Source modeling, detection and science of gravitational waves emitted by precessing compact binaries

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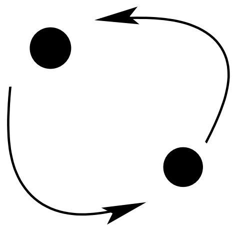
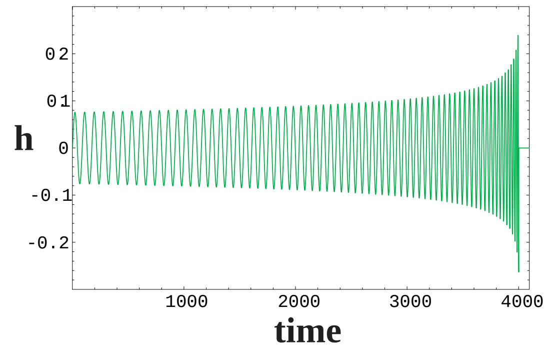
Content:

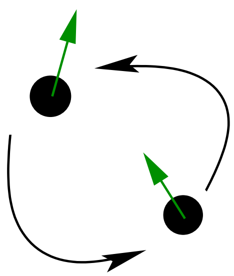
- **Main features of dynamics in precessing compact binaries**
- **Phenomenological and physical templates in frequency and time domain**
- **Transition from adiabatic inspiral to plunge in precessing binaries**
- **Estimation of binary parameters and testing GR**
- **Do we need spin couplings at higher post-Newtonian orders?**

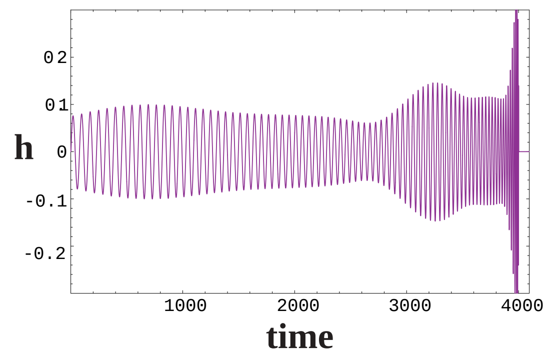
Precessing versus non-precessing compact binaries





- **Non spinning: Inspiral [$f_{\rm GW} = 2 f_{\rm orb}, f_{\rm end}(m_1, m_2)$], plunge, merger and ring down**





Many more parameters!

- **Precessing: Inspiral [$f_{\rm GW} = (2 f_{\rm orb}, f_{\rm prec}), f_{\rm end}(\boldsymbol{S}, m_1, m_2)$], plunge (?) merger, ring down**

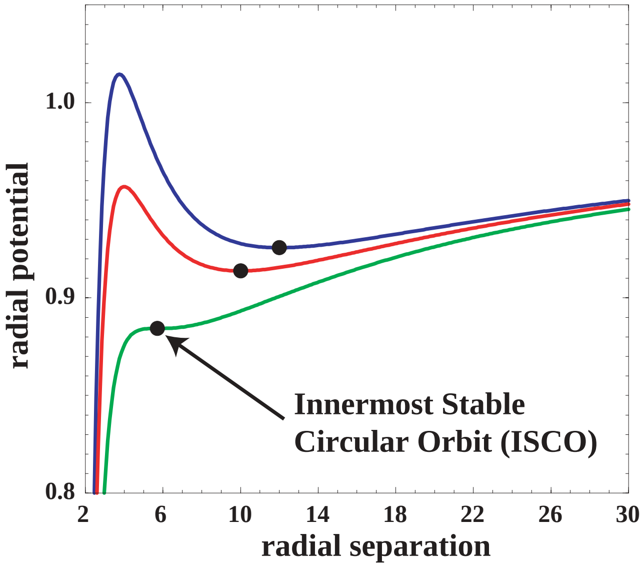
Precession of the orbital plane modulates both amplitude and phase of gravity-wave

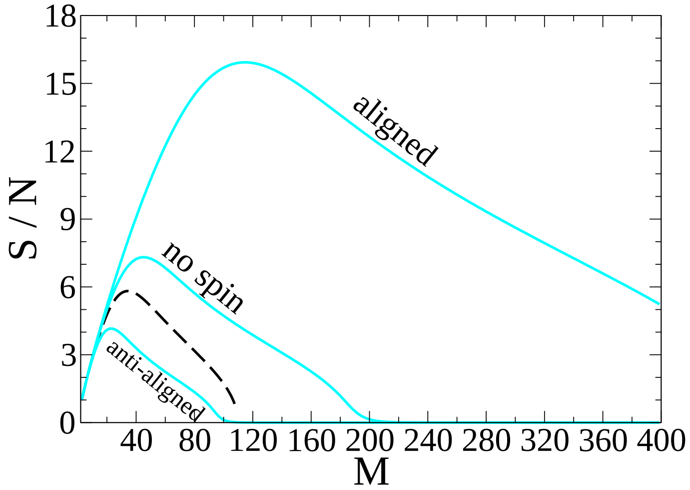
Detectibility of inspiraling spinning binaries with GW interferometers

Spin-orbit coupling makes two-body gravitational interaction more (less) repulsive when spins are aligned (anti-aligned)

$$V(r) = -\frac{mM}{r} + \frac{L^2}{2mr^2} - \frac{L^4}{m^3 r^4} + \cdots + \frac{2}{r^3}\boldsymbol{L}\cdot\boldsymbol{S} + \cdots$$

Duration of inspiral (and signal-to-noise ratio) modified by spin effects





Equal-mass binaries at 100 Mpc

Predictions for spin misalignement and spin magnitude

- **Simulations considering NS-BH and BH-BH in Galactic field**

  [misalignement due to "kick" (recoil velocity) imparted to NSs at birth]

  [Kalogera 00; Grandéclement et al. 04]

  – $50\% - 80\%$ **of NS-BH have tilt angle smaller than** 40^o **and** $10 - 20\%$ **between** 40^o **and** 50^o

  – **more than** 90% **of BH-BH binaries have tilt angle smaller than** 30^o

- **Spin properties of NS-BH and BH-BH in centers of globular clusters may be very different**

- **No constraints on the orientation and magnitude of spins in massive and supermassive black-hole binaries**

- $S_{\rm BH}/m_{\rm BH}^2 \sim 0 - 1$ **and** $S_{\rm NS}/m_{\rm NS}^2 \sim 0.005 - 0.01$

Time scales characterizing compact binary dynamics

- **Inspiral time scale:**

$$E = -\frac{\mu M}{2r} \qquad -\frac{dE}{dt} = \frac{32}{5}\frac{\eta^2 M^5}{r^5} \qquad \Rightarrow \qquad \frac{T_{\rm insp}}{M} \propto \frac{1}{\eta}\left(\frac{r}{M}\right)^4$$

- **Orbital time scale:**

$$\omega^2 = \frac{M}{r^3} \qquad \Rightarrow \qquad \frac{T_{\rm orb}}{M} = \frac{2\pi}{M\omega} \propto \left(\frac{r}{M}\right)^{3/2}$$

- **Precession time scale (spin-orbit coupling):**

$$\dot{\boldsymbol{S}} \propto \overbrace{\frac{1}{r^3}}^{\Omega^L_{\rm prec}}\boldsymbol{L}\times\boldsymbol{S}\,, \qquad \dot{\hat{\boldsymbol{L}}} \propto \overbrace{\frac{1}{r^3}}^{\Omega^S_{\rm prec}}\boldsymbol{S}\times\hat{\boldsymbol{L}}$$

$$\frac{T^L_{\rm prec}}{M} = \frac{2\pi}{M\Omega^L_{\rm prec}} \propto \frac{1}{\eta}\left(\frac{r}{M}\right)^{5/2}\,, \qquad \frac{T^S_{\rm prec}}{M} = \frac{2\pi}{M\Omega^S_{\rm prec}} \propto \left(\frac{r}{M}\right)^3$$

$$T_{\rm orb} \ll T^L_{\rm prec}, T^S_{\rm prec} \ll T_{\rm insp}$$

Inspiraling dynamics averaging over orbital period: adiabatic limit

[Barker & O'Connell 75; Damour 82; Thorne & Hartle 85; Kidder 95; Kidder, Wiseman & Will 96]

$$\dot{\omega} = -[dE(\omega)/d\omega]/\mathcal{F}_{\rm GW}(\omega)$$

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\,\eta\,(M\omega)^{5/3}\,\Big[1 + c_{\rm 1PN}(m_1,m_2)\,(M\omega)^{2/3} + c_{\rm 1.5PN}(SO,m_1,m_2)\,(M\omega) + c_{\rm 2PN}(SS,m_1,m_2)\,(M\omega)^{4/3} + c_{\rm 2.5PN}(m_1,m_2)\,(M\omega)^{5/3} + c_{\rm 3PN}(m_1,m_2)\,(M\omega)^2 + c_{\rm 3.5PN}(m_1,m_2)\,(M\omega)^{7/3}\Big]$$

$$\dot{\boldsymbol{S}}_1 = \frac{(M\omega)^2}{2M}\left\{\left(4+3\frac{m_2}{m_1}\right)\boldsymbol{L} + \frac{1}{M^2}\left[\boldsymbol{S}_2 - 3(\boldsymbol{S}_2\cdot\hat{\boldsymbol{L}})\hat{\boldsymbol{L}}\right]\right\}\times\boldsymbol{S}_1$$

$$\dot{\boldsymbol{S}}_2 = \frac{(M\omega)^2}{2M}\left\{\left(4+3\frac{m_1}{m_2}\right)\boldsymbol{L} + \frac{1}{M^2}\left[\boldsymbol{S}_1 - 3(\boldsymbol{S}_1\cdot\hat{\boldsymbol{L}})\hat{\boldsymbol{L}}\right]\right\}\times\boldsymbol{S}_2\,,$$

$$\dot{\hat{\boldsymbol{L}}} = -\frac{(M\,\omega^{1/3})}{\eta\,M^2}\,\dot{\boldsymbol{S}} \qquad \boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$$

Two classes of binaries: single spin

- **Equal-mass binary and SS couplings neglected:**

$$\Rightarrow \boldsymbol{S} = \boldsymbol{S}_{\rm tot} = \boldsymbol{S}_1 + \boldsymbol{S}_2 \quad \text{and} \quad d(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)/dt = 0$$

- **Small mass-ratio binary:**

$$\Rightarrow m_2 \ll m_1 \quad \Rightarrow \quad |\boldsymbol{S}_2| = m_2^2 \chi_2 \ll |\boldsymbol{S}_1| = m_1^2 \chi_1 \quad \Rightarrow \quad \boldsymbol{S} = \boldsymbol{S}_1$$

$$\dot{\boldsymbol{S}} = \frac{\eta (M\omega)^{5/3}}{2M} \left(4 + \frac{3m_2}{m_1}\right) \hat{\boldsymbol{L}} \times \boldsymbol{S} \qquad \dot{\hat{\boldsymbol{L}}} = \frac{\omega^2}{2M} \left(4 + \frac{3m_2}{m_1}\right) \boldsymbol{S} \times \hat{\boldsymbol{L}}$$

$$\dot{\omega} = \dot{\omega}(\omega, \boldsymbol{S} \cdot \boldsymbol{L}, m_1, m_2)$$

$\alpha^J_{\rm prec} \propto \omega^{-1}$ if $L \sim \eta M^{5/3} \omega^{-1/3} \gg S$ [large separations, comparable masses]

$\alpha^J_{\rm prec} \propto \omega^{-2/3}$ if $S \gg L \sim \eta M^{5/3} \omega^{-1/3}$ [small mass ratio, last stages of inspiral]

$d\alpha^J_{\rm prec}/dt = \Omega^J_{\rm prec}$ [Apostolatos, Cutler, Sussman and Thorne 95]

Precession around the direction of the total angular momentum $\widehat{\boldsymbol{J}}$

- **Single spin binary (simple precession)**

$t_2 > t_1$

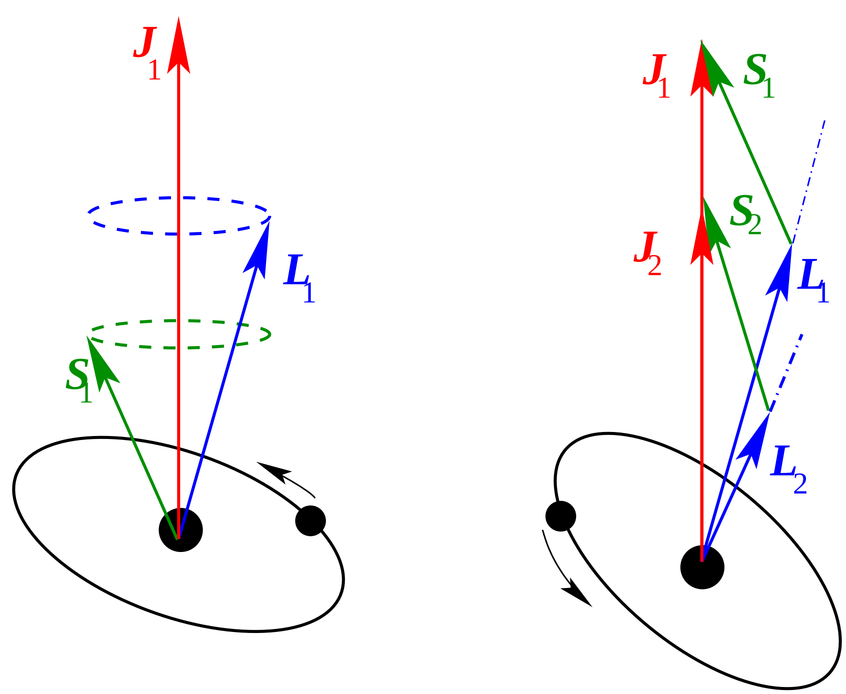
$$\dot{\boldsymbol{S}} \propto \frac{J}{r^3}\widehat{\boldsymbol{J}} \times \boldsymbol{S}$$

$$\dot{\widehat{\boldsymbol{L}}} \propto \frac{J}{r^3}\widehat{\boldsymbol{J}} \times \widehat{\boldsymbol{L}}$$

$$\Omega^J_{\rm prec} \propto \frac{J}{r^3}$$

$$\dot{L} \sim -\frac{1}{r^{7/2}}\widehat{\boldsymbol{L}}$$

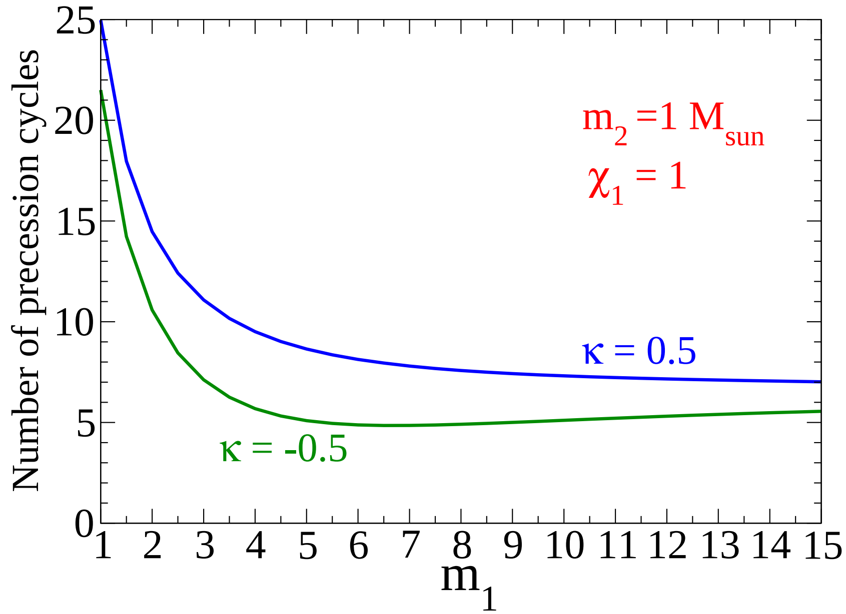
$$\widehat{\boldsymbol{L}} \cdot \widehat{\boldsymbol{S}} = {\rm const}$$



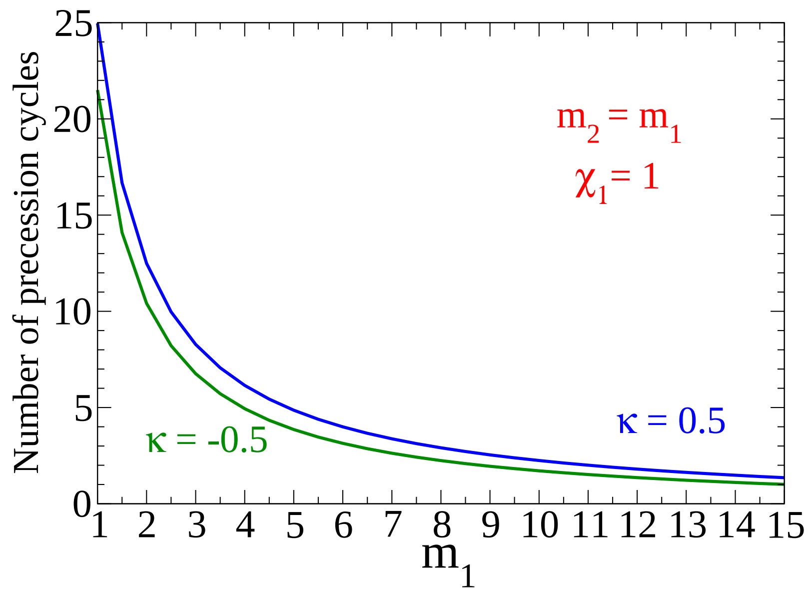
- **Transitional precession** ($\boldsymbol{L} \simeq -\boldsymbol{S}$ the binary tumbles in space)

Number of precessing cycles

small mass ratio binary



equal-mass binary



Phenomenological templates in adiabatic limit: Apostolatos ansatz

Phenomenological waveforms: introducing few *new* parameters and having reasonably good matches with the signal

Apostolatos' ansatz: add modulations to SPA phase [Apostolatos 96]

$$\psi_{\rm SPA}(f) = f^{-5/3}\,\left(\psi_0 + \psi_1\,f^{2/3} + \psi_{3/2}\,f + \cdots\right) + \mathcal{C}\,\cos(\delta + \alpha^J_{\rm prec})$$

$$\alpha^J_{\rm prec} = \mathcal{B}\,f^{-2/3} \quad \text{or} \quad \mathcal{B}\,f^{-1} \quad \text{where} \quad d\alpha^J_{\rm prec}/dt = \Omega^J_{\rm prec}$$

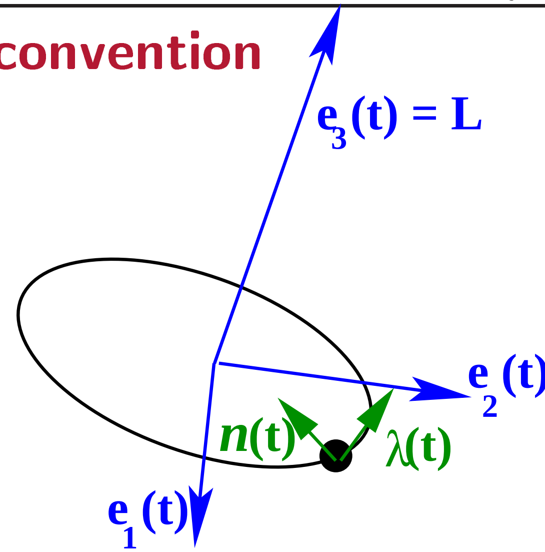
– **Not very satisfactory performances (why?)** [Grandéclement, Kalogera & Vecchio 96]

– **three *new* intrinsic parameters** $(m_1, m_2, \mathcal{C}, \delta, \mathcal{B})$

– **high computational cost**

Performances can improve using "spike" waveforms [Grandéclement & Kalogera 03]

Gravity-wave in precessing convention



[AB, Chen & Vallisneri 03]

$$h^{ij} \propto \ddot{Q}^{ij}\,, \qquad \ddot{Q}^{ij} = 2(\lambda^i\lambda^j - n^i n^j)$$

$$\hat{\boldsymbol{n}}(t) = \boldsymbol{e}_1(t)\,\cos\Phi(t) + \boldsymbol{e}_2(t)\,\sin\Phi(t)$$
$$\hat{\boldsymbol{\lambda}}(t) = -\boldsymbol{e}_1(t)\,\sin\Phi(t) + \boldsymbol{e}_2(t)\,\cos\Phi(t)$$

- **adiabatic sequence of spherical orbits** $\Rightarrow \dot{\hat{\boldsymbol{n}}} = \omega\hat{\boldsymbol{\lambda}}$ **but in general** $\dot{\Phi} \neq \omega$
- ω **almost non-modulated**

Precessing convention: $\dot{e}_i(t) = \Omega_e(t) \times e_i(t)$ **such that** $\dot{\Phi} = \omega$ **!**

$$\ddot{Q}^{ij} \propto \left[\boldsymbol{e}_+^{ij}(t)\,\cos 2(\Phi(t) + \Phi_0) + \boldsymbol{e}_\times^{ij}(t)\,\sin 2(\Phi(t) + \Phi_0)\right]$$

Gravity-wave in precessing convention (continued)

Clean separation of time-dependent and time-independent parameters in waveforms

$$h_{\rm GW}(t) = \underbrace{-\frac{2\mu}{D}\frac{M}{r(t)}\left[\boldsymbol{e}_+^{ij}(t)\,\cos 2(\Phi(t)+\Phi_0) + \boldsymbol{e}_\times^{ij}(t)\,\sin 2(\Phi(t)+\Phi_0)\right]}_{\text{Q(t): wave generation}} \times$$

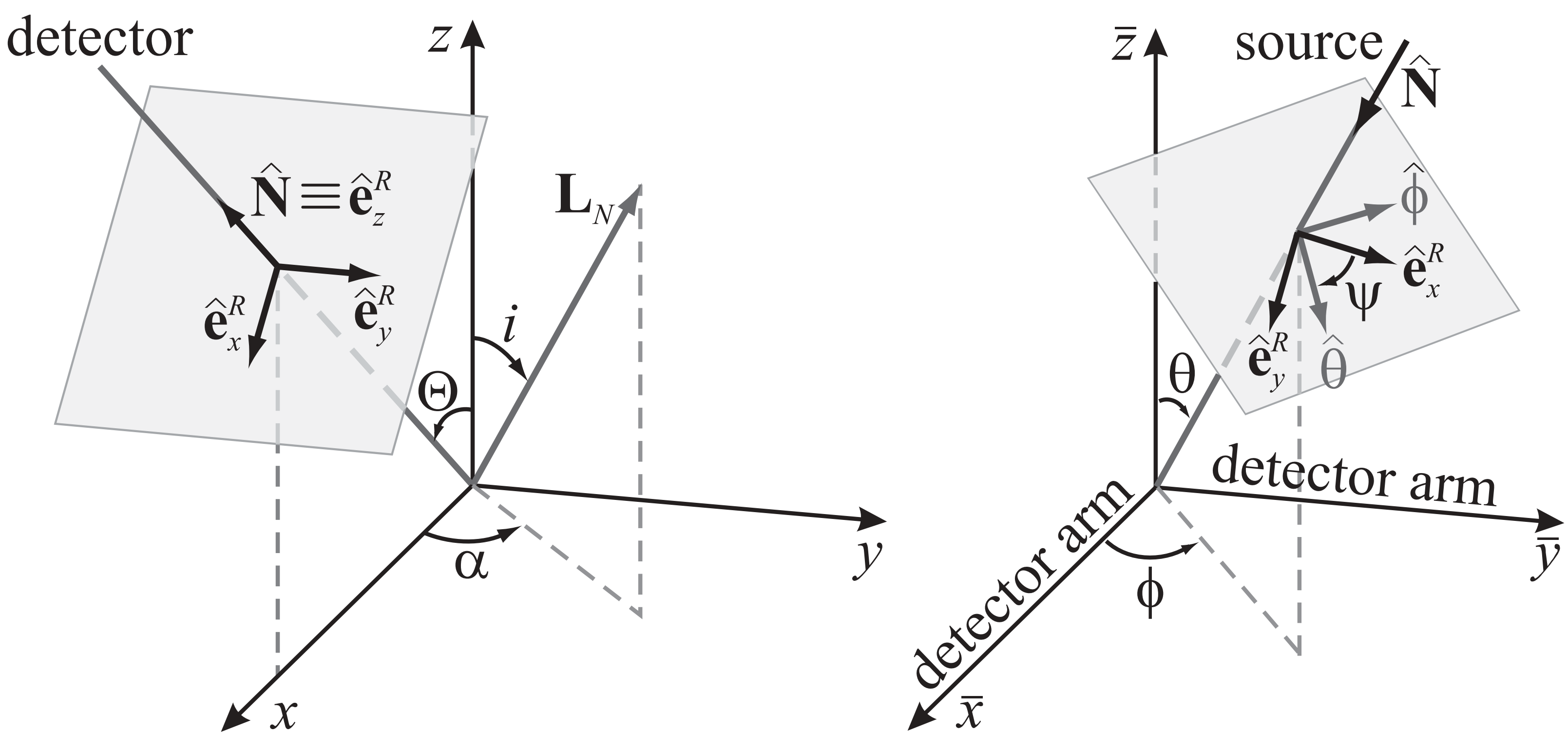
$$\underbrace{[T_{+\,ij}(\Theta,\varphi)\,F_+(\theta,\phi,\psi) + T_{\times\,ij}(\Theta,\varphi)\,F_\times(\theta,\phi,\psi)]}_{\text{P: detector projection}}$$

$$\tilde{h}_{\rm SPA}(f) = -\tilde{h}_{\rm non\,mod}(f)\left([\mathbf{e}_+(t_f)]^{jk} + i\,[\mathbf{e}_\times(t_f)]^{jk}\right)\,([\mathbf{T}_+]_{jk}\,F_+ + [\mathbf{T}_\times]_{jk}\,F_\times)$$

$$\sim f^{-7/6}\,e^{i\psi_{\rm non\,mod}(f)}\left[\mathcal{C}_0^{ij} + \mathcal{C}_1^{ij}\,\cos(\delta^{ij} + \mathcal{B}f^{-p})\right]\,([\mathbf{T}_+]_{jk}\,F_+ + [\mathbf{T}_\times]_{jk}\,F_\times)$$

Binary-detector orientation

$$\mathbf{T}_+ \equiv \boldsymbol{e}^R_x \otimes \boldsymbol{e}^R_x - \boldsymbol{e}^R_y \otimes \boldsymbol{e}^R_y \quad \text{and} \quad \mathbf{T}_\times \equiv \boldsymbol{e}^R_x \otimes \boldsymbol{e}^R_y + \boldsymbol{e}^R_y \otimes \boldsymbol{e}^R_x$$



Phenomenological frequency-domain template family

[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri (to appear)]

$$h(f) = \mathcal{A}(f)\, e^{i\psi(f)}$$

$$\mathcal{A}(f) = f^{-7/6}\,\theta(f_{\rm cut} - f)\times$$

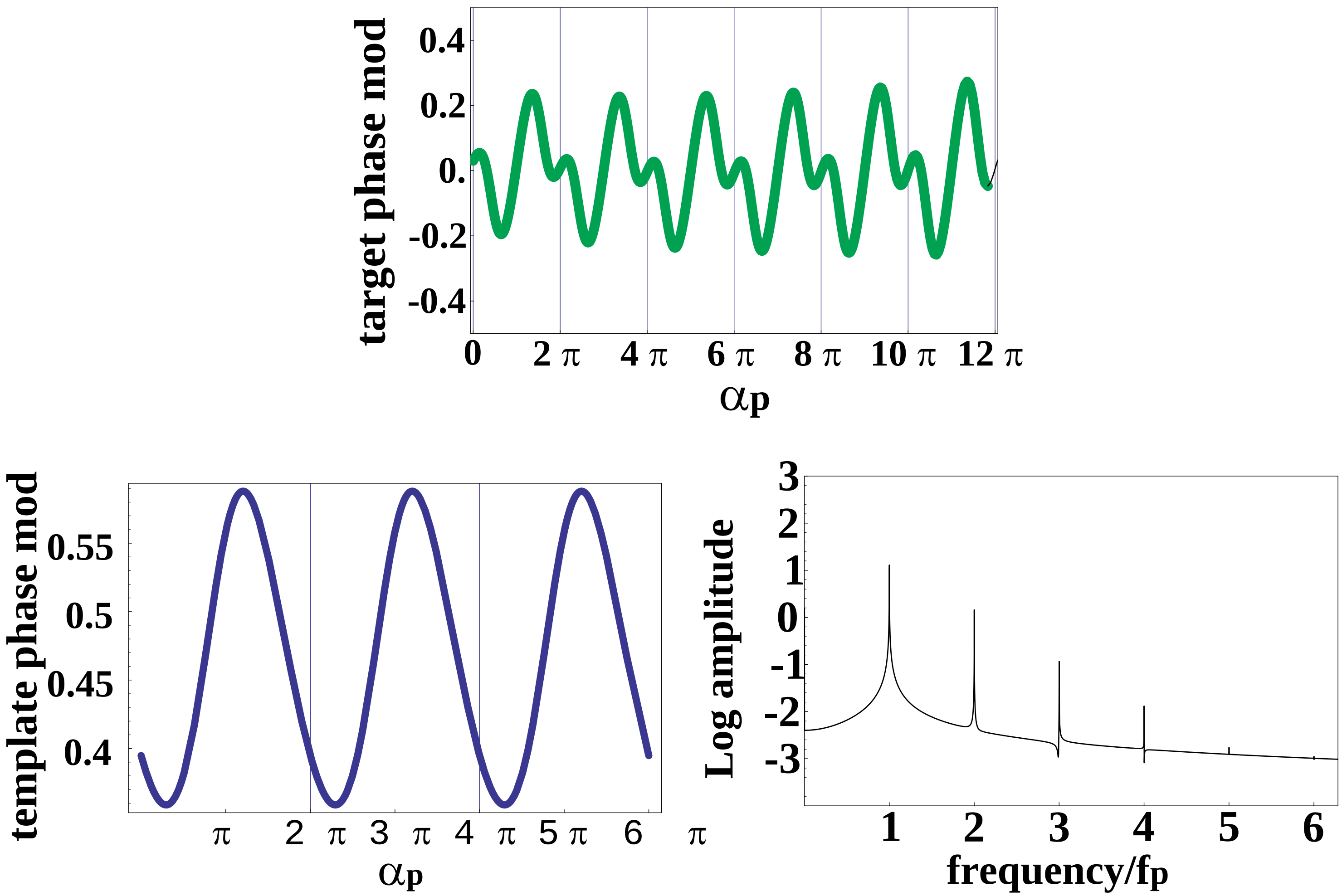
$$[(\mathcal{C}_1 + i\mathcal{C}_2) + (\mathcal{C}_3 + i\mathcal{C}_4)\,\cos\left(\mathcal{B} f^{-p}\right) + (\mathcal{C}_5 + i\mathcal{C}_6)\,\sin\left(\mathcal{B} f^{-p}\right)]$$

$$\psi(f) = f^{-5/3}\,\left(\psi_0 + \psi_{1/2}\, f^{1/3} + \psi_1\, f^{2/3} + \psi_{3/2}\, f + \cdots\right) + 2\pi f\, t_0$$

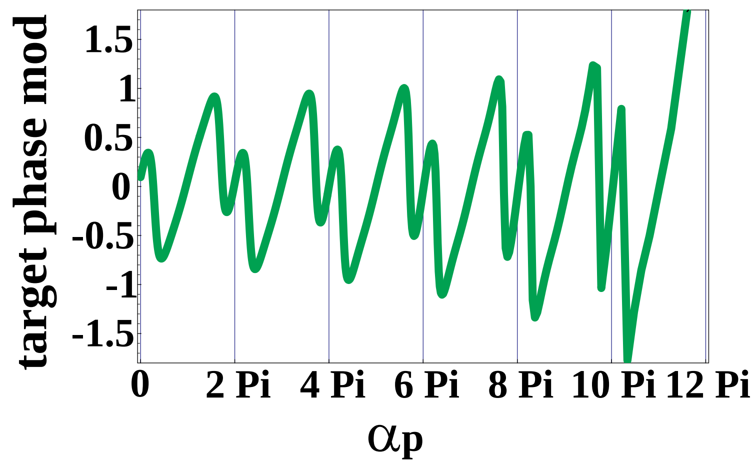
To get high signal matches, sufficient to set $\psi_0, \psi_{3/2} \neq 0$ and all $\psi_i = 0$

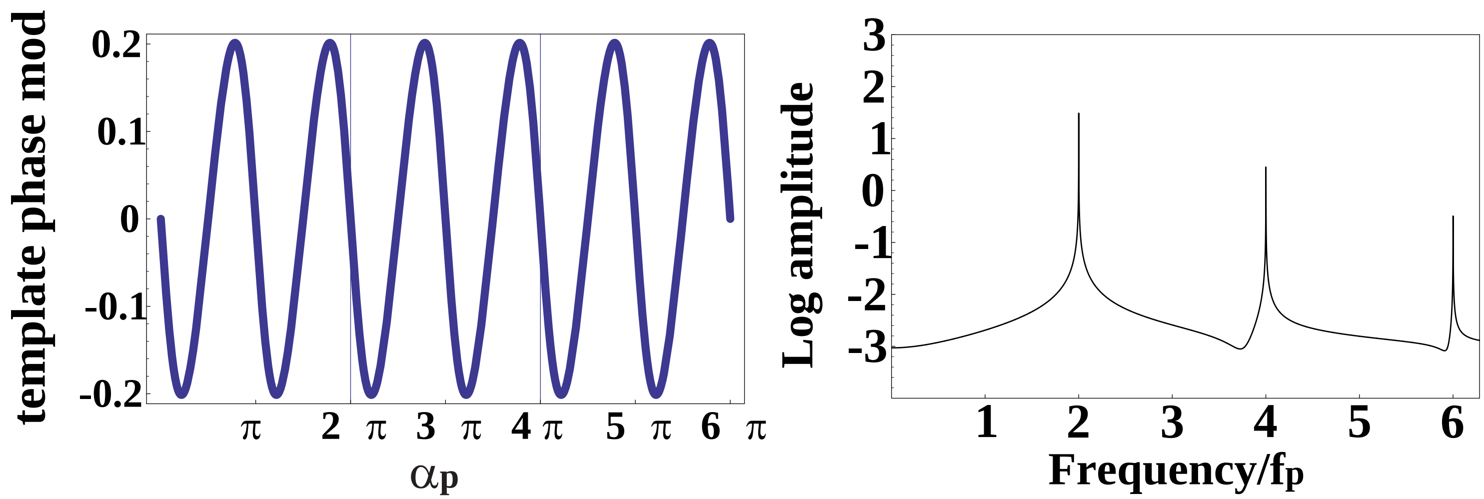
- 3 intrinsic parameters $(\psi_0, \psi_{3/2}, \mathcal{B})$ + $f_{\rm cut}$ and 7 extrinsic parameters $(\mathcal{C}_1, \mathcal{C}_2, \ldots)$
- Fast search on the seven extrinsic parameters $\Rightarrow$ low computational cost

Phase modulations in target and template



Phase modulations in target and template (continued)





Performances for high mass and small mass-ratio binaries

Averaging over uniform distribution of $S_{1,2}$ and L directions

	High-mass binaries				
	$(7+5)M_\odot$ $\overline{\mathrm{FF}}$	$(10+10)M_\odot$ $\overline{\mathrm{FF}}$	$(15+15)M_\odot$ $\overline{\mathrm{FF}}$	$(20+5)M_\odot$ $\overline{\mathrm{FF}}$	$(20+10)M_\odot$ $\overline{\mathrm{FF}}$
non-mod SPA	0.903	0.894	0.811	0.858	0.826
$(\psi_0\psi_{3/2}\alpha f_{\mathrm{cut}})$	0.929	0.948	0.955	0.899	0.942
$(\psi_0\psi_{3/2}\mathcal{B} f_{\mathrm{cut}})$	0.975	0.986	0.986	0.974	0.984

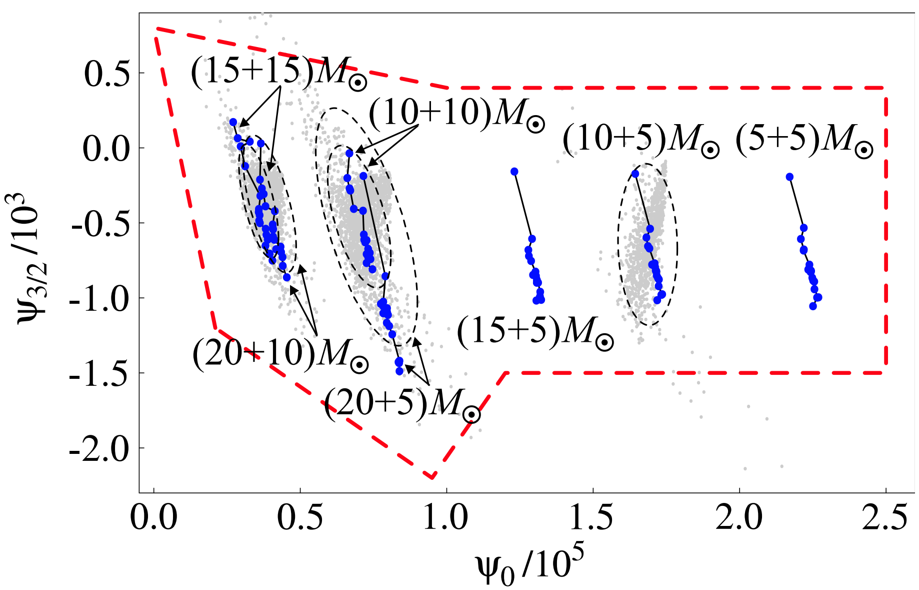
Increase of overlap can be lower than increase in threshold but templates are closer to signal

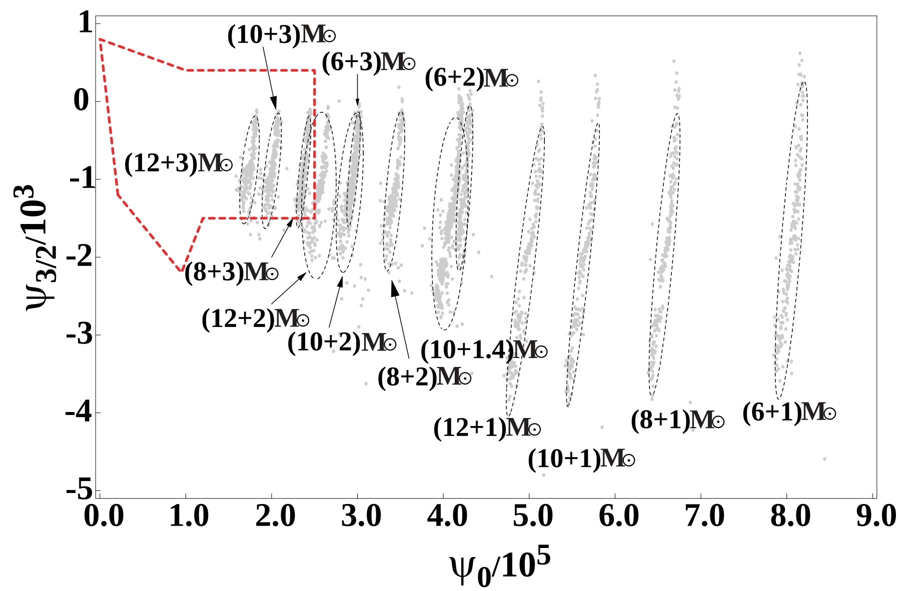
	Small mass-ratio and equal–low-mass binaries					
	$(10+1.4)M_\odot$ $\overline{\mathrm{FF}}$	$(12+1)M_\odot$ $\overline{\mathrm{FF}}$	$(12+2)M_\odot$ $\overline{\mathrm{FF}}$	$(6+3)M_\odot$ $\overline{\mathrm{FF}}$	$(6+1)M_\odot$ $\overline{\mathrm{FF}}$	$(2+2)M_\odot$ $\overline{\mathrm{FF}}$
non-mod SPA	0.780	-	-	-	-	-
$(\psi_0\psi_{3/2}\mathcal{B})$	0.933	0.932	0.960	0.975	0.937	0.964

[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri (to appear)]

Template space

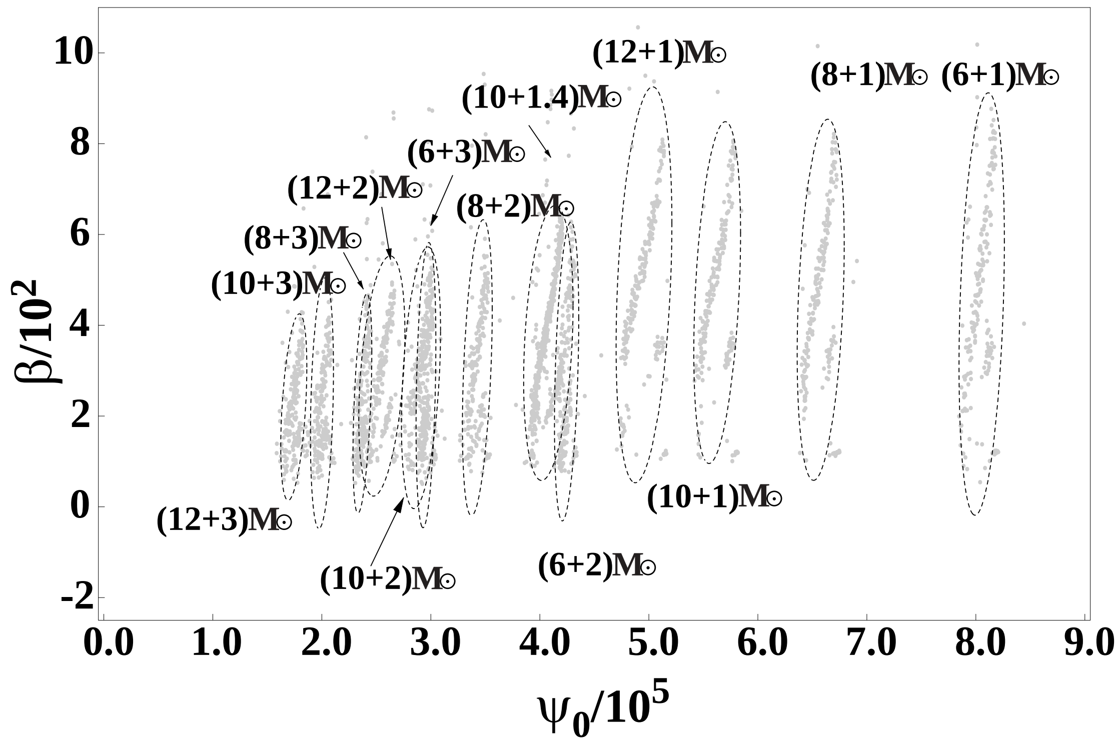
[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri (to appear)]





Template space

[AB, Chen & Vallisneri 02; AB, Chen, Pan & Vallisneri (to appear)]



Mismatch template metric and projected metric

[Balasubramanian, Dhurandhar, Sathyaprakash 91, 94, 96; Owen 96]

- $1 - \langle \hat{h}(\lambda^A), \hat{h}(\lambda^A + \Delta\lambda^A)\rangle \equiv \delta[\lambda^A, \lambda^A + \Delta\lambda^A] = g_{BC}\Delta\lambda^B\Delta\lambda^C$

$$g_{BC} = -\frac{1}{2}\frac{\partial^2 \left\langle \hat{h}(\lambda^A), \hat{h}(\lambda^A + \Delta\lambda^A)\right\rangle}{\partial(\Delta\lambda^B)\partial(\Delta\lambda^C)}$$

[Pan, AB, Chen & Vallisneri 04; AB, Chen, Pan & Vallisneri + Tagoshi (to appear)]

$X^\alpha \Rightarrow$ **intrinsic parameters** $(\psi_0, \psi_3, \mathcal{B})$, $\Xi^\alpha \Rightarrow$ **extrinsic parameters** $(\mathcal{C}_i, t_0)$

- $\delta(X^i, \Xi^\alpha; X^i + \Delta X^i, \Xi^\alpha + \Delta\Xi^\alpha) = \begin{pmatrix} \Delta X^i & \Delta\Xi^\alpha \end{pmatrix} \begin{pmatrix} G_{ij} & C_{i\beta} \\ C_{\alpha j} & \gamma_{\alpha\beta} \end{pmatrix} \begin{pmatrix} \Delta X^j \\ \Delta\Xi^\beta \end{pmatrix}$

- $\min_{\Delta\Xi^\alpha} \delta(X^i, \Xi^\alpha; X^i + \Delta X^i, \Xi^\alpha + \Delta\Xi^\alpha) = \left[G_{ij} - C_{i\alpha}(\gamma^{-1})^{\alpha\beta}C_{\beta j}\right]\Delta X^i \Delta X^j$

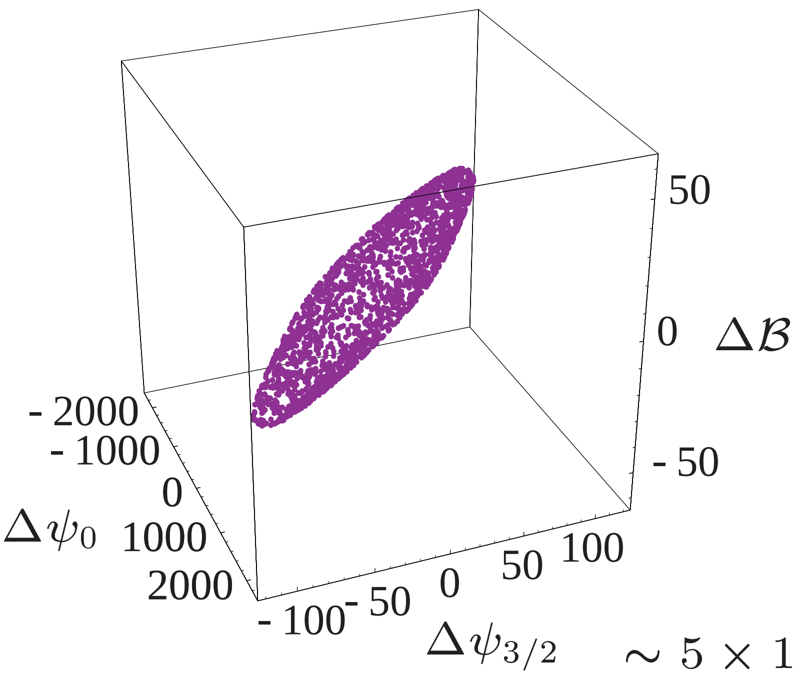
  $\equiv g^{\rm proj}_{ij}\Delta X^i \Delta X^j$ $\qquad g^{\rm proj}_{ij}$ **depends only on** $\mathcal{B}$ **and** $\mathcal{C}_i$

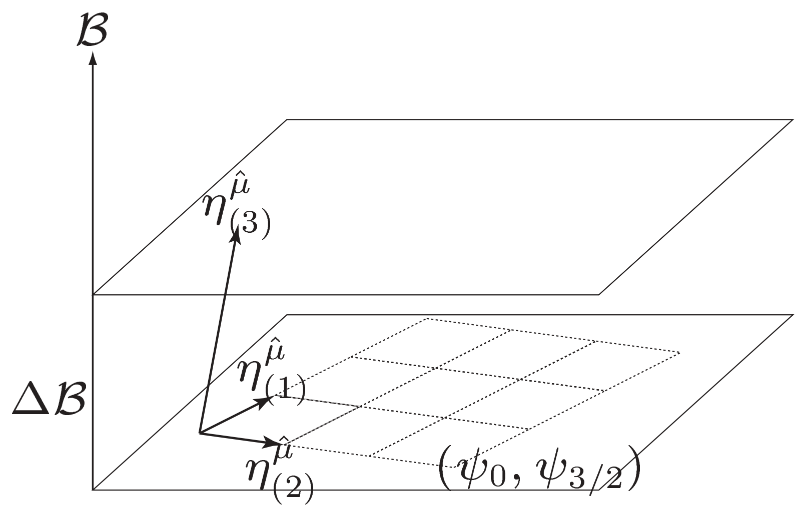
Projected metric (continued)

[Damour, Iyer and Sathyaprakash 98]

[Pan, AB, Chen & Vallisneri 04; AB, Chen, Pan & Vallisneri + Tagoshi (to appear)]

$$\max_{\Xi^\alpha} \min_{\Delta\Xi^\alpha} \delta(X^i, \Xi^\alpha; X^i + \Delta X^i, \Xi^\alpha + \Delta\Xi^\alpha) \simeq \hat{g}^{\rm proj}_{ij}(\mathcal{B})\Delta X^i \Delta X^j = 1 - {\rm MM}$$





$\sim 5 \times 10^5$ **templates for** $m_1 = 6$–$12 M_\odot$ **and** $m_2 = 1$–$3 M_\odot$

Quasi-physical frequency-domain template family

[AB, Chen, Pan & Vallisneri (to appear)]

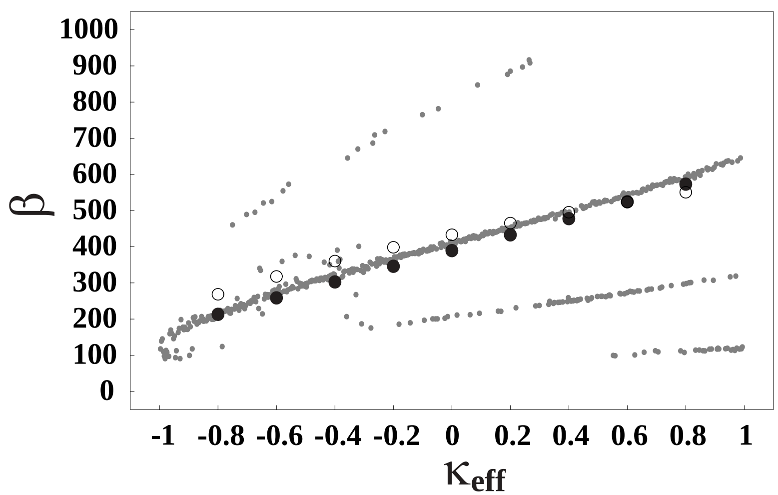
- **The three** *phenomenological* **parameters** $(\psi_0, \psi_{3/2}, \mathcal{B})$ **traded with four** *physical* **parameters** (M, η, κ, χ)
- $Q_c^{ij} \sim \left[\mathcal{C}_0^{ij} + \cos(\delta^{ij} + \alpha^J_{\rm prec}(M, \eta, \kappa, \chi))\, \mathcal{C}_1^{ij}\right] \; f^{-7/6}\, e^{i\psi_{\rm SPA}(M,\eta,\kappa,\chi)}$
- $\alpha^J_{\rm prec}(M, \eta, \kappa, \chi)$ **known analytically !**

  $\alpha^J_{\rm prec} \sim \mathcal{B}\, f^{-p}$

– **The template metric** *is not* **analytical**

– **4D template space**

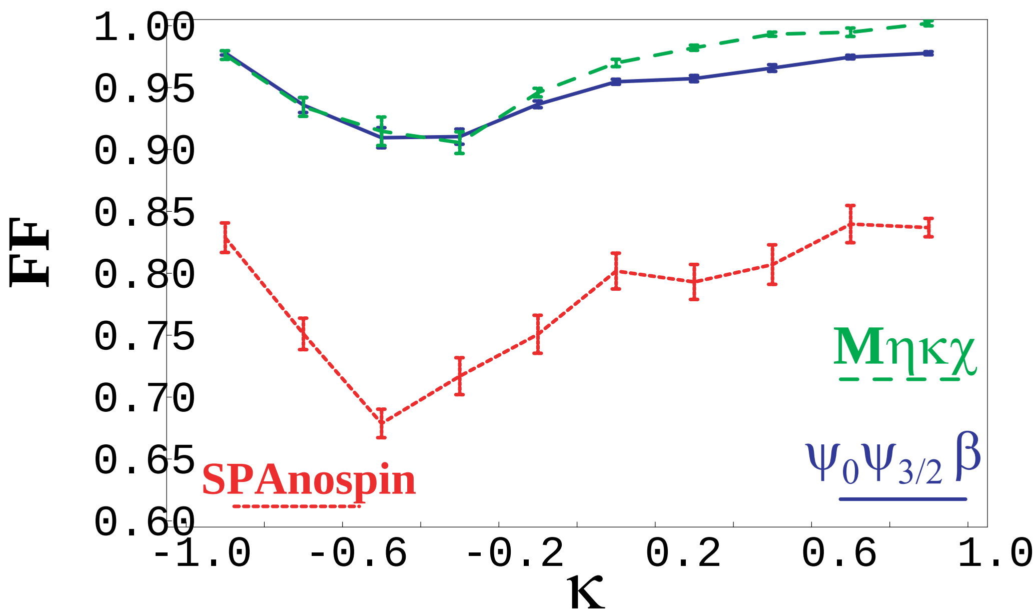
– **to be used after detection to estimate binary parameters**



Systematic errors: $\Delta\mathcal{M}/\mathcal{M} \sim 1\%,\ \Delta M/M, \Delta\eta/\eta \sim 10\%, \Delta\chi/\chi \sim 20\%, \Delta\kappa \sim 2\%$

Performances for NS/BH binary $(10 + 1.4)M_\odot$

[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri (to appear)]



Physical time-domain templates for single-spin precessing binaries

[Pan, AB, Chen & Vallisneri 04]

$$h_{\rm GW}(t) = -\frac{2\mu}{D}\frac{M}{r}\underbrace{\left[\boldsymbol{e}_{+}^{ij}(t)\,\cos 2(\Phi(t)+\Phi_0) + \boldsymbol{e}_{\times}^{ij}(t)\,\sin 2(\Phi(t)+\Phi_0)\right]}_{\rm Q_{ij}(t):\,wave\,generation}\times$$

$$\underbrace{[T_{+\,ij}(\Theta,\varphi)\,F_{+}(\theta,\phi,\psi) + T_{\times\,ij}(\Theta,\varphi)\,F_{\times}(\theta,\phi,\psi)]}_{\rm P_{ij}:\,detector\,projection}$$

Parameters in $\mathrm{Q}_{ij}(t)$**:** $(m_1, m_2, S_1/m_1^2, \boldsymbol{L}\cdot\boldsymbol{S}_1; t_0, \Phi_0)$ $Q_{ij}, P_{ij} \Rightarrow$ **3D STF tensors**

Parameters in P_{ij}**:** $(\Theta, \varphi, \alpha = \arctan(F_+/F_\times))$ $P_I \Rightarrow$ **the 5 components are constrained**

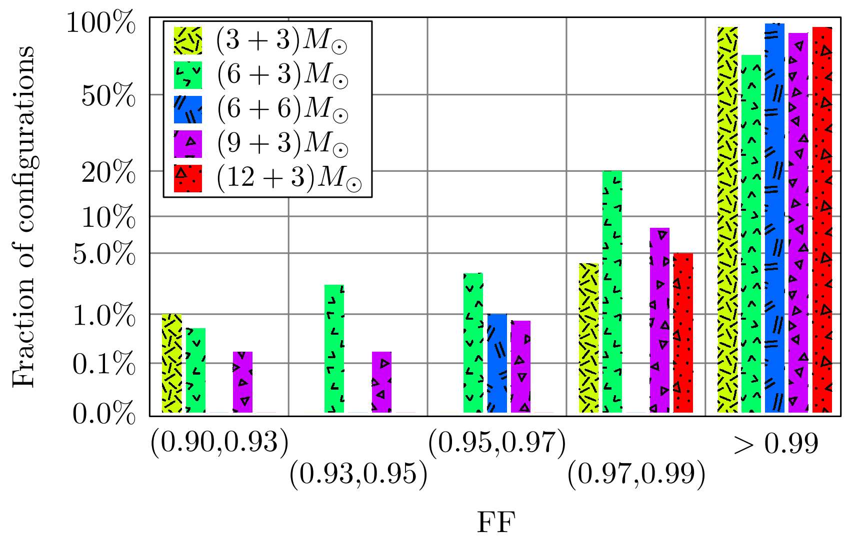
- **Analytic maximization over** Φ_0, α **and numerical maximization over** (Θ, φ)
- **Two-stage detection scheme using first the fully algebraic unconstrained statistics**
- **Projected and reduced metric:**
  - **one very small eigenvalue** $\Rightarrow$ **reduction 4D to 3D!**
  - $\sim 2\times 10^5$ **templates for** $m_1 = 7\text{–}12 M_\odot$ **and** $m_2 = 1\text{–}3 M_\odot$

Physical time-domain templates for double-spin precessing binaries

[Apostolatos et al. 94, 96; AB, Chen, Pan & Vallisneri 04]

- **Equal-mass binaries without SS and small mass-ratio binaries described by single spin**
- **What about low total masses and intermediate mass-ratio binaries with SS?**
- **Spin-spin and mass difference modify S and L monotonic evolution by superimposing oscillations**

  $\Rightarrow$ **Single effective-spin description**



$$\dot{\boldsymbol{S}}_{1s} = \frac{\eta_s (M_s\omega)^{5/3}}{2M_s}\left(4 + \frac{3m_{2s}}{m_{1s}}\right)\hat{\boldsymbol{L}} \times \boldsymbol{S}_{1s}$$

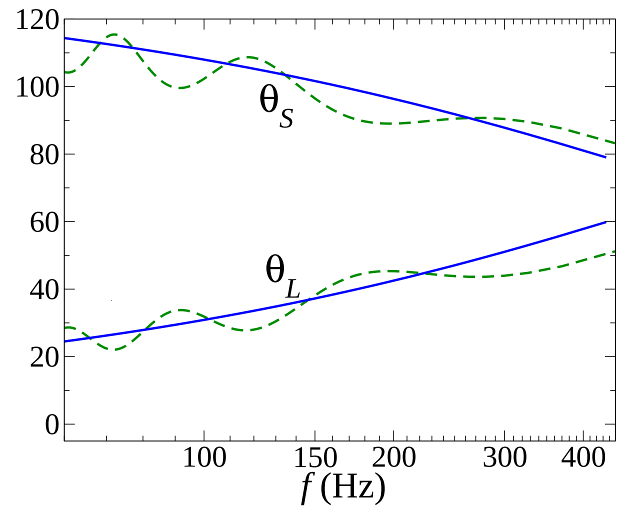
$$\dot{\hat{\boldsymbol{L}}} = \frac{\omega^2}{2M_s}\left(4 + \frac{3m_{2s}}{m_{1s}}\right)\boldsymbol{S}_{1s} \times \hat{\boldsymbol{L}}$$

$$\dot{\omega} = \dot{\omega}(\omega, \boldsymbol{S}_{1s}\cdot\boldsymbol{L}, m_{1s}, m_{2s})$$

Physical template family (continued)

$$(m_1 + m_2) = (6 + 3) M_\odot$$

$$(\theta_S)_{\rm targ} = \arccos(\widehat{\boldsymbol{S}}_{\rm tot} \cdot \widehat{\boldsymbol{J}})$$
$$(\theta_S)_{\rm templ} = \arccos(\widehat{\boldsymbol{S}}_{1s} \cdot \widehat{\boldsymbol{J}})$$
$$\theta_L = \arccos(\widehat{\boldsymbol{L}} \cdot \widehat{\boldsymbol{J}})$$

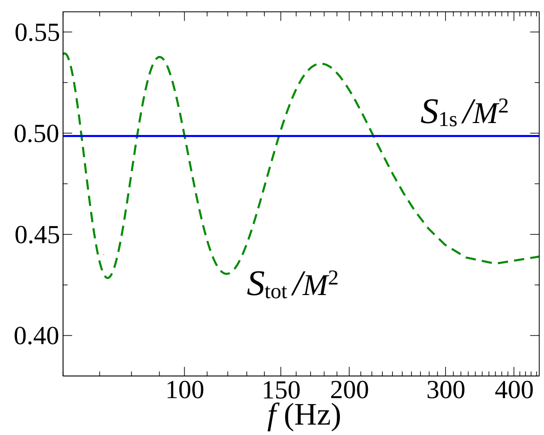


Projected and reduced metric:

$\sim 10^5$ **templates for**

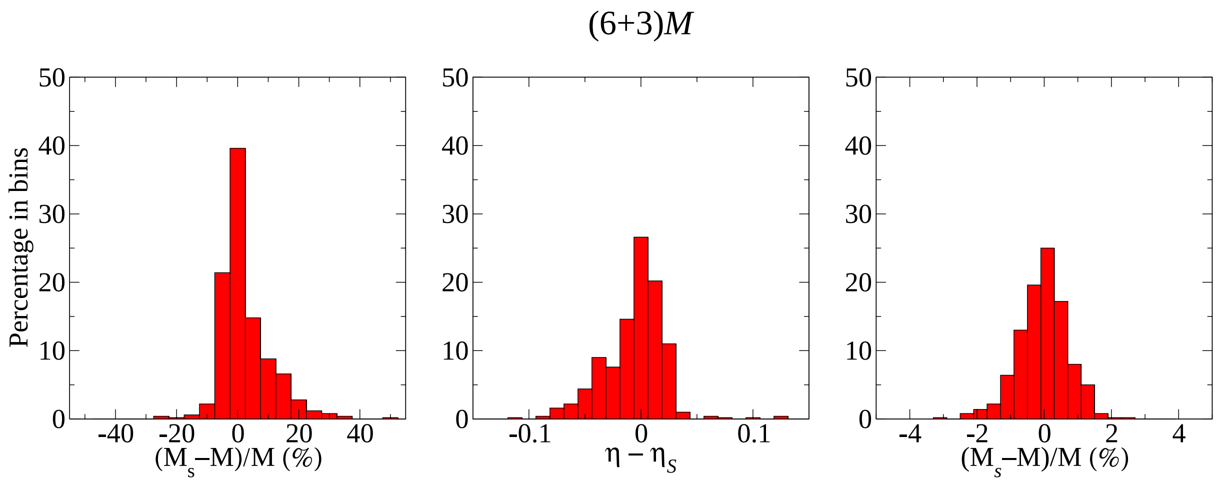
$m_1 = 3$–$15 M_\odot$ **and**

$m_2 = 3$–$15 M_\odot$

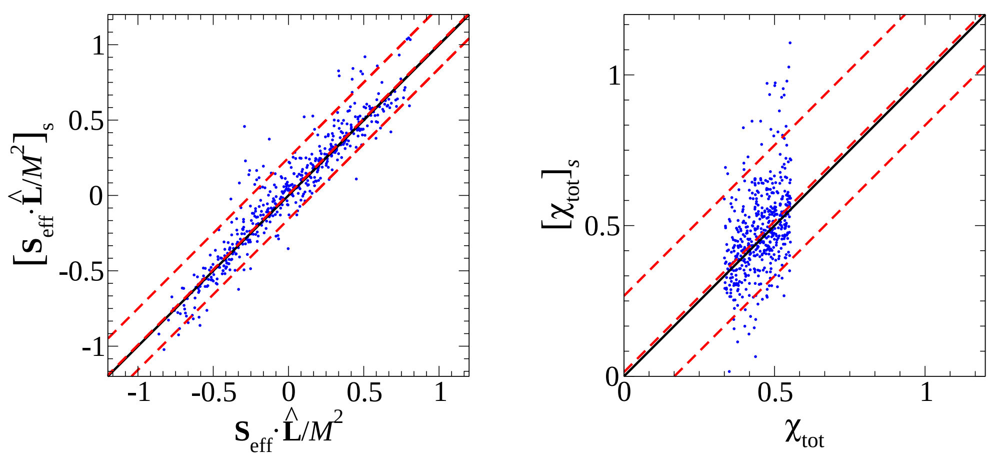


Physical template family (continued)

- **Systematic biased for mass parameters**



- **Estimation of spin-related parameters**



Going beyond the adiabatic approximation

Hamiltonian framework: $H(\boldsymbol{R}, \boldsymbol{P}, \boldsymbol{S}_1, \boldsymbol{S}_2), \quad \dot{\boldsymbol{S}}_{1,2} = \{\boldsymbol{S}_{1,2}, H\}, \quad \dot{\boldsymbol{L}}_i = \{\boldsymbol{L}_i, H\}$

[Barker & O'Connell 71; Damour & Schaefer 88]

- **Effective-one-body resummation of the conservative two-body dynamics to speed up the convergence of PN expansion** [AB & Damour 99, 00; Damour 01]

- **Padé resummation of the GW flux**

  [Damour, Iyer & Sathyaprakash 98; Porter & Sathyaprakash 04; AB, Chen & Damour]

- **Radiation-reaction force including spin couplings with matches known rates of energy and angular momentum loss for quasi-adiabatic orbits**

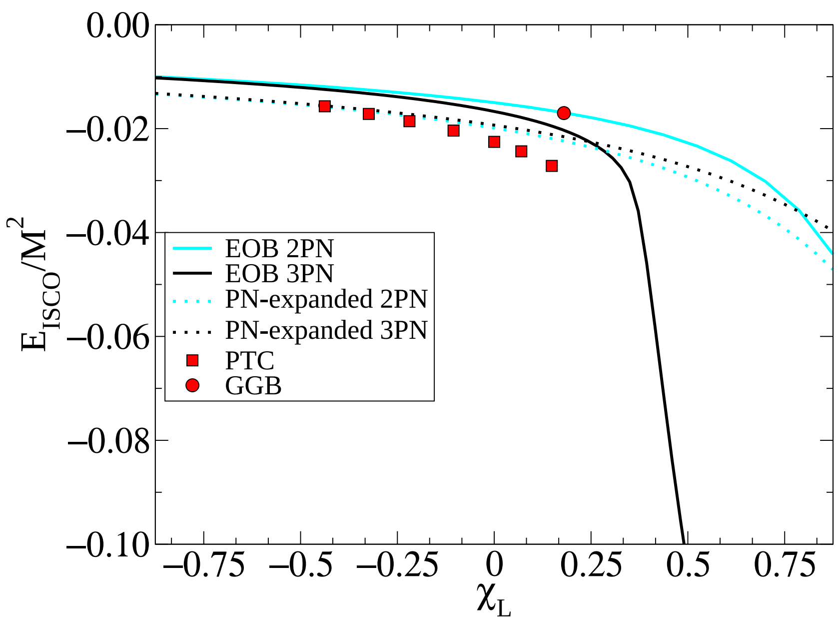
  [AB, Chen & Damour (to appear)]

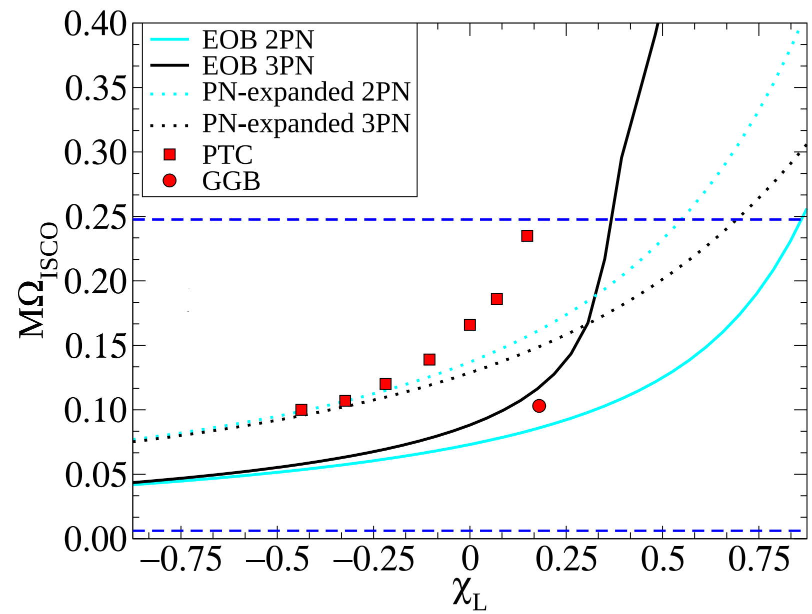
$$F_i = \frac{1}{\omega\,|\boldsymbol{L}|}\,\frac{dE}{dt}\,P_i + \frac{8}{15}\,\eta^2\,\frac{v^8}{R\boldsymbol{L}^2}\left\{\left(61 + 48\,\frac{m_2}{m_1}\right)\boldsymbol{P}\cdot\boldsymbol{S}_1 + \left(61 + 48\,\frac{m_1}{m_2}\right)\boldsymbol{P}\cdot\boldsymbol{S}_2\right\} L_i$$

Comparing ISCO predictions for energy and frequency

Equal-mass and equal-spin binaries

[AB, Chen & Damour (to appear)]

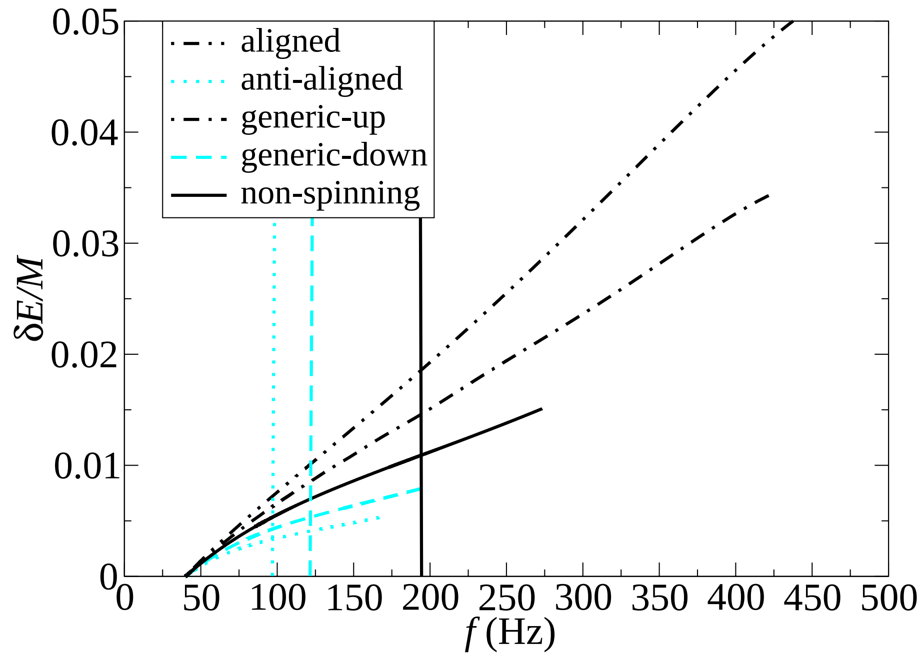




For spins aligned with angular momentum $\Rightarrow$ non-linear effects dominate $\Rightarrow$ predictions differ, but for LIGO this would affect *only* binaries with mass $\gtrsim 40 M_\odot$

Energy and angular-momentum release during inspiral and plunge

- **Maximal spins and $(15 + 15) M_\odot$** [AB, Chen & Damour (to appear)]
- **Energy release before $40\,\mathrm{Hz}$ is $\sim 0.008/M$**



Rotation parameter J/E^2 smaller than one at the end of inspiral

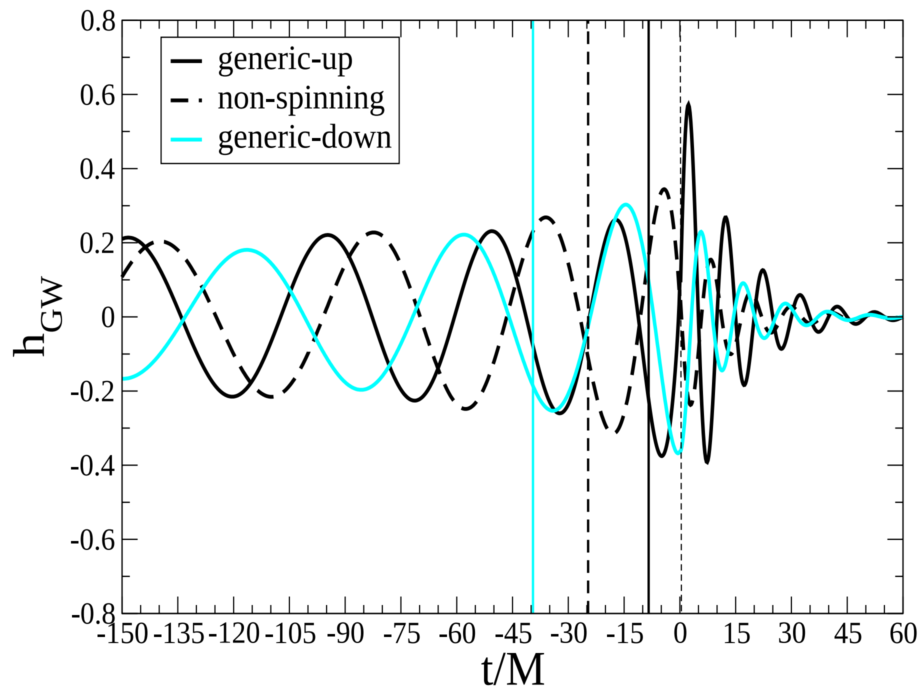
$\Rightarrow$ Kerr black hole could already form

Gravity-wave signal from inspiral-plunge(–ring-down)

- $\chi_1 = \chi_2 = 0.5$
  and $(15 + 15)M_\odot$

`[AB, Chen & Damour (to appear)]`

- Comparison with `Baker et al. 04`



- **More study to match end of plunge with ring-down for larger spins**
- **Evaluate match performances of adiabatic templates against EOB waveforms**

Parameter estimation: non-precessing versus precessing binaries

- For aligned/antialigned configurations, the inclusion of spin-orbit and SS couplings degrade parameter estimation [Poisson & Will 95; Krolak et al. 95; Berti, AB & Will 03]
- Addition of higly-correlated parameters dilute available information
- Precession helps in decorrelating the parameters [Vecchio 03]
- However, there are very small eigenvalues $\Rightarrow$ some parameters do not change significantly the waveform $\Rightarrow$ only a reduced set of parameters can be estimated

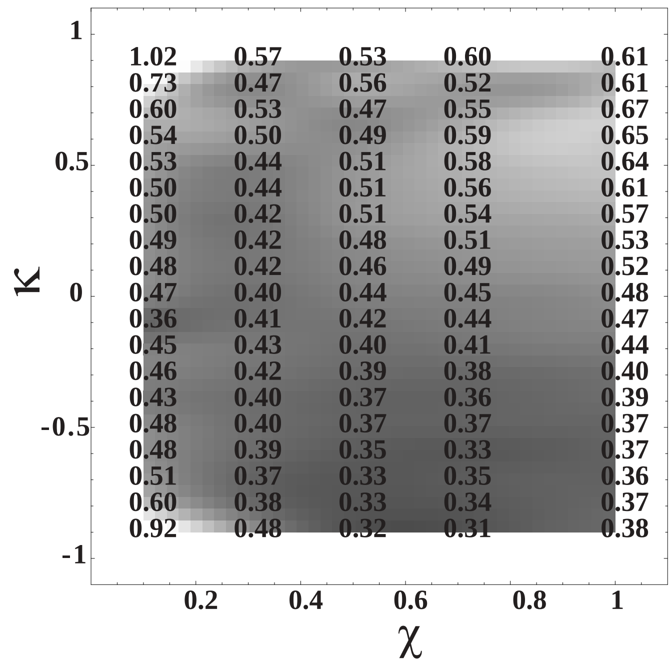
  [Schutz and Sathyaprakash 01]

  [AB, Chen, Pan & Vallisneri (work in progress)]

  - $(10 + 1.4)M_\odot$

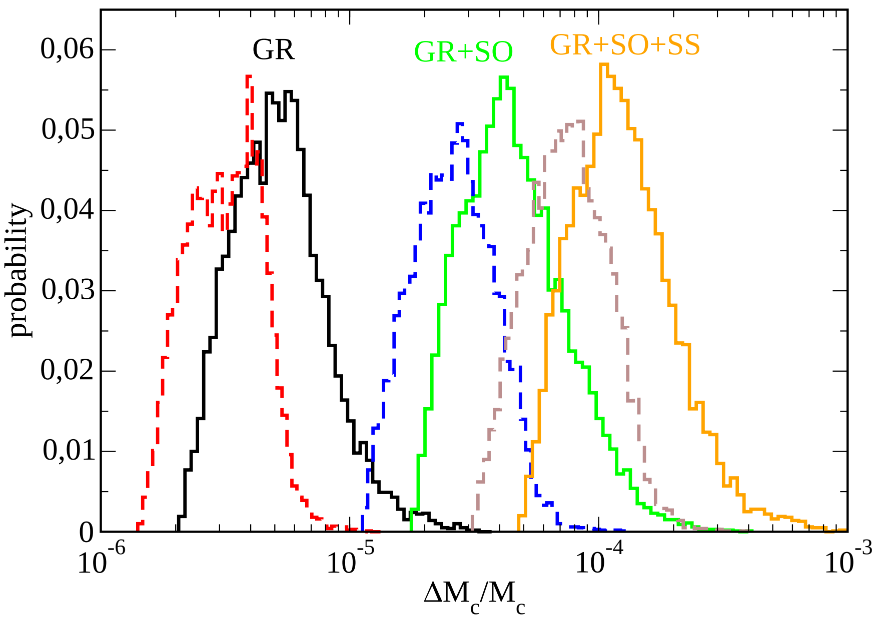
No spins: $\Delta\mathcal{M}/\mathcal{M} = 0.002 \quad \Delta\eta/\eta = 0.04$

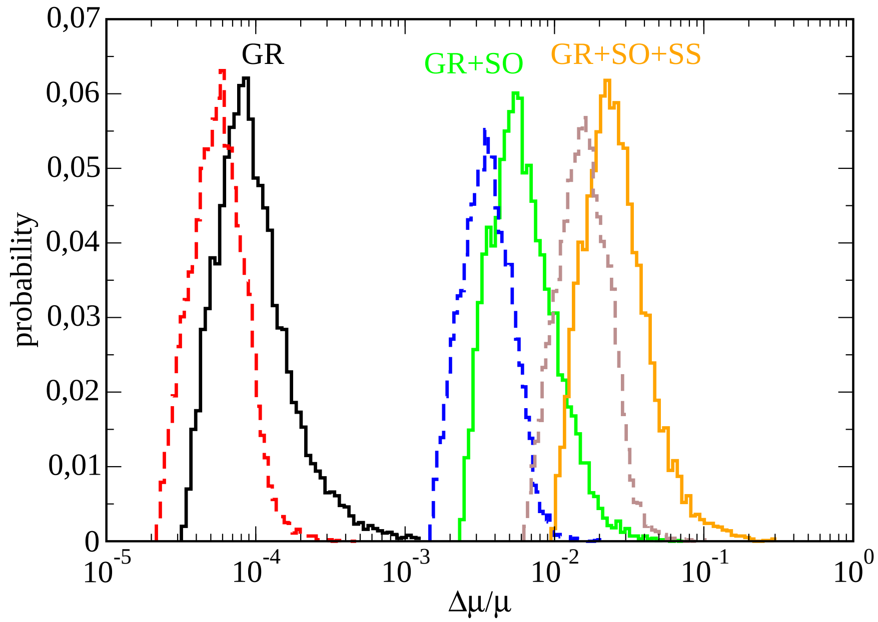
With spins aligned: $\Delta\mathcal{M}/\mathcal{M} = 0.04 \quad \Delta\eta/\eta = 4.9$



Results in Einstein theory when including spin couplings

$$M = (10^6 + 10^6)M_\odot \text{ at } 3 \text{ Gpc}$$





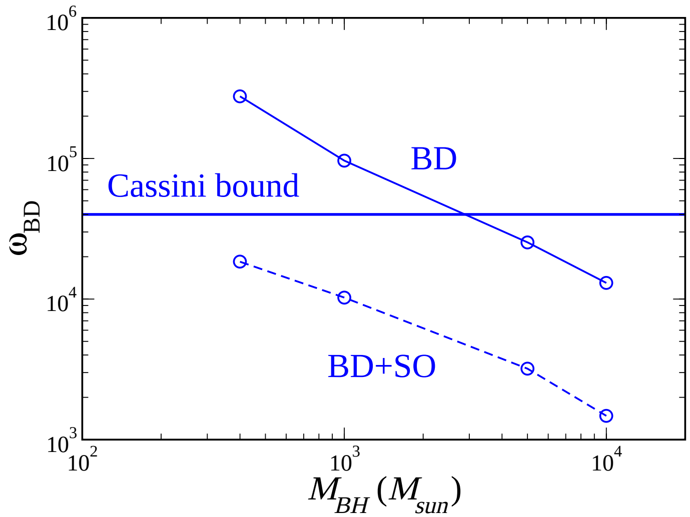
[Berti, AB & Will 04]

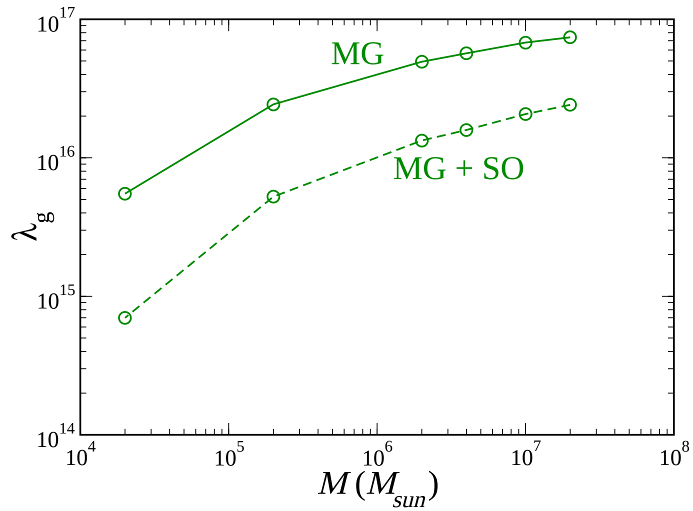
Testing Einstein general relativity

[Will 94; Krolak et al. 95; Will 98; Scharre & Will 02; Will & Yunes 04, Berti, AB & Will 05]

- **Scalar-tensor theories: phasing modified by GW dipole radiation**
- **Massive graviton theories: GW–propagation-speed depends on wavelength $\Rightarrow$ distortion in time of arrival with respect to GR**

$$\dot{\omega} = \frac{96}{5\mathcal{M}^2}\,(\mathcal{M}\omega)^{11/3}\left\{1 + \frac{5\,\hat{\alpha}^2\,\eta^{2/5}}{192\,\omega_{\rm BD}}\,(\mathcal{M}\,\omega)^{-2/3} + \frac{96\,\pi^2\,\mathcal{M}\,D}{5\,(1+z)\,\lambda_g^2}\,(\mathcal{M}\omega)^{2/3} + \text{PN corr.}\right\}$$





Effect of systematics: how well do we know the signal?

$M = (10^6 + 10^6) M_\odot$ **at** 3 **Gpc**		$M = (10^6 + 10^5) M_\odot$ **at** 3 **Gpc**
$f_{\rm in} = 4.5 \times 10^{-5}$ Hz; $f_{\rm fin} = 2.2 \times 10^{-3}$ Hz		$f_{\rm in} = 9.9 \times 10^{-5}$ Hz; $f_{\rm fin} = 4.0 \times 10^{-3}$ Hz

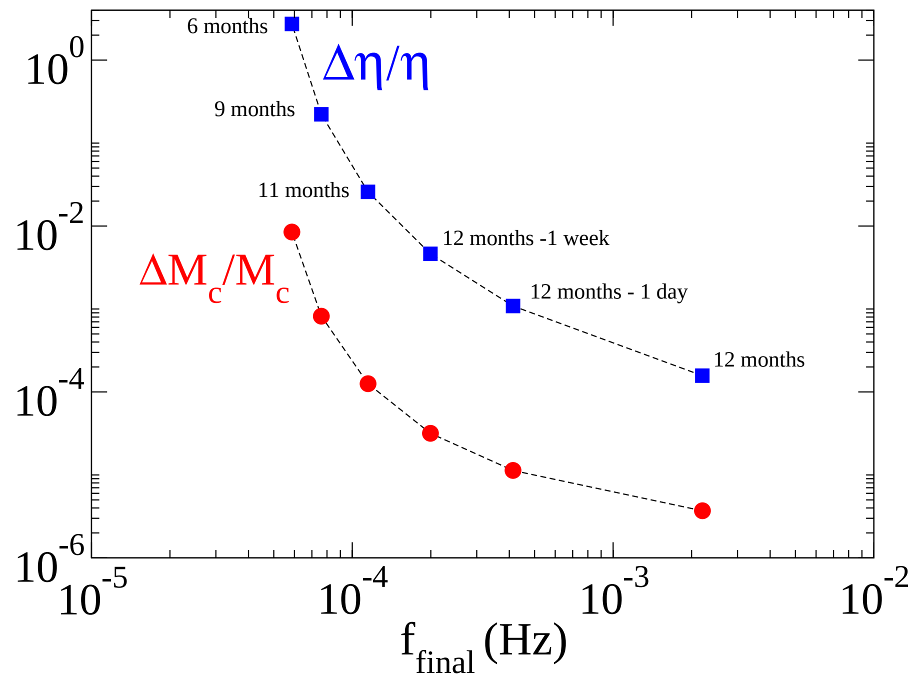
	Number of cycles	Number of cycles:
Newtonian:	2266	4986
1PN:	134	281
1.5PN	-92	-243
Spin-orbit:	$\pm 29\chi_1 \pm 29\chi_2$	$\pm 161\chi_1 \pm 12\chi_2$
2PN	6	12
Spin-spin:	$2\chi_1^2 \pm 3\chi_1\,\chi_2 + 2\chi_2^2$	$15\chi_1^2 \pm 3\chi_1\,\chi_2 + 0.1\chi_2^2$
2.5PN	$-9\,[\pm 5\chi_1 \pm 5\chi_2]$	$-27\,[\pm 29\chi_1 \pm 2\chi_2]$
3PN:	$2\,[-6, 8]$	$2\,[-18, 31]$
3.5PN:	$-1\,[-3, 3]$	$-2\,[-13, 7]$

Statistical errors as function of observation time

Observing for 11 months

$M = (10^6 + 10^6) M_\odot$ **at** 3 **Gpc**

	Number of cycles
Newtonian:	1790
1PN:	83
1.5PN	-46
Spin-orbit:	$\pm 14\chi_1 \pm 14\chi_2$
2PN	2
Spin-spin:	$0.6\chi_1^2 \pm 1.2\chi_1\,\chi_2 + 0.6\chi_2^2$
2.5PN	$-2.2\ [\pm 1.3\chi_1 \pm 1.3\chi_2]$
3PN:	$0.2\ [-0.9, 1.1]$
3.5PN:	$-0.1\ [-0.1, 0.2]$



Higher order spin couplings using PN expansion

- **Spin-orbit effects at 2.5PN order in equations of motion and at 2PN order in GW amplitude** [Owen, Tagoshi & Ohashi 98, Tagoshi, Ohashi & Owen 01]
- **Spin-orbit effects at 2.5PN order in GW phase evolution**

[Blanchet, AB, Faye & Nissanke(work in progress)]

Determination of source multipole moments I_L and J_L:

$$I_L \sim \int d^3x\, x_L \left[\sigma + \frac{1}{c^2}\dot{\sigma}_k + \frac{1}{c^4}\ddot{\sigma}_{kl}\right] \qquad J_L \sim \epsilon \int d^3x\, x\, \left[\sigma_i + \frac{1}{c^2}\dot{\sigma}_{kl}\right]$$

$$\sigma = (T^{00} + T^{ii})/c^2, \qquad \sigma_i = T^{0i}/c, \qquad \sigma_{ij} = T^{ij}$$

$$\mathcal{F}_{\rm GW} \sim I_2 + \frac{1}{c^2}(I_3 + J_2) + \frac{1}{c^4}(I_4 + J_3) \quad \Rightarrow \quad \dot{\omega} = -[dE(\omega)/d\omega]/\mathcal{F}_{\rm GW}(\omega)$$

$$\dot{\omega} = \dot{\omega}_{\rm Newt}\, \left[1 + c_{\rm 1PN}\,\omega^{2/3} + c^{\rm SO}_{\rm 1.5PN}\,\omega + c^{\rm SS}_{\rm 2PN}\,\omega^{4/3} + c^{\rm SO}_{\rm 2.5PN}\,\omega^{5/3} + \cdots\right]$$

Summary

- **Frequency-domain templates: phenomenological and quasi-physical**
  - low computational cost
  - simple template placement
  - too many unphysical waveforms? Ok for NS-BH; some warnings for high mass BH-BH
- **Time-domain templates for single-spin and double precessing binaries:**
  - very high signal-matching perfomances
  - two-stage detection scheme
  - template metric is not analytical; more complicated template placement
- **Beyond adiabatic approximation: waveforms from last stages of inspiral and plunge**
- **Which source modeling for large and almost aligned spins in very massive binaries?**
- **Parameter-estimation–formalism in presence of spins needs to be revisited**
- **More accurate spinning waveforms probably needed for space-based detectors to extract accurately the parameters**