Gravitational waves from precessing binaries

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Outline

Subtitle: Planned search of LIGO data
(some tricks relevant to LISA)

1. Context and motivation
2. Basic binary search (NS/NS)
3. Complications for BH/BH binaries
4. Complications for precessing binaries (mainly BH/NS)

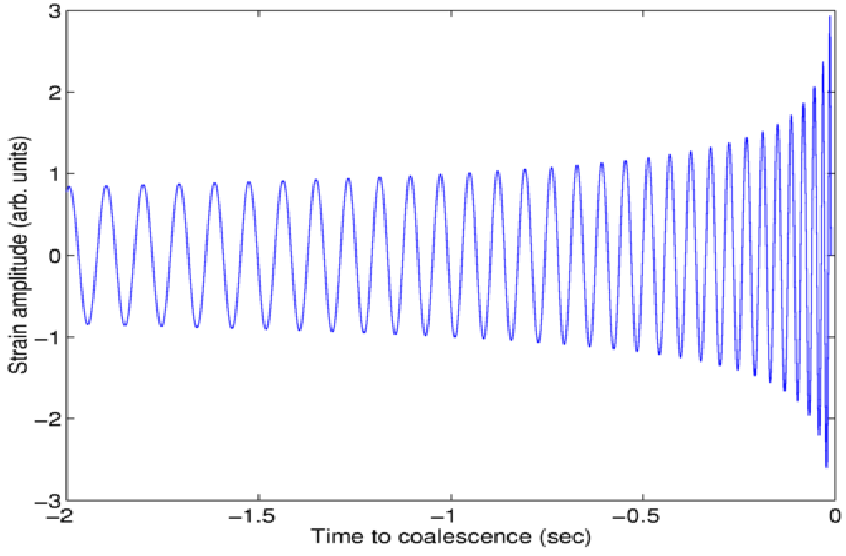
Inspiraling compact binaries

Two objects orbit each other

Compact objects $\Rightarrow$ close orbit $\Rightarrow$ high velocity

Emission of gravitational waves shrinks orbit

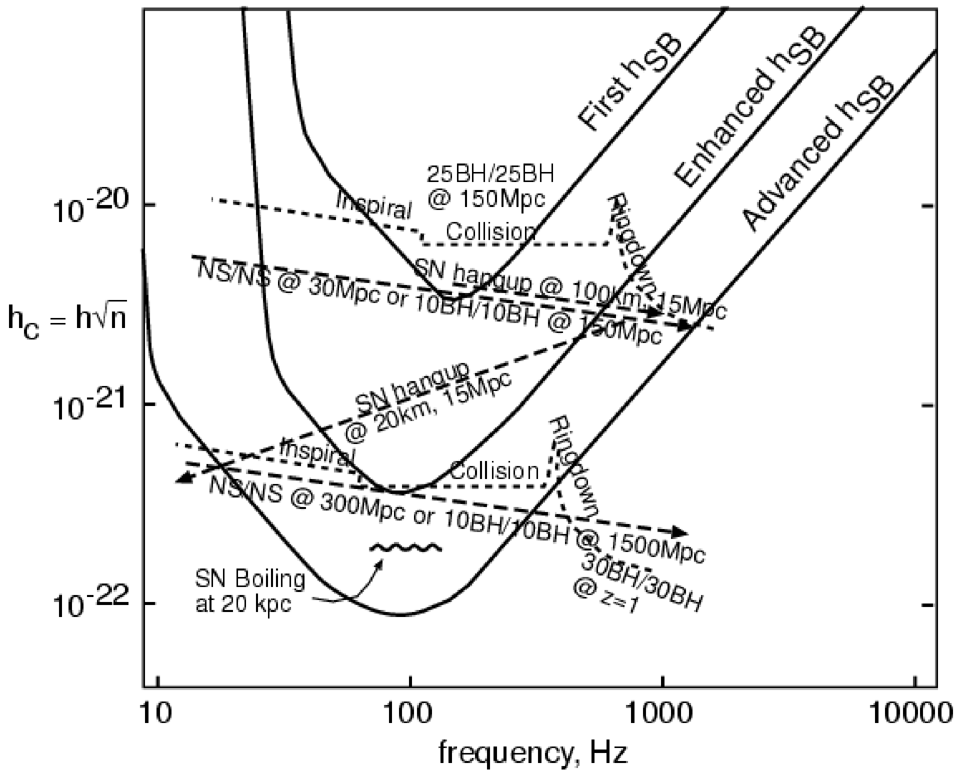
$$h \propto \frac{M^{5/3} f^{2/3}}{D} \cos\psi$$



$$\tilde{h} \propto f^{-7/6} e^{i\psi(f)}$$

LIGO sources

(graph from Thorne)



Boundary between inspiral and plunge/collision scales as 1/mass.

Why binaries are important

Known to exist: PSR B1913+16 (Hulse & Taylor ApJL 1974)
Event rate predictions for initial LIGO (Kalogera's group):

- NS/NS up to 1/few yr at range 20 Mpc based on observations of PSR J0737-3039
- BH/NS and BH/BH up to 1/few yr at range 40 Mpc and 100 Mpc based on stellar evolution codes

Best modeled source: post-Newtonian approximation carried to $O(v/c)^7$ (cf. perturbative QED).

Good modeling lets you use matched filtering, which is the best technique for detecting faint signals in noise.

Three kinds of binaries - 1

NS/NS:

- "Solid" event rates based on observations
- Masses 1–3 $M_\odot$ each (1.4 $M_\odot$ observed)
- Plunge $\simeq$1400 Hz $\Rightarrow$ post-Newtonian at 150 Hz
- Spins, quadrupoles, etc don't matter at 150 Hz

Upper limit searches have been done:

- Algorithms stable, developed during 1990s
- Caltech 40m (Allen et al. PRL 1999)
- TAMA (Tagoshi et al. PRD 2001)
- LIGO S1 w/GEO (Abbott et al. PRD 2004)

Matched filtering - basic

Define inner product between functions a and b as

$$\langle a, b\rangle = 4\text{Re}\int_0^\infty df \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_h(f)},$$

where S_h is the power spectral density of strain noise.

Signal-to-noise ratio for linear filter is defined by

$$\rho^2 = \langle h, u\rangle^2 / \langle u, u\rangle,$$

where h is strain data and u is template (predicted waveform).

Optimal filter (highest ρ^2) is then $u \propto h$; set $\langle u, u\rangle = 1$ since amplitude drops out.

Sinusoidal signals: Filtering amplifies ρ^2 by number of cycles, more sensitive to phase than amplitude (mostly).

Matched filtering - nuisance parameters

Sinusoidal signal w/unknown time & phase shifts t_0 & δ:

$h(t) \propto \cos[\psi(t-t_0)+\delta]$

Maximize over all values of shifts ("nuisance parameters").

Maximize over δ by redefining signal-to-noise ratio functional

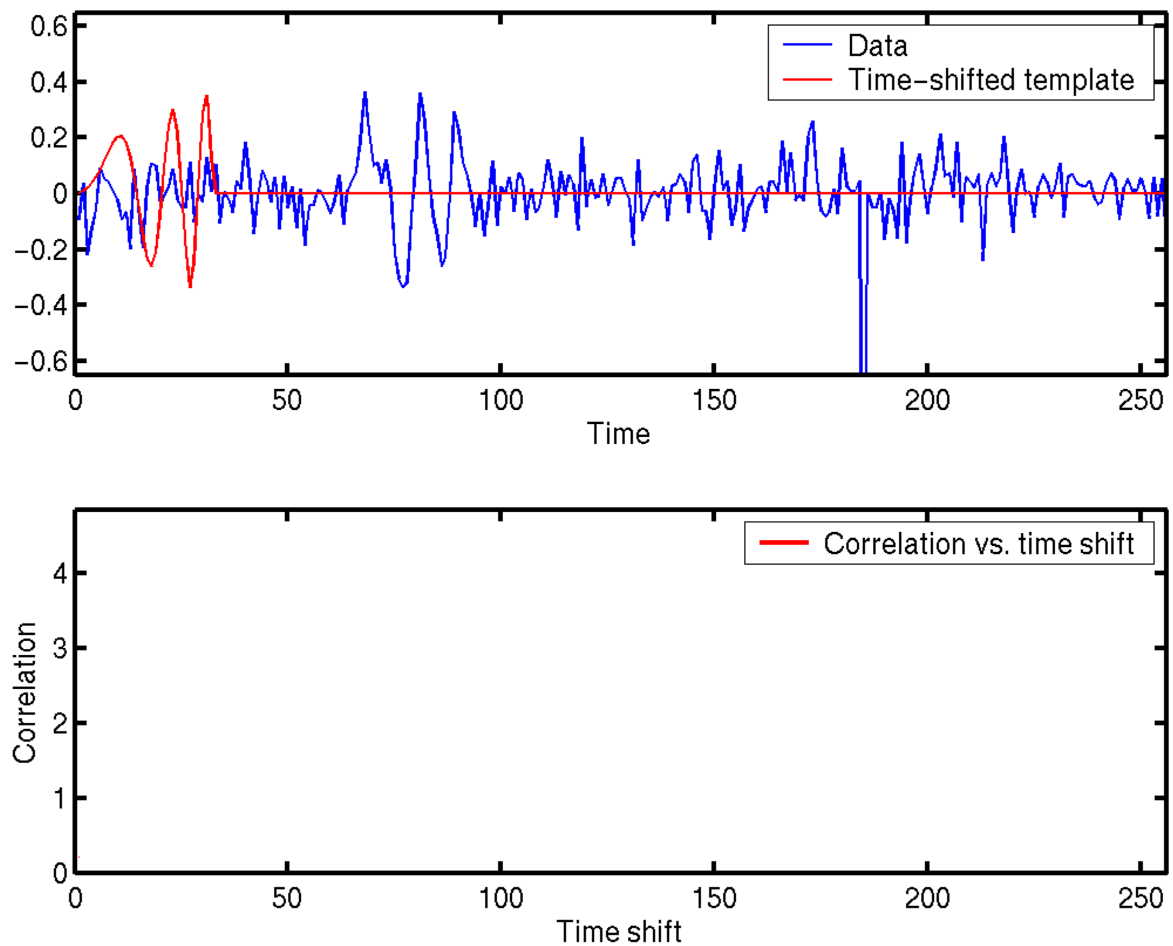
$\rho^2[h] = \langle h,u\rangle^2 + \langle h,v\rangle^2, \quad u \propto \cos\psi, \quad v \propto \sin\psi.$

Adds power in sine and cosine filters;

Requires $(u,v)=0$ to avoid double-counting.

Maximize over t_0 by hunting for peaks in Fast Fourier Transform.

(graphics by Peter Shawhan)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Time−shifted template
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Time
Correlation vs. time shift
Correlation
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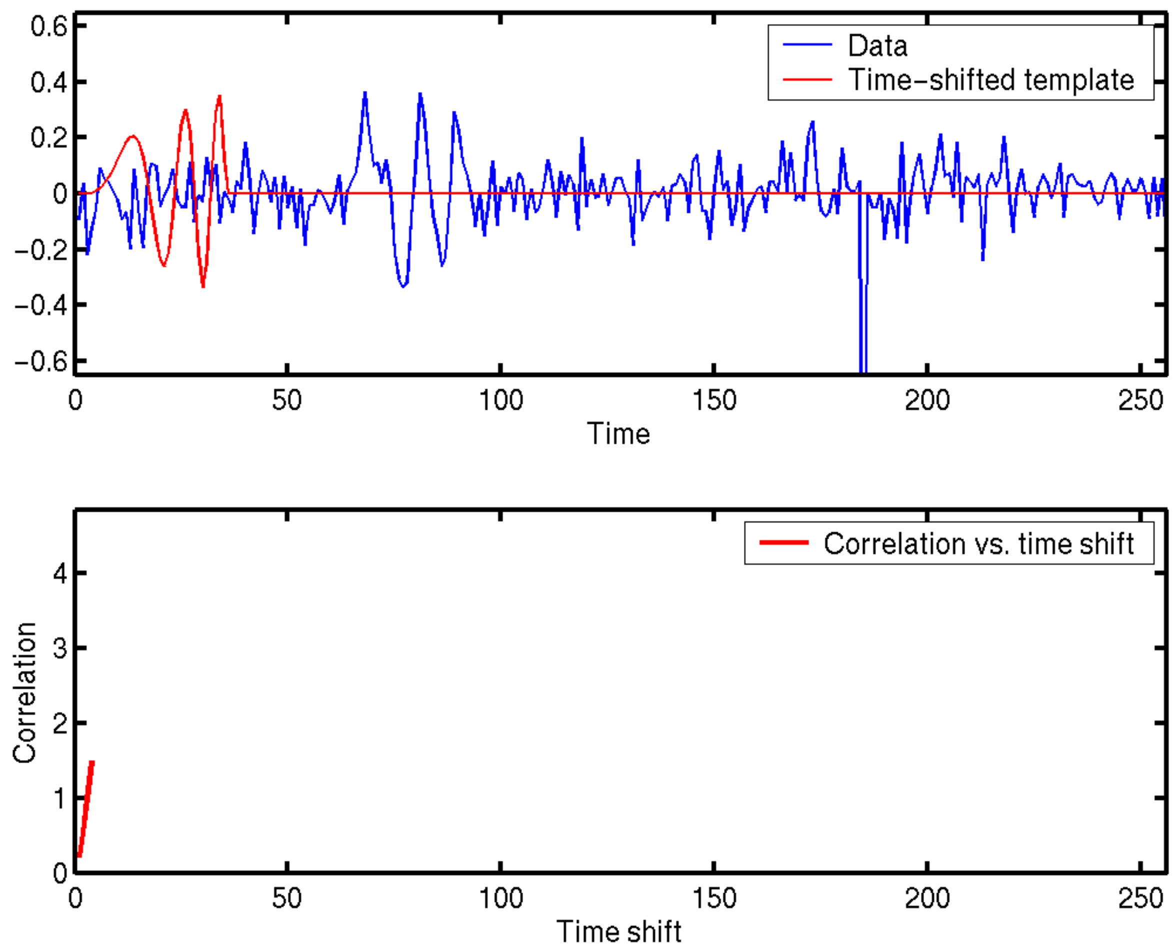
Time shift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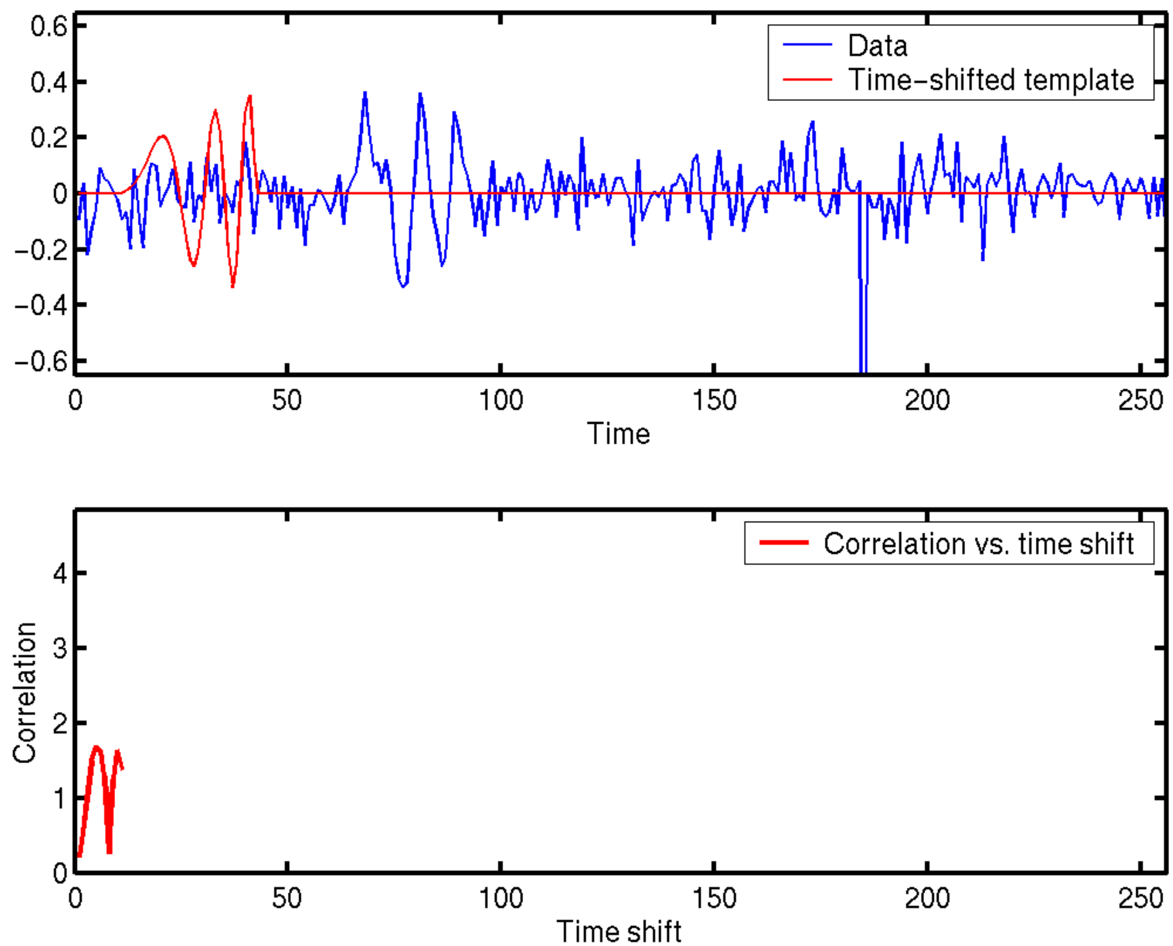

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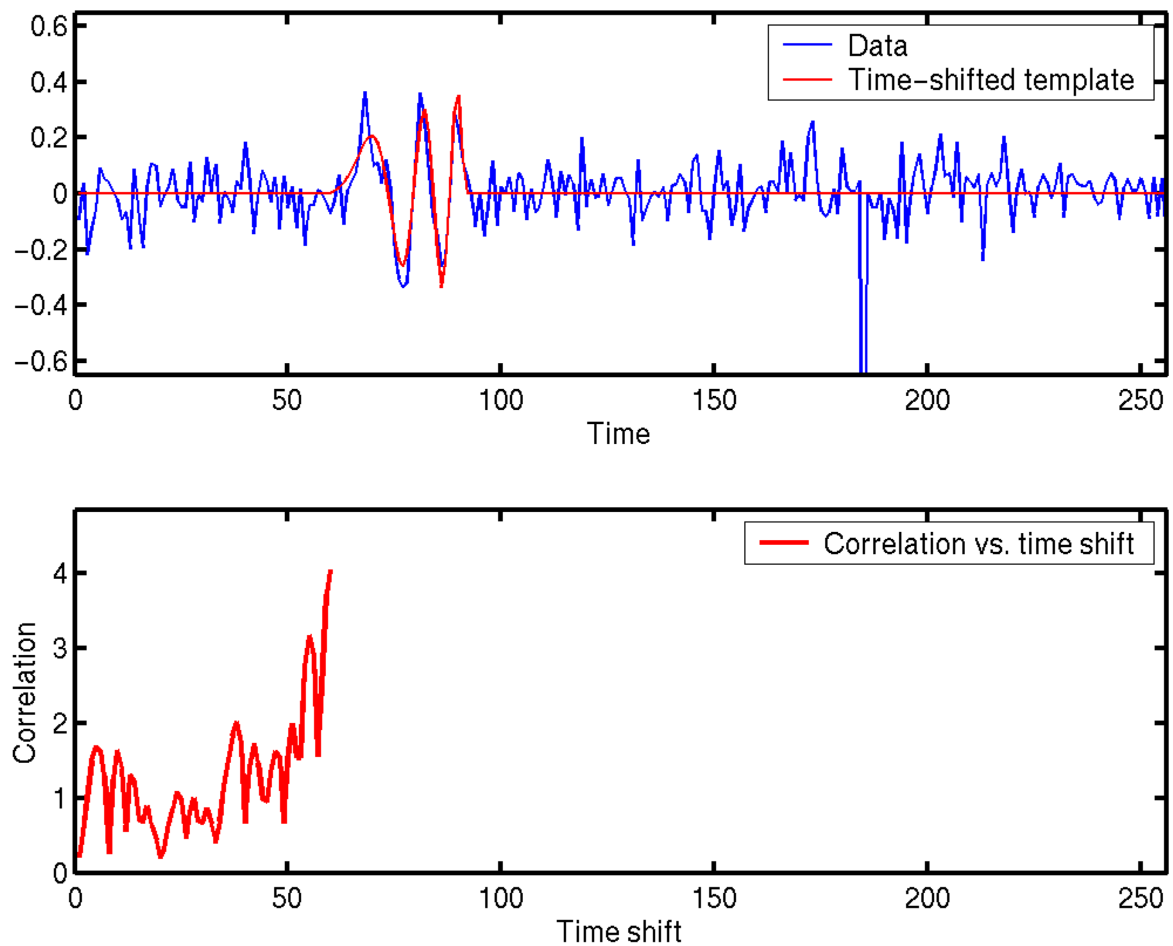
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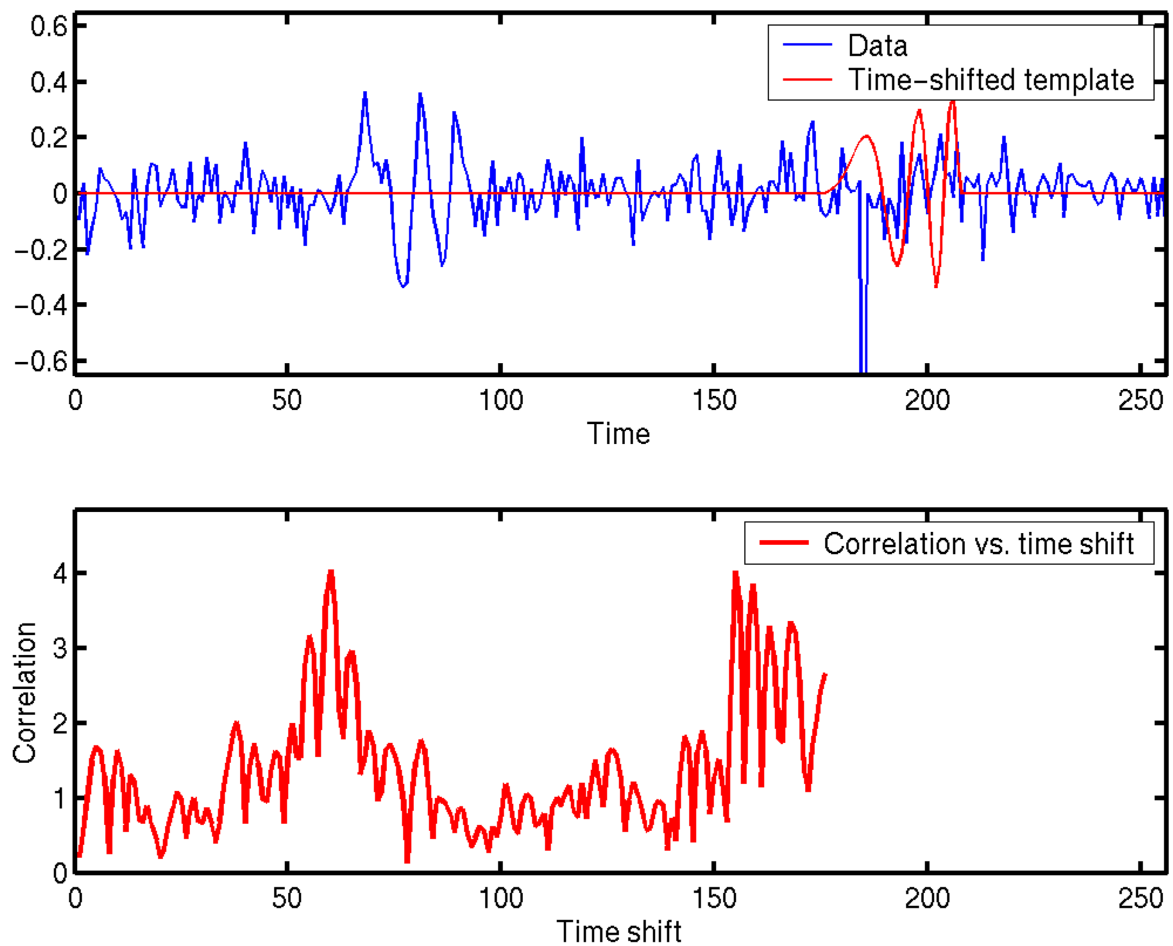

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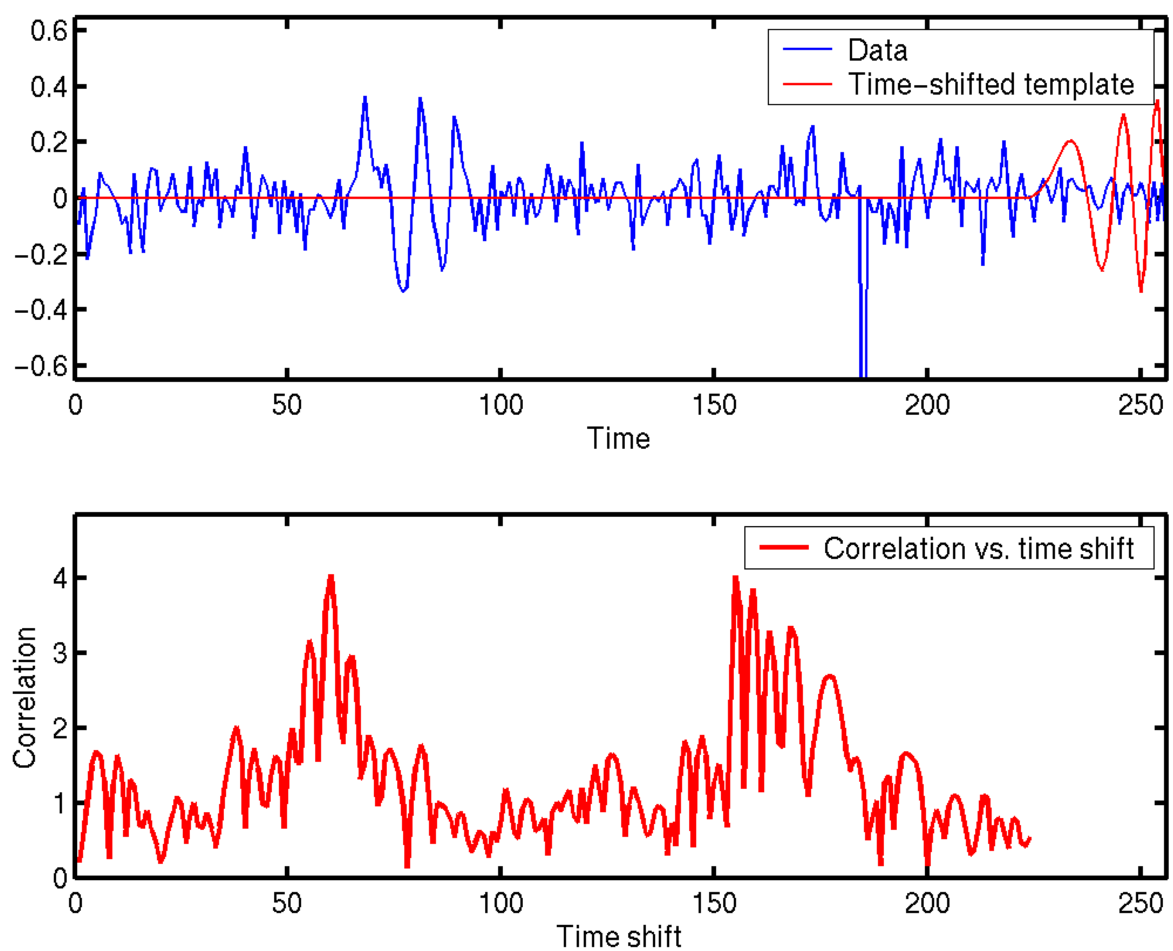


Data
Time−shifted template
Time
Correlation vs. time shift
Correlation
Time shift

Matched filtering - shape parameters

Templates form manifold with shape parameters $\vec{\lambda}$ as coordinates. Not covered by a few orthogonal functions: what to do?

How much ρ do we lose filtering a signal $u(\vec{\mu})$ with a nearby template $u(\vec{\lambda})$? Falls off quadratically in $\vec{\lambda} - \vec{\mu}$.

Define metric so that (Owen PRD 1996)

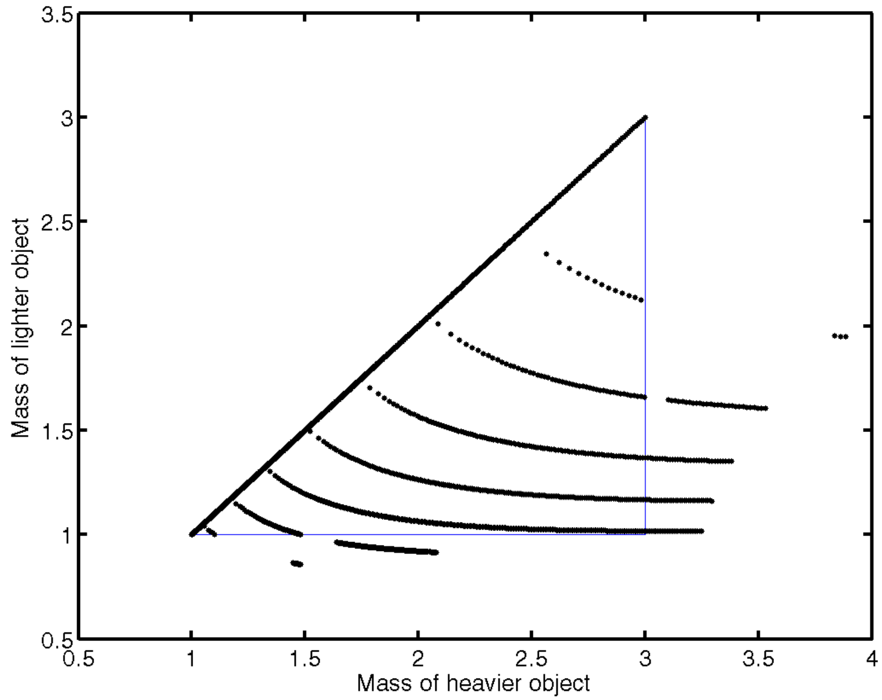
(proper distance)^2 = fractional loss in ρ:

$$g_{ab} = -\frac{1}{2}\frac{\partial}{\partial\lambda^a}\frac{\partial}{\partial\lambda^b}\langle u(\vec{\mu}), u(\vec{\lambda})\rangle|_{\lambda=\mu}$$

(or equivalently by projecting from information matrix).

Post-Newtonian template metric can be written in terms of a dozen integrals of the form $I_p = \int_0^\infty df/f^{p/3}/S_h(f)$.

NS/NS parameter space



Lay grid with constant *proper* spacing $ds^2 = \mu$ (mismatch);
Number of templates = proper volume / volume per template.

Signal-based vetoes

Signal-to-noise isn't everything: high ρ^2 from kicking a test mass.
Why? $\langle a, u \rangle$ can be large even if $\langle a, u \rangle / \sqrt{\langle a, a \rangle} \ll 1$.

Get rid of transients with χ^2 veto: (Allen et al. PRL 1999)
Divide spectrum into p bands of equal power if signal is present;
Test if observed power ρ_i really is distributed equally:

$$\chi^2 = p \sum_{i=1}^{p} [\rho_i - \rho/p]^2$$

Veto something with large ρ^2 if it's also got large χ^2.

Three kinds of binaries - 2

BH/BH (w/o precession):

- Can neglect spins if small or parallel to orbit
- Event rates less certain, but signals are stronger
- Masses 3–25 $M_\odot$ each (not MACHOs)
- Plunge $\simeq$150 Hz $\Rightarrow$ too relativistic for post-Newtonian

Search strategies being implemented:

- Use Caltech *ad hoc* waveforms
- Complications due to mismatched filtering
- Algorithms still not fully worked out
- Implement them anyway on S2 & S3 data!

BH/BH waveforms

Late inspiral/early plunge: Post-Newtonian gradually going bad, waveforms still too long for numerical relativity (10s of cycles). Different versions of post-Newtonian expansion diverge in different ways at end.

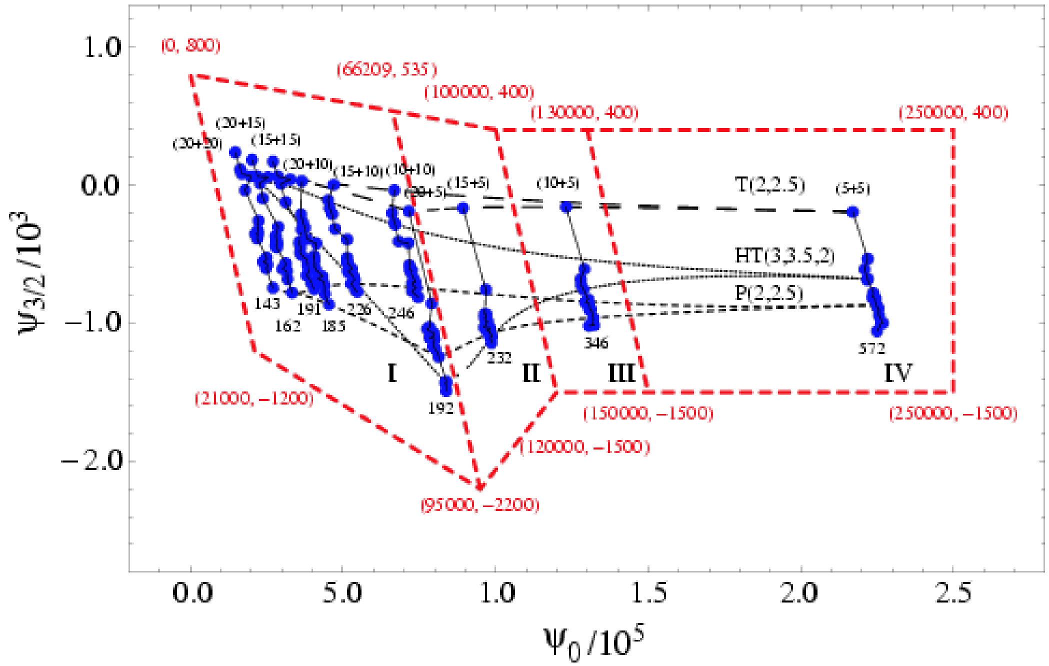
Solution: Buonanno, Chen, and Vallisneri PRD 2003

$$\tilde{h}(f) = \tilde{h}_{\mathrm{old}}(f) \times \left(1 - \alpha f^{2/3}\right) \times \Theta(f - f_{\mathrm{cut}})$$

Fits all versions of late inspiral ($< 10\%$ loss due to systematics); Requires two new shape parameters (not too many templates).

BH/BH search

Parameter mapping becomes uncertain: (BCV 2003)



Search under development at Cardiff.

Three kinds of binaries - 3

Precessing BH/BH or BH/NS:

- No direct observations, but range is high
- Many BH spins misaligned w/orbit $\Rightarrow$ precession
- Modulates waveforms, esp. at "extreme" mass ratios
- Plunge 150 Hz (BH/BH) to 900 Hz (BH/NS)
- Post-Newtonian still OK for BH/NS, *ad hoc* for BH/BH

Data analysis was thought hopeless:

- Carries more information, but makes detection harder
- Clever mathematical handling of modulation
- Computationally feasible, though expensive

Three regimes of precession

Very complicated dynamics (Apostolatos et al. PRD 1994)
Not generally closed-form expressions for waveforms.

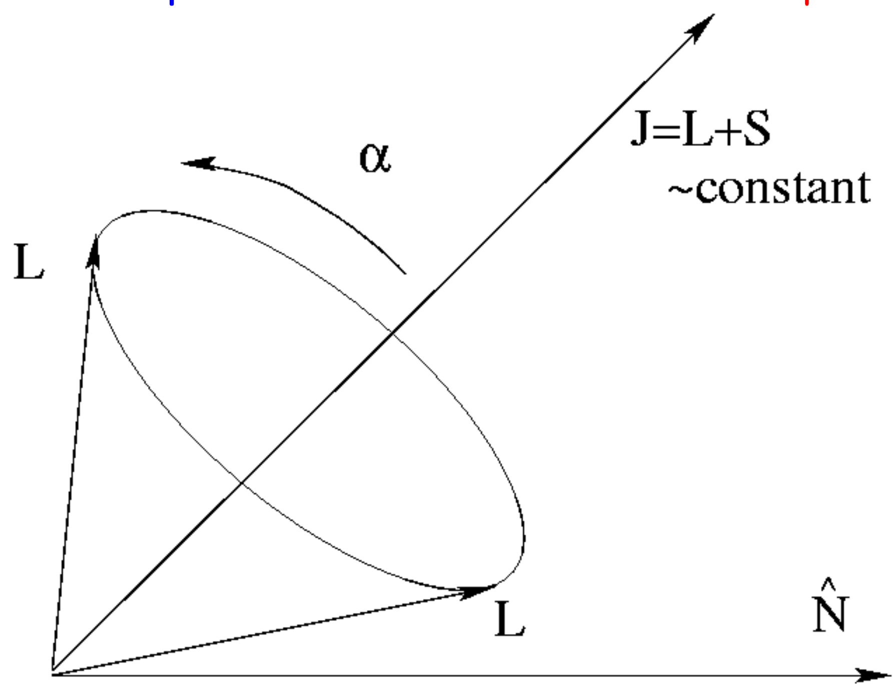
$L \gg S$: little modulation

$L \simeq S$: possible "transitional precession" (chaos?)

$L \ll S$: strong modulation
Several more parameters (9 total), was thought to be hopeless search (Apostolatos PRD 1996).

Why $L \ll S$ is important

Detector response depends on $\hat{L} \cdot \hat{N} = \hat{J} \cdot \hat{N} + \text{const.} \times \cos\alpha$,



which varies most for $L \ll S \Rightarrow m_2 \ll m_1$.
So focus on BH/NS binaries (also nice parameter mapping).

Detector response

$$h = F_+ A_+ \cos\psi + F_\times A_\times \sin\psi = A\cos(\psi + \delta),$$
$$A = \sqrt{(F_+ A_+)^2 + (F_\times A_\times)^2},$$
$$\delta = \tan^{-1}[F_\times A_\times / (F_+ A_+)],$$

where $F_+, F_\times$ depend on sky position and $A_+, A_\times$ on source.

For binaries: $A_+ = 1 + (\hat{L} \cdot \hat{N})^2, \qquad A_\times = 2\hat{L} \cdot \hat{N}$
Precession changes $\hat{L} \cdot \hat{N}$, modulates at angular frequency $d\alpha/dt$.

For pulsars $F_+, F_\times$ change and $A_+, A_\times$ are fixed; but same type of modulation.

For freely precessing pulsars or precessing binaries with LISA, all four quantities change; but same problem again.

The $\mathcal{F}$-statistic

Developed for pulsars: A's fixed, F's change
(Jaranowski, Krolak & Schutz PRD 1998)
Applied to precessing binaries: A's change, F's fixed
(Buonanno, Chen, & Vallisneri PRD 2003)

Full power of matched filtering by including amp. modulation;
Maximizes quickly over extra nuisance parameters (angles);
Must mean something physically?

How the F-statistic works

Complex modulation in frequency domain:

$$\tilde{h}(f) \propto [Z_0 + Z_1 \cos\alpha(f) + Z_2 \sin\alpha(f)]\, f^{-7/6} e^{i\psi(f)}$$

Carrier $e^{i\psi}$ gains sidebands $e^{i(\psi\pm\alpha)}$.
Spacing between carrier & sidebands is given by α.
Relative amplitudes depend on $\hat{J}\cdot\hat{N}$ and α.

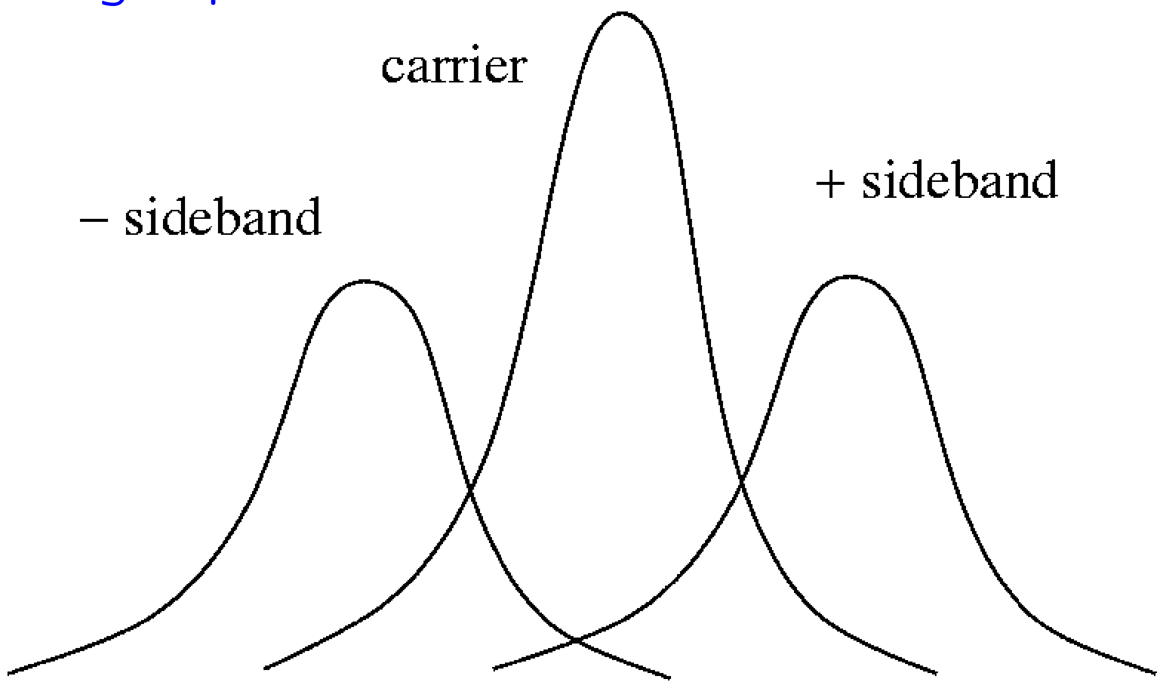
For strong modulation, carrier & sidebands are orthogonal:

$$\rho^2 = \mathcal{F} = \langle h, \cos\psi\rangle^2 + \langle h, \cos(\psi+\alpha)\rangle^2 + \langle h, \cos(\psi-\alpha)\rangle^2 + \text{sine filters}^2$$

adds power in carrier & sidebands, sine & cosine of each.

Carrier and sidebands

If they're not orthogonal, make them so with Gram-Schmidt.
But then you get problems. . . Pan *et al.* 2004



Parameter-space metric

Strong modulation makes metric simple: (Owen & Vecchio 2004)

$$2g_{ab} = 3(\partial_a\psi, \partial_b\psi) + 2(\partial_a\alpha, \partial_b\alpha) - \text{ projections}$$

Block diagonal, two mass parameters (ψ_0, ψ_3) and one spin parameter$\beta : \alpha = \beta f^{-2/3}$ (good for $m_1 \gg m_2$).

Not-so-strong modulation:

Still pretty simple, only need sine & cosine moment functions

$$S_p(\beta) = \int \frac{df\ \sin(\beta f^{-2/3})}{f^{p/3} S_h(f)}, \qquad C_p(\beta) = \int \frac{df\ \cos(\beta f^{-2/3})}{f^{p/3} S_h(f)}$$

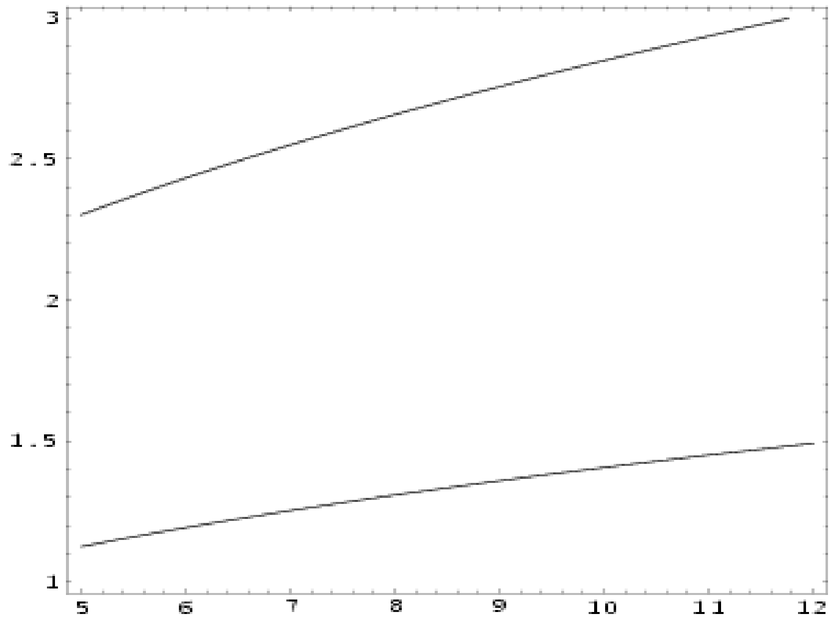
These can be stored in an interpolation table (up to $2\beta_{\text{max}}$).

Infinitesimal modulation:

Singular metric (physics is breaking down) so forget it.

Which systems have strong modulation?

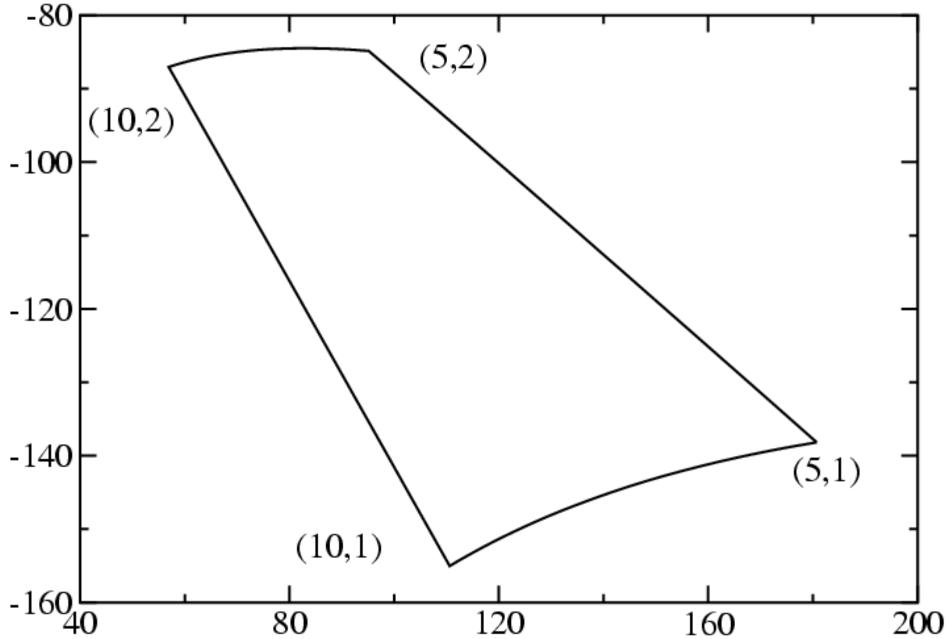
Or "where is $\langle$ carrier,sideband $\rangle < 0.1$ in m_{NS}-m_{BH} plane?"



Below top line: Strong for $S_1/m_1^2 = 1$ (GR maximum).
Below bottom line: Strong for $S_1/m_1^2 = 0.5$ (believable).

2D space: mass parameters

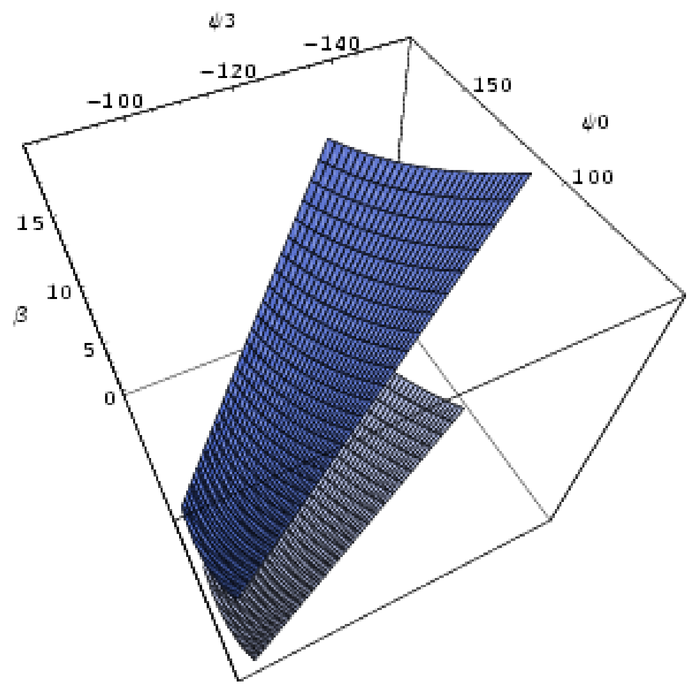
Astrophysical parameter range: $m_1 \in (5, 10) M_\odot; m_2 \in (1, 2) M_\odot$



Rectangle in $m_1 m_2$ plane $\longrightarrow$ wedge in $\psi_0 \psi_3$ plane.

3D space: Masses and spin

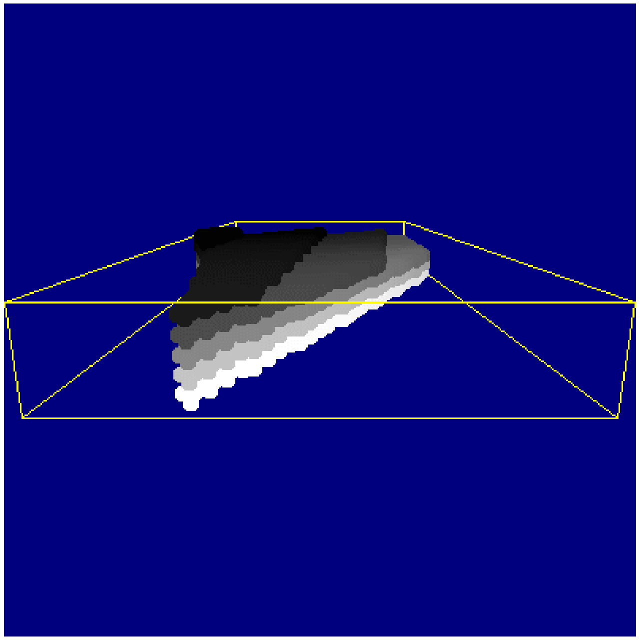
Only large body's spin matters: $S_1/m_1^2 \in (0,1)$



Strongest modulation (highest β_{max}) is at extreme mass ratio.

Template bank (LAL)

LIGO-I SRD noise, worst mismatch 100% ($\mu = 1$) for legibility:



Template count is $120\,000\,(0.03/\mu)^{3/2}$. (PRELIMINARY: wrong by factor 2?)

Future issues

Too many templates for brute force search.
(in practice limited by computation of χ^2, not ρ^2)

Better go hierarchical:
First pass scans data w/o sidebands, low ρ threshold.
Second pass includes sidebands on promising candidates from first pass w/higher threshold to weed out false alarms.

Need clever 3D tiling algorithms (basic one in place).

What will this do to χ^2 veto?