Quantum nondemolition theory with LIGO-II

Alessandra Buonanno* **& Yanbei Chen**
Caltech, Pasadena

based on:

• *Optical noise correlations and beating the standard quantum limit in LIGO-II* [GRP/00/549, gr-qc/0010011]

• *Quantum noise in second generation, signal-recycled laser interferometric* gravitational-wave detectors [GRP/00/553, gr-qc/0102012]

• *Laser interferometric gravitational-wave detectors* as optical springs [GRP/00/554, in preparation]

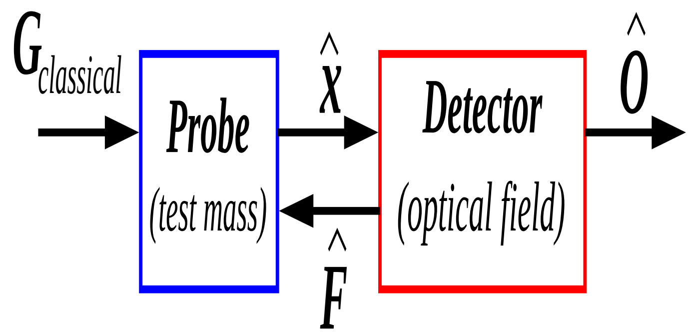
2/8/01

Gravitational Wave Advanced Detectors Workshop, Aspen 2001

Quantum nondemolition (QND) devices [Braginsky 68-77; Braginsky & Khalili, 92; Braginsky, Gorodetsky, Khalili, Matsko, Thorne & Vyatchanin, 01]

How to preclude quantum properties of detector and probe from demolishing information we want to extract?

How to monitor classical force G so weak to change test-mass displacement by amount less than zero-point quantum fluctuations?



Beat the quantum limit due to probe and detector

Naive argument for GW interferometers: (free test mass $\widehat{x}(t) = \widehat{x}_o + \widehat{p}_o/\mu\, t$)

$$[\widehat{x}(t), \widehat{x}(t')] = i\,(t'-t)/\mu \quad \Rightarrow \quad \text{Reduction of state} \quad \Rightarrow \quad S_{\rm SQL}(\Omega) = 2\hbar/(\mu\,\Omega^2\,L^2)$$

Origin of the SQL in GW interferometers [Braginsky & Khalili, 92; Braginsky, Gorodetsky, Khalili, Matsko, Thorne, Vyatchanin, 01]

$$\textit{Output:}\quad \widehat{\mathcal{O}}(\Omega) = \overbrace{\left[\widehat{\mathcal{Z}}(\Omega) - \frac{4}{m\Omega^2}\,\widehat{\mathcal{F}}(\Omega)\right]}^{\text{optical}-\text{field operator}} + \overbrace{\widehat{x}^{(0)}(\Omega)}^{\text{test}-\text{mass operator}} + \overbrace{h(\Omega)\,L}^{\text{GW signal}}$$

$\widehat{\mathcal{Z}} \rightarrow$ shot noise $\quad \widehat{\mathcal{F}} \rightarrow$ radiation-pressure force $\quad \widehat{x}^{(0)}(t) \rightarrow$ displacement of antisymmetric mode

$h(\Omega) \rightarrow$GW strain $\quad m \rightarrow$ arm-cavity mirror's mass $\quad L \rightarrow$ arm-cavity's length

- The test-mass initial quantum state affects only very low frequencies ($\widehat{x}(t) = \widehat{x}_o + \frac{4\widehat{p}_o}{m}\,t$) $\Rightarrow$ dependence on initial quantum state removed filtering the data [BGKMTV, 01]

- The SQL enforced by the light's quantum noise *not* by the test mass, *but* the test mass feeds the GW signal and the light through back action [BGKMTV, 01]

SQL for GW interferometers and how to beat it

Noise spectral density: $S_h(\Omega) = \frac{1}{L^2}\left[S_{\mathcal{Z}\mathcal{Z}} - \frac{8}{m\,\Omega^2}\,S_{\mathcal{F}\mathcal{Z}} + \frac{16}{m^2\,\Omega^4}\,S_{\mathcal{F}\mathcal{F}}\right]$

Constraint: $S_{\mathcal{Z}\mathcal{Z}}\,S_{\mathcal{F}\mathcal{F}} - S_{\mathcal{Z}\mathcal{F}}\,S_{\mathcal{F}\mathcal{Z}} \geq \hbar^2$ [Braginsky & Khalili, 92]

Conventional interferometer (LIGO-I/VIRGO)

$$S^{\rm conv}_{\mathcal{Z}\mathcal{F}} = 0 = S^{\rm conv}_{\mathcal{F}\mathcal{Z}} \quad \Rightarrow \quad S_h^{\rm conv}(\Omega) \geq S_h^{\rm SQL}(\Omega) = \frac{8\hbar}{m\,\Omega^2\,L^2} = h^2_{\rm SQL}(\Omega)$$

- LIGO-I/VIRGO: correlations can be built up *statically* during the readout process

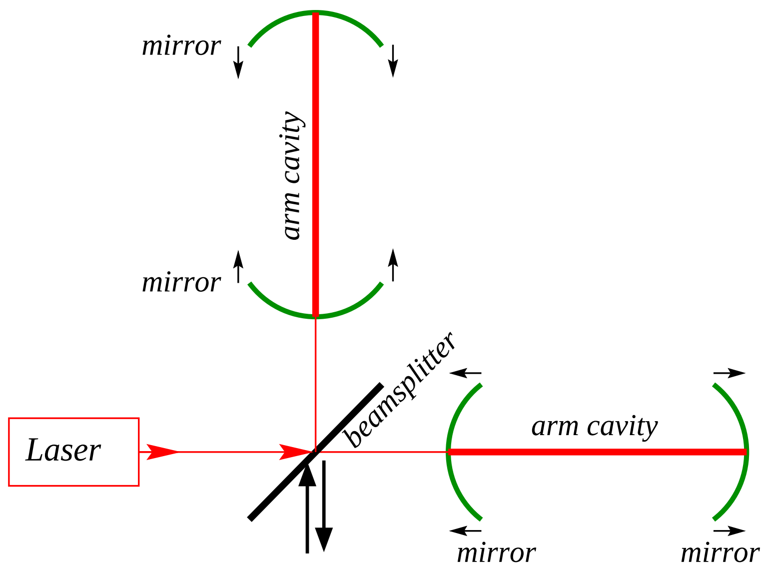
  [Matsko, Vyatchanin & Zubova, 93-98]

- LIGO-III: modification input-output optics $\Rightarrow \sqrt{S_h^{\rm LIGOIII}}/\sqrt{S_h^{\rm SQL}} \simeq 0.24$ at $f = 100\,{\rm Hz}$ over band of $\Delta f \sim f$ [Kimble, Levin, Matsko, Thorne & Vyatchanin, 00]

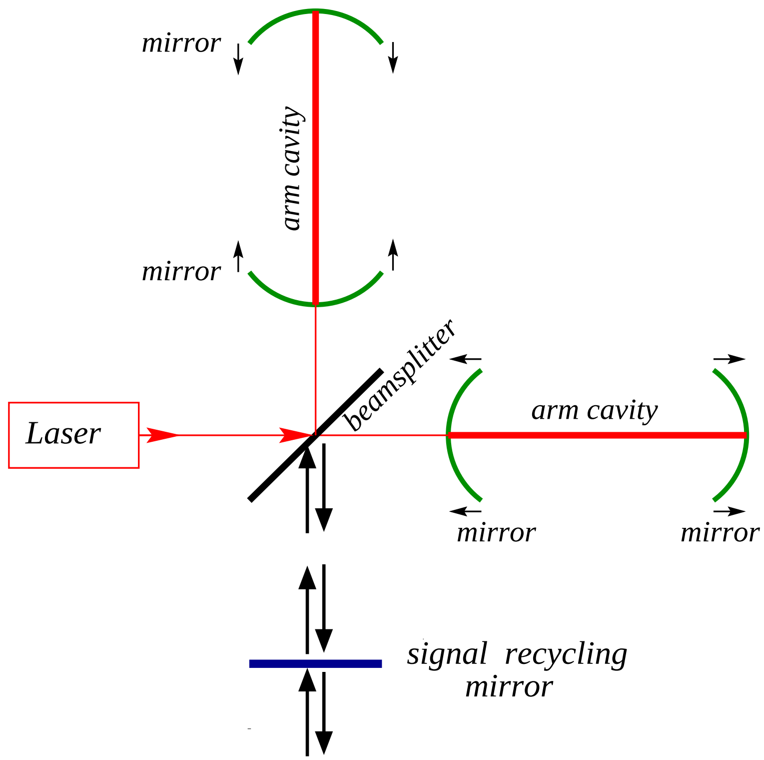
- Optical bar detectors: test mass behaves effectively as an oscillator . . .

  [Braginsky, Gorodetsky, Khalili, 97; Braginsky, Khalili, 99]

Conventional interferometer's design (LIGO-I/VIRGO)



Signal recycled interferometer's design (LIGO-II) [Drever, 82; Vinet, Meers, Man & Brillet 88; Meers 88; Mizuno, Strain et al. 93]



Signal recycled interferometer [Drever, 82; Vinet, Meers, Man & Brillet 88; Meers 88; Mizuno, Strain et al. 93]

Old picture: $S_h^{\rm shot} = S_{\mathcal{Z}\mathcal{Z}}/L^2$ assuming (naively) $S_{\mathcal{Z}\mathcal{F}} = 0 = S_{\mathcal{F}\mathcal{Z}} \Rightarrow$

$$S_h^{\rm back} = (S_h^{\rm SQL})^2/(4\, S_h^{\rm shot}) \Rightarrow S_h^{\rm tot} = S_h^{\rm shot} + S_h^{back}$$

New picture [AB & Chen, 00-01]**:**

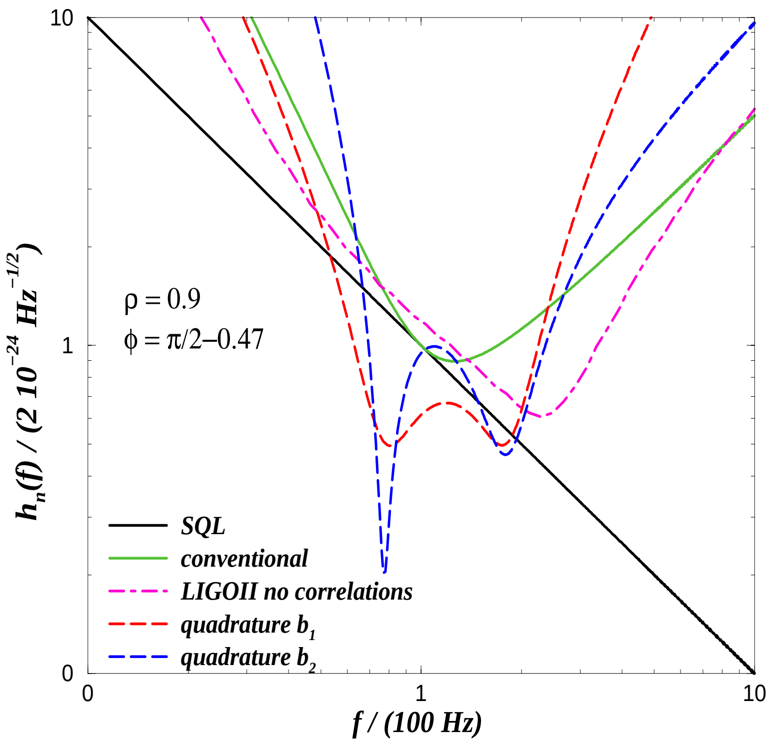
- Signal-recycling cavity builds up *dynamical* correlations between shot noise and radiation-pressure noise $\Rightarrow$

$$S_{\mathcal{Z}\mathcal{F}} = S_{\mathcal{F}\mathcal{Z}} \neq 0$$

- Resonant amplification of GW signal as in optical bar detectors [Braginsky et al. 97, 99]

$\sqrt{S_h^{\rm LIGOII}}/\sqrt{S_h^{\rm SQL}} \simeq 0.5$ over band of $\Delta f \sim f$

if thermoelastic noise can be pushed low enough



Signal recycling interferometer: input-output relation [AB & Chen, 00-01]

- Quantum formalism introduced for LIGO-I/VIRGO [Kimble, Levin, Matsko, Thorne & Vyatchanin,00]

$$E(t) = \cos(\omega_o t)\, E_1(t) + \sin(\omega_o t)\, E_2(t), \qquad E_i(t) \propto \int_0^\infty (\widehat{d}_i e^{-i\Omega t} + \widehat{d}_i^\dagger e^{i\Omega t}) \frac{d\Omega}{2\pi}$$

$\omega_o \simeq 10^{15}$ Hz, $\Omega (= 2\pi f) \simeq 10 - 10^3$ Hz

Input-output relation: $\widehat{d}_i \Leftrightarrow \widehat{c}_i$

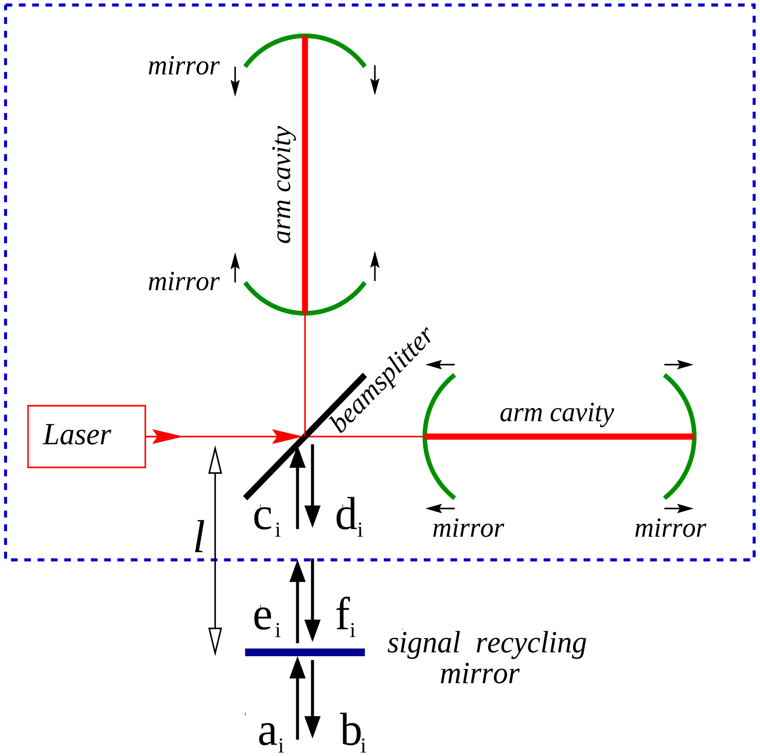
$\gamma = \frac{Tc}{4L} \simeq 2\pi \times 100\,\text{Hz}$ $\quad T \simeq 0.033$ $\quad m = 30$ kg

$I_o \to$ laser power $\qquad I_{\rm SQL} = \frac{m\, L^2\, \gamma^4}{4\omega_o} \simeq 10^4$ Watt

$\phi \equiv [\frac{\omega_o l}{c}]_{\rm mod\, 2\pi} \to$ phase gained by ω_o in SR cavity

$\rho, \tau \to$ SR amplitude reflectivity, transmissivity

SR input-output relation: $\widehat{b}_i \Leftrightarrow \widehat{a}_i$



SR noise spectral density

Homodyne detection ($\zeta = \text{const}$): $\qquad b_\zeta(\Omega) = b_1(\Omega)\sin\zeta + b_2(\Omega)\cos\zeta$

Signal-to-noise ratio:

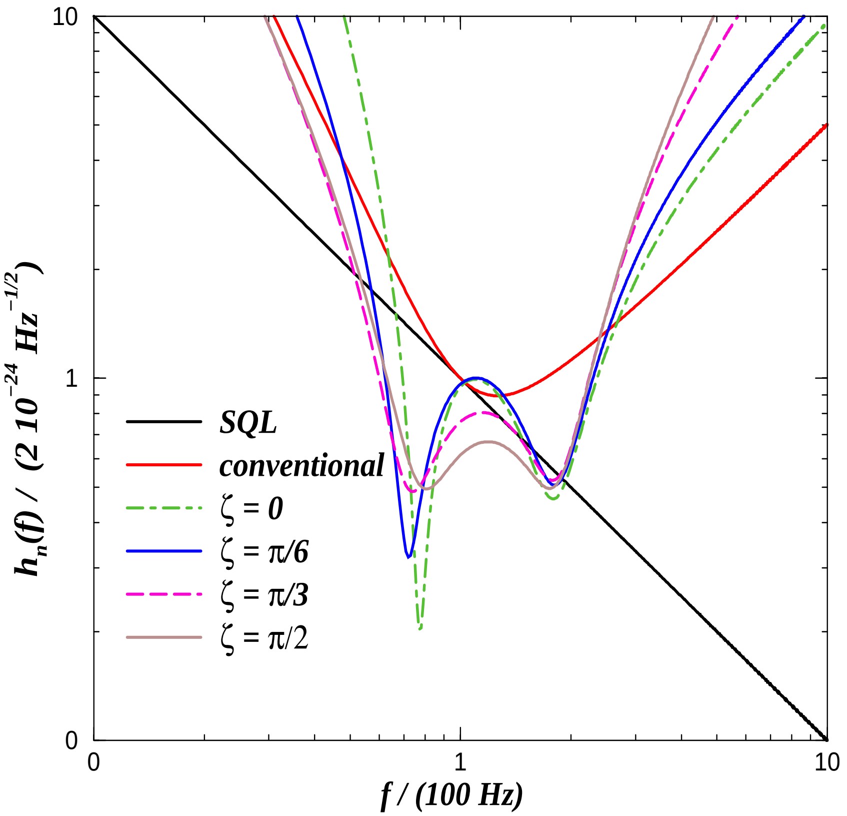
GWs from binary black hole systems

$h_s(f) \;\rightarrow\;$ Newtonian template

$$\left(\frac{S}{N}\right)^2_\zeta = 4\int_0^\infty \frac{|h_s(f)|^2}{S_h^\zeta(f)}df$$

for $\rho = 0.9$, $\phi = \frac{\pi}{2} - 0.47$, $I_o = I_{\rm SQL} \Rightarrow$

$$\frac{(S/N)_{\zeta=\pi/2}}{(S/N)_{\rm conv}} \simeq 1.83, \quad \frac{(S/N)_{\zeta=0}}{(S/N)_{\rm conv}} \simeq 1.98$$

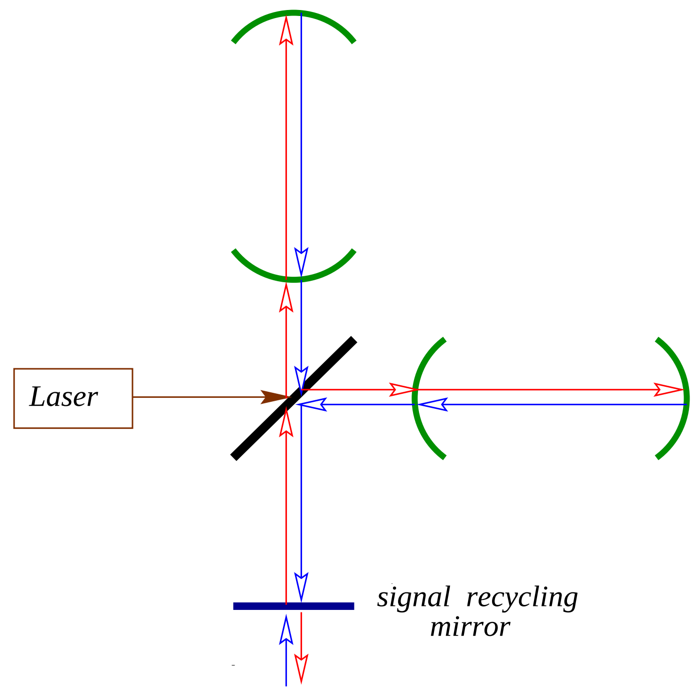


Remarks

Part of light leaks out through SR mirror contributing to shot noise but another portion correlated to it, is fed back into arm cavities contributing to radiation-pressure noise at some later time. Effectively: $S_{\mathcal{Z}\mathcal{F}} = S_{\mathcal{F}\mathcal{Z}} \neq 0$.

Optical field fed back into arm cavities also contains classical GW signal $\Rightarrow$ back-action force depends on history of test-mass motion

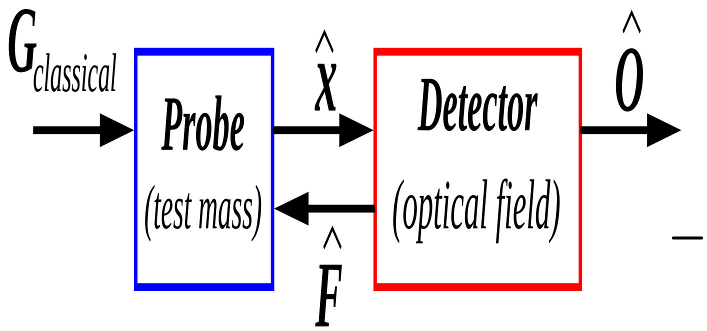
$$\Rightarrow \widehat{F}_{\rm back-action} = \widehat{F}_0(\Omega) + R_{FF}(\Omega)\,\widehat{x}(\Omega)$$



Nontrivial coupling between asymmetric mode of motion and signal-recycled optical field

LIGO-II as an optical spring [Braginsky et al. 97,99; AB, Chen, 01]

Dynamical system formed by SR optical field and mirrors [Braginsky & Khalili, 92; AB & Chen, 01]



$$\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_{\mathcal{P}} + \widehat{\mathcal{H}}_{\mathcal{D}} - \widehat{x}\,[G + \widehat{F}]$$

$$-\tfrac{m}{4}\,\Omega^2\,\widehat{x}(\Omega) = -\tfrac{m}{4}\,\Omega^2\,h(\Omega)\,L + \widehat{F}_0(\Omega) + R_{FF}(\Omega)\,\widehat{x}(\Omega)$$

Test-mass mirrors buffeted by Poissonian radiation pressure $\widehat{F}_0$, but also subject to harmonic restoring force with frequency-dependent spring constant:

$$K_{\text{spring constant}}(\Omega) = -R_{FF}(\Omega) \propto I_o\,\rho\,\sin 2\phi$$

$$\overbrace{[-\frac{m}{4}\,\Omega^2 - R_{FF}(\Omega)]}^{\text{new resonances!}}\,\widehat{x}(\Omega) = -\tfrac{m}{4}\,\Omega^2\,h(\Omega)\,L + \widehat{F}_0(\Omega)$$

Free test-mass SQL has no longer relevance for SR interferometer

The two resonances have different origin [Braginsky et al. 97,99; AB, Chen, 01]

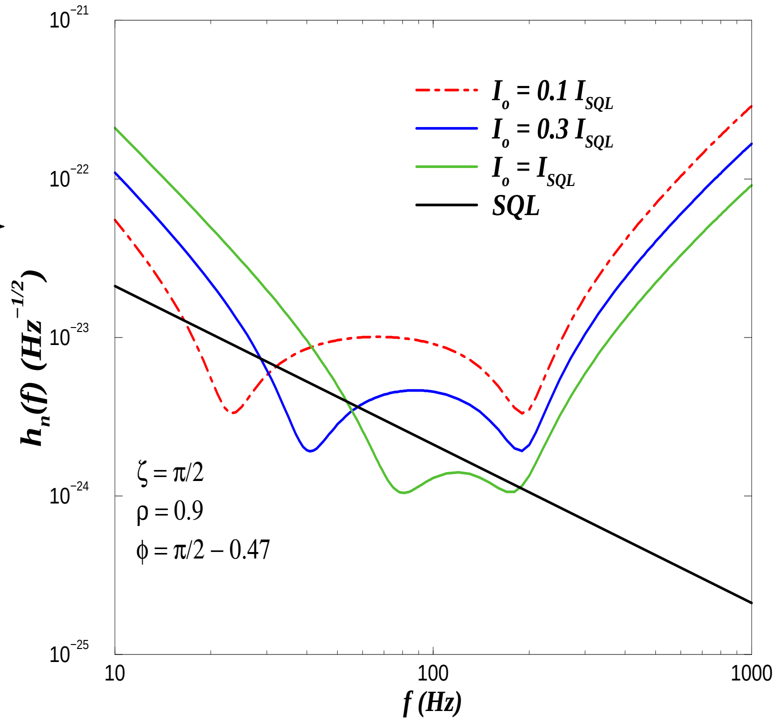
Totally reflecting SR mirror ($\rho = 1$): $\Omega^2_{\rm res} = \frac{\gamma^2}{2}\,[\tan^2\phi \pm \sqrt{\tan^4\phi - \frac{4 I_O}{I_{\rm SQL}}\,\tan\phi}]$

- $I_o \ll I_{\rm SQL} \Rightarrow$ resonances decoupled into:

$\Omega^0_{\rm res} \simeq 0, \quad \Omega^{1,2}_{\rm res} \simeq \pm\gamma\,\tan\phi$ [Meers,88; Strain et al., 93]

$2\phi \pm 2\arctan\frac{\Omega}{\gamma} \to$ total round-trip phase

- $R_{FF} \propto I_o\,\rho\,\sin 2\phi \Rightarrow$ increasing I_o up to $I_{\rm SQL} \Rightarrow$ test masses and optical field coupled more and more

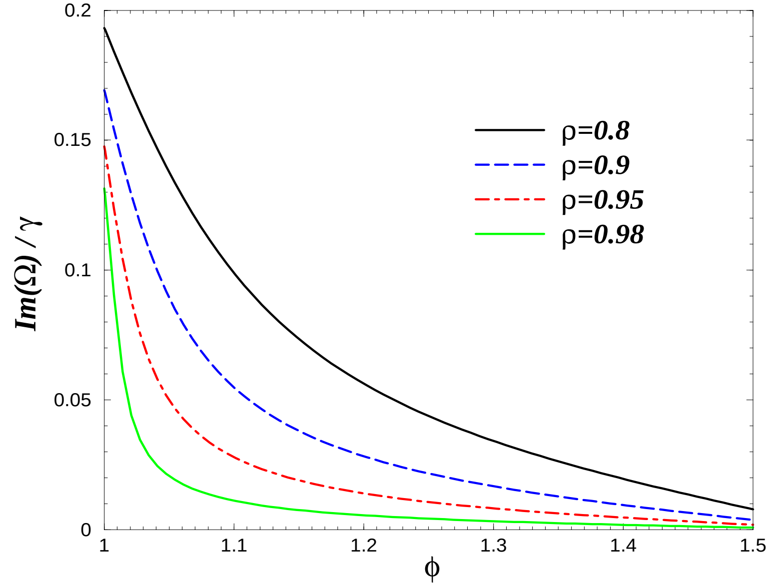


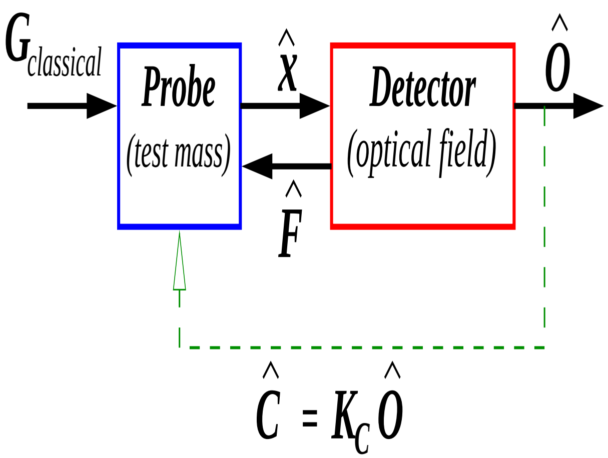
Generic SR-mirror reflectivity ($\rho \neq 1$):

$$\Omega^2\,(\Omega - \Omega^\rho_1)\,(\Omega - \Omega^\rho_2) + \frac{I_O\,\gamma^3}{2 I_{\rm SQL}}\,(\Omega^\rho_1 - \Omega^\rho_2) = 0$$

One resonance always unstable !

Instability and control system





$$\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_{\mathcal{P}} + \widehat{\mathcal{H}}_{\mathcal{D}} - \widehat{x}\,[G + \widehat{F} + \widehat{\mathcal{C}}]$$

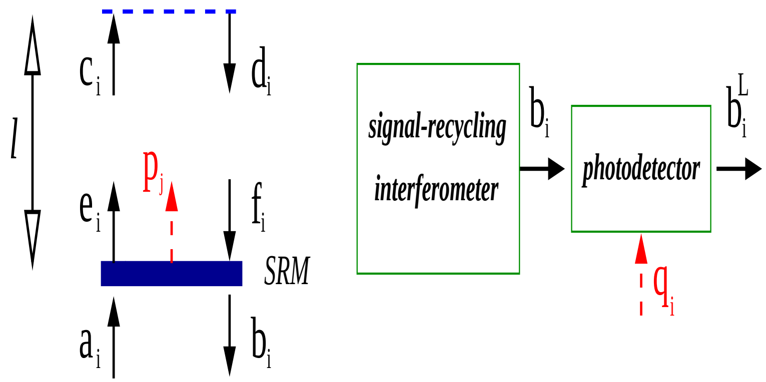
$$\widehat{\mathcal{C}} = \int dt'\, K_{\mathcal{C}}(t-t')\, \widehat{O}(t')$$

GW signal and noise fed back into arm-cavity end mirrors in same way

$\Rightarrow$ relative amplitude not altered $\Rightarrow$ noise spectral density unchanged

except for extra noise coming from electronics

Inclusion of losses [Kimble, Levin, Matsko, Thorne & Vyatchanin,00; AB & Chen, 00]



$p_i, q_i \rightarrow$ additional quantum noises

- $\mathcal{L} \sim 200 \times 10^{-6} \rightarrow$ loss coefficient in arm-cavity round trip
  - $\lambda_{\rm SR} \sim 0.02 \rightarrow$ fraction of photons lost at each bounce off SR mirror

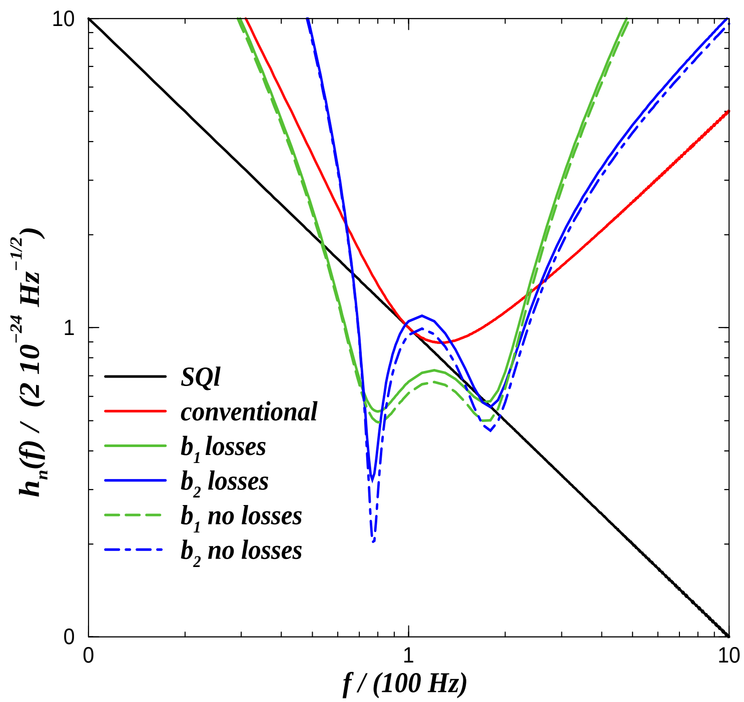
- $\lambda_{\rm PD} \sim 0.1 \rightarrow$ photodetection efficiency 90%

Signal-to-noise ratio: $(S/N)^2_\zeta = 4 \int_0^\infty |h_s(f)|^2 / S_h^\zeta(f) df$

GWs from BBHs for $\rho = 0.9$, $\phi = \frac{\pi}{2} - 0.47$, $I_o = I_{\rm SQL}$

Fractional loss in S/N for inspiraling binaries:

8% (for b_1), 21% (for b_2)



Conclusions and future directions

- If thermal noise pushed low enough, LIGO-II can beat the free mass SQL for modest amounts: roughly a factor two over a bandwidth $\Delta f \sim f$.

- Dynamical system composed by SR optical field and arm-cavity mirrors originates doublet of resonances $\Rightarrow$ optical spring detector with instability.

- Effects of new results better clarified and sharpened once the readout scheme is specified. More careful study of control system.

- Best SR configuration that optimizes the signal-to-noise ratio for inspiraling binaries, for low-mass X-ray binaries, and for other astrophysical GW sources.

- Back-action force on arm-cavity mirrors does not include time-delay effects.

- Investigate other optical configurations which capture main features of SR interferometer.