they are at the level of the secondary noise sources.
- Since X accounted for the case of unequal arms (and equal velocities), by further "differencing X" it is possible to find a new observable (X_1) that does not depend on the difference of the relative velocities.
- X_1 does depend on the relative accelerations, which however are 8 orders of magnitude smaller than the terms derived by Cornish & Hellings.

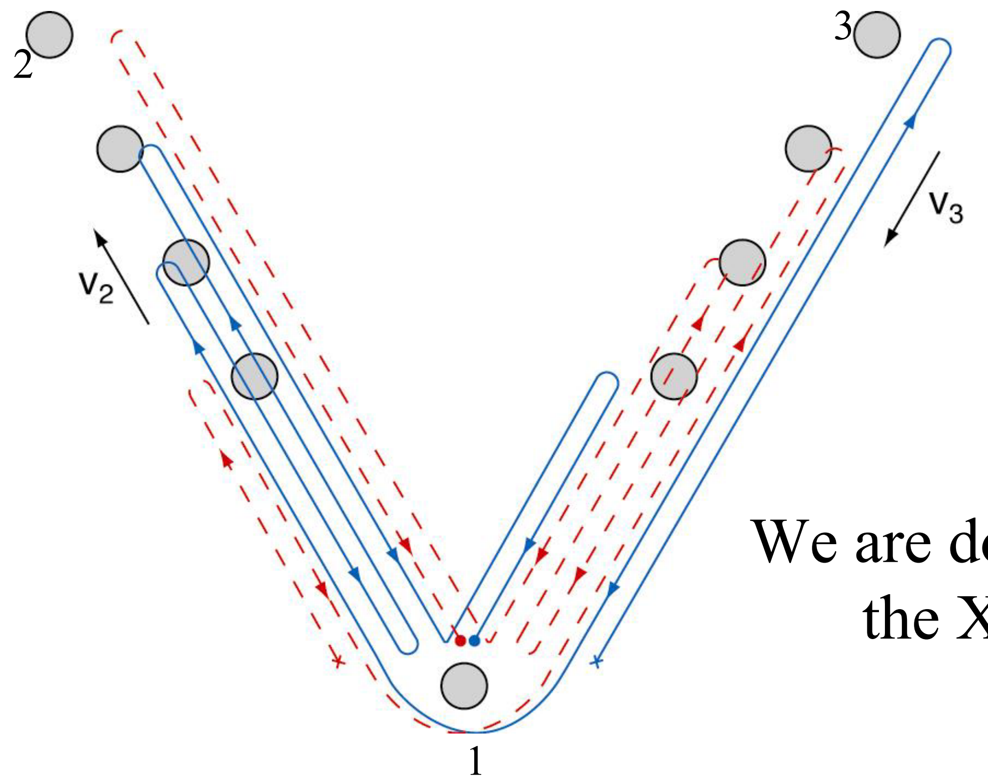
Solution to "flexy": Graphical Approach



This is the same expression used for removing the Sagnac effect.

D.A. Shaddock, M. Tinto, F.B. Estabrook & J.W. Armstrong, *Phys. Rev. D*, **68**, 061303 (R) (2003).

Solution to "flexy": Graphical Approach (Cont.)



We are double-differencing the X combination!

$$X_1(t) = [(\eta_{31} + \eta_{13;2}) + (\eta_{21} + \eta_{12;3'})_{;2'2} + (\eta_{21} + \eta_{12;3'})_{;33'2'2} + (\eta_{31} + \eta_{13;2})_{;33'33'2'2}]$$
$$- [(\eta_{21} + \eta_{12;3'}) + (\eta_{31} + \eta_{13;2})_{;33'} + (\eta_{31} + \eta_{13;2})_{;2'233'} + (\eta_{21} + \eta_{12;3'})_{;2'22'233'}]$$

$$\eta_{12;3'2} \equiv D_2 D_{3'} \eta_{12}(t) = \eta_{12}(t - L_2 - L_{3'}(t - L_2)) \neq D_{3'} D_2 \eta_{12}(t)$$

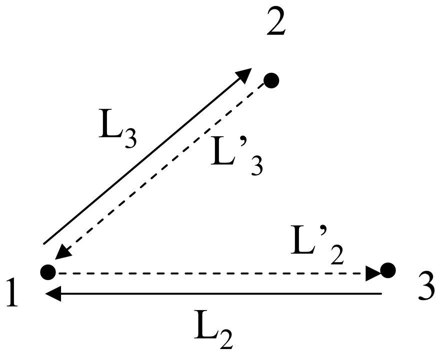
Systematic Approach

- Unlike X and α, the combinations ζ, E, P, U, do not allow for a “loop interpretation”.
- Is there a general procedure for deriving all the TDI combinations that are now unaffected by the Sagnac and Flexy problem?
- YES!

Systematic Approach (Cont.)

$$\eta_{12}(t), \eta_{21}(t), \eta_{13}(t), \eta_{31}(t)$$

$$\eta_{21} + \eta_{12;3'} = [D_{3'} D_{3} - I] p_{1}$$
$$\eta_{31} + \eta_{13;2} = [D_{2} D_{2'} - I] p_{1}$$



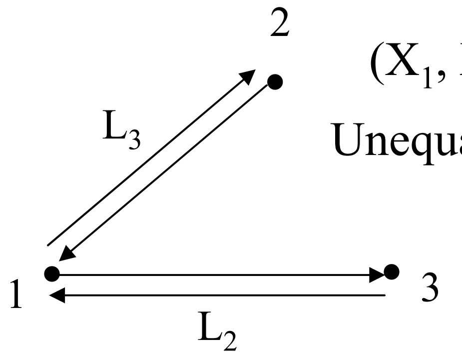
$$X = [D_{2} D_{2'} - I](\eta_{21} + \eta_{12;3'}) - [D_{3'} D_{3} - I](\eta_{31} + \eta_{13;2})$$
$$= [D_{2} D_{2'} - I][D_{3'} D_{3} - I] p_{1} - [D_{3'} D_{3} - I][D_{2} D_{2'} - I] p_{1} \neq 0$$

$$(\eta_{31} + \eta_{13;2}) + (\eta_{21} + \eta_{12;3'})_{;2'2} = [D_{2} D_{2'} D_{3'} D_{3} - I] p_{1}$$
$$(\eta_{21} + \eta_{12;3'}) + (\eta_{31} + \eta_{13;2})_{;33'} = [D_{3'} D_{3} D_{2} D_{2'} - I] p_{1}$$

$$\Downarrow$$

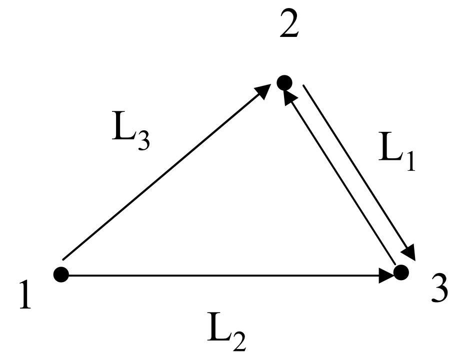
$$X_{1} = [D_{3'} D_{3} D_{2} D_{2'} - I][(\eta_{31} + \eta_{13;2}) + (\eta_{21} + \eta_{12;3'})_{;2'2}]$$
$$- [D_{2} D_{2'} D_{3'} D_{3} - I][(\eta_{21} + \eta_{12;3'}) + (\eta_{31} + \eta_{13;2})_{;33'}] \cong 0$$

Systematic Approach (Cont.)



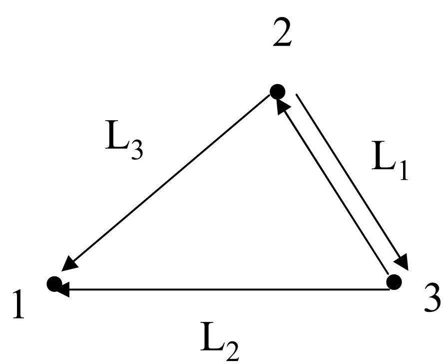
(X_1, X_2, X_3)

Unequal-arm Michelson



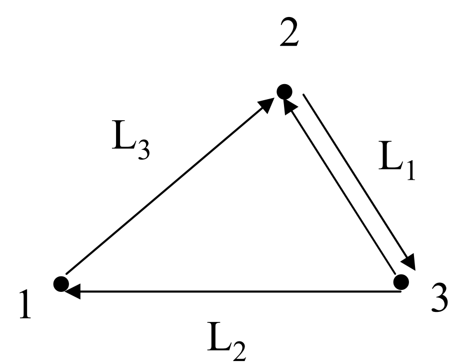
(P_1, P_2, P_3)

Beacon



(E_1, E_2, E_3)

Monitor



(U_1, U_2, U_3)

Relay

We have identified flex-free combinations for the above beam configurations.

M. Tinto, F.B. Estabrook, & J.W. Armstrong, gr-qc/0310017, October 6, 2003

How Does the LISA Sensitivity Change?

- Once the laser frequency fluctuations are removed, the corrections to the signal and the secondary noises (optical path, proof-mass, etc.), introduced by the extra delays due to the Sagnac (14 km) and flexy (~ 300 m) effects, are orders of magnitude below the signals and secondary noises determined by the "1st generation" TDI expressions:

$$\longrightarrow \quad \widetilde{X}_1(f) = \widetilde{X}(f)\left[1 - e^{8\pi i f L}\right]$$

Conclusions

- T.D.I. provides a robust method for canceling the leading noise source – laser phase fluctuations – in an interferometer with unequal, time-variable arms.
- It relies on specific use of time-delays between the LISA spacecraft.
- It allows analysis of signals, noises, achievable sensitivities, and architectural design (including system-level tradeoffs between, e.g. laser stability, arm length accuracy, stability of optical bench, Doppler shifts due to chosen orbits, USO stability).
- It provides robustness of the mission with respect to failures of subsystems.
- It shows existence of alternate LISA configurations offering potential design, implementation, or cost advantages.
- It gives a data combination (ζ) for assessing the LISA on-orbit instrumental noise performance.
- Modifications of the 1st generation TDI expressions are required to avoid stricter laser frequency stability requirements.
- 2nd generation TDI combinations **do** exist, which remove the laser noise way below the secondary noise sources.
- The sensitivities of the next generation TDI combinations are the same as those derived for the stationary configuration.