



*LIGO Laboratory / LIGO Scientific Collaboration*

LIGO-T070170-00-E

*LIGO*

July 26, 2007

---

LIGO I mirror scattering loss  
by non smooth surface structure

---

Hiro Yamamoto

Distribution of this document:  
LIGO Science Collaboration

This is an internal working note  
of the LIGO Project.

**California Institute of Technology**  
**LIGO Project – MS 18-34**  
**1200 E. California Blvd.**  
**Pasadena, CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

**Massachusetts Institute of Technology**  
**LIGO Project – NW17-161**  
**175 Albany St**  
**Cambridge, MA 02139**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: [info@ligo.mit.edu](mailto:info@ligo.mit.edu)

**LIGO Hanford Observatory**  
**P.O. Box 1970**  
**Mail Stop S9-02**  
**Richland WA 99352**  
Phone 509-372-8106  
Fax 509-372-8137

**LIGO Livingston Observatory**  
**P.O. Box 940**  
**Livingston, LA 70754**  
Phone 225-686-3100  
Fax 225-686-7189

<http://www.ligo.caltech.edu/>

## 1 Introduction

Understanding of the scattering loss mechanisms in the core optics component system is very important for the design of Advanced LIGO. The performance of Initial LIGO was studied in various ways [4] and it was found that there is unexpected loss of  $\sim 50$  ppm per mirror, possibly due to large angle scattering or short spatial wavelength ( $\lambda < 5\text{mm}$ ) surface structure. The loss assumed for the Advanced LIGO is around 70ppm per arm or 35ppm per mirror, and it is very important to understand this extra loss and to reduce it to the acceptable level.

The estimation of this extra loss always came with a big systematic uncertainty because some measurements varied from time to time, possibly due to actual hardware setup changes, and some measurements had large systematic errors. Recently there were several new measurements done [5,6], now the power recycling gains can be measured with smaller systematic errors [7], and some misconception was clarified regarding the specification of microroughness [1].

In this note, one scenario of estimating the extra loss is discussed. First, the size of the extra loss is reevaluated using the latest recycling gain and the updated estimation of the loss due to microroughness. Then the analysis of the RTS data measured at Caltech OTF is used to evaluate the average loss due to point scattering observed using an integrating sphere. Lastly, the measurement of the scattered light at large angles off from LLO ITM done by V. Frolov [9] is compared with the CSIRO LIGO I mirror polished surface PSD.

To date, not all data are consistent or well understood nor have been fully investigated. The estimation given in this analysis indicates that the extra loss is around 10~15ppm. This note does not discuss the issues related to two ITMs (2ITM04 and 4ITM07) replaced because of surface anomalies [4].

## 2 Indirect estimation of LIGO I mirror scattering loss

Based on the FFT-based simulation using mock phase maps [2] (see Appendix), the LIGO I test mass surface requirement was set to be

- $\text{rms}(\lambda_s > 2.3\text{mm}) < 0.8\text{nm} \sim \lambda(\text{Nd:YAG})/1200$  and
- $\text{rms}(\lambda_s < 2.3\text{mm}) < 0.2\text{nm}$

for each spatial wave length ( $\lambda_s$ ) region [3].

CSIRO analyzed the microroughness [8] and reported the rms in this high frequency region to be 0.17nm (e.g., [9]). Based on this number, the loss due to this high frequency region was estimated to be 5ppm.

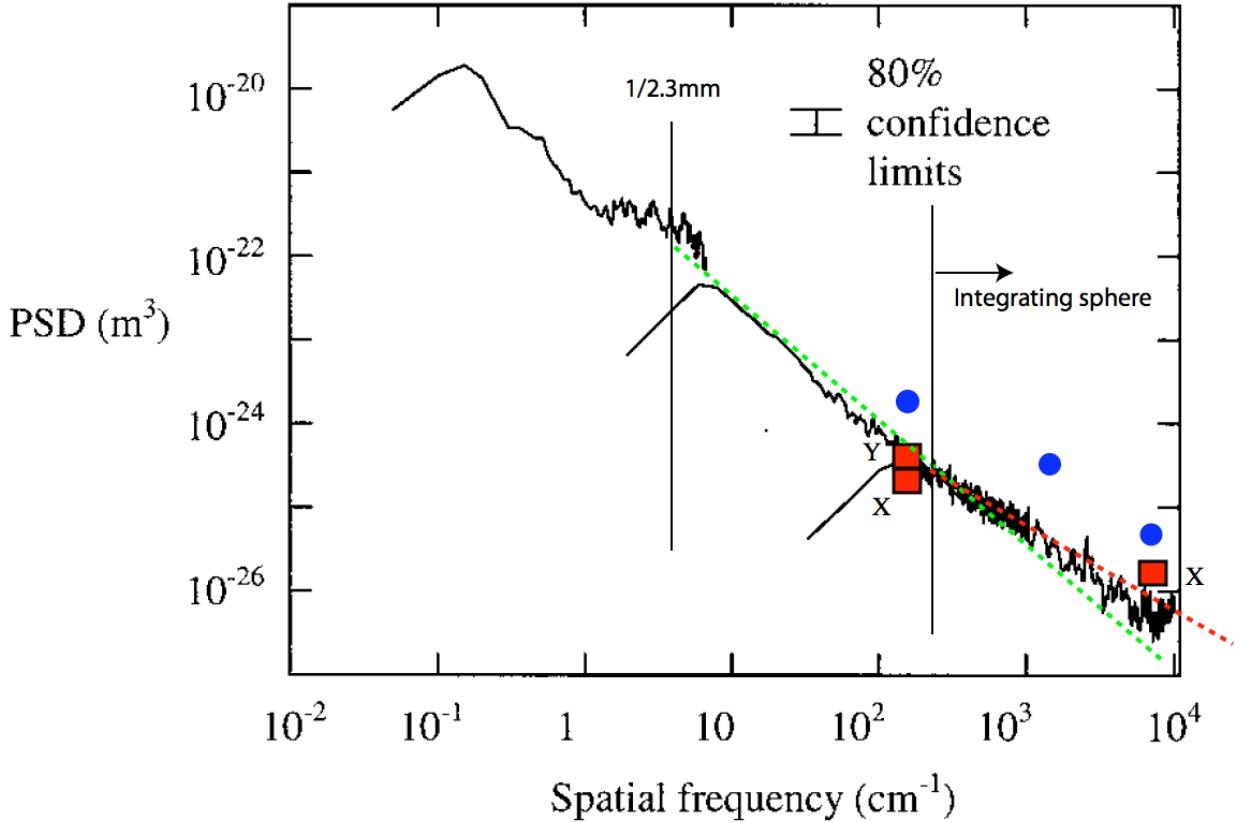
When various quantities like the power recycling gain and the arm visibility were analyzed, known loss (see Appendix about the definition of “know” and “extra” loss ) was not enough to explain the measurement, and had to introduce extra ad hoc loss of around 50 ppm per mirror.

As the various analyses went on, it was found that some losses were not properly estimated. One was the “measurement” of the rms [11,1] of the microroughness. The bent seen in Figure 1 at  $f = 5\text{ cm}^{-1}$  is a side effect due to the subtraction of the tilt and curvature to minimize the measurement bias. This side effect gave a misleading result. The correct estimation of the rms turned out to be

0.33nm, using a fit based on different sets of measurements to avoid this bias, and the loss due to microroughness turned out to be 19ppm, instead of 5ppm using the following formula:

$$loss = 2\pi D \left( \frac{4\pi\sigma}{\lambda} \right)^2, D \equiv \Gamma\left(\frac{C+1}{2}\right) / (2\sqrt{\pi}\Gamma\left(\frac{C}{2}\right)) \quad (1)$$

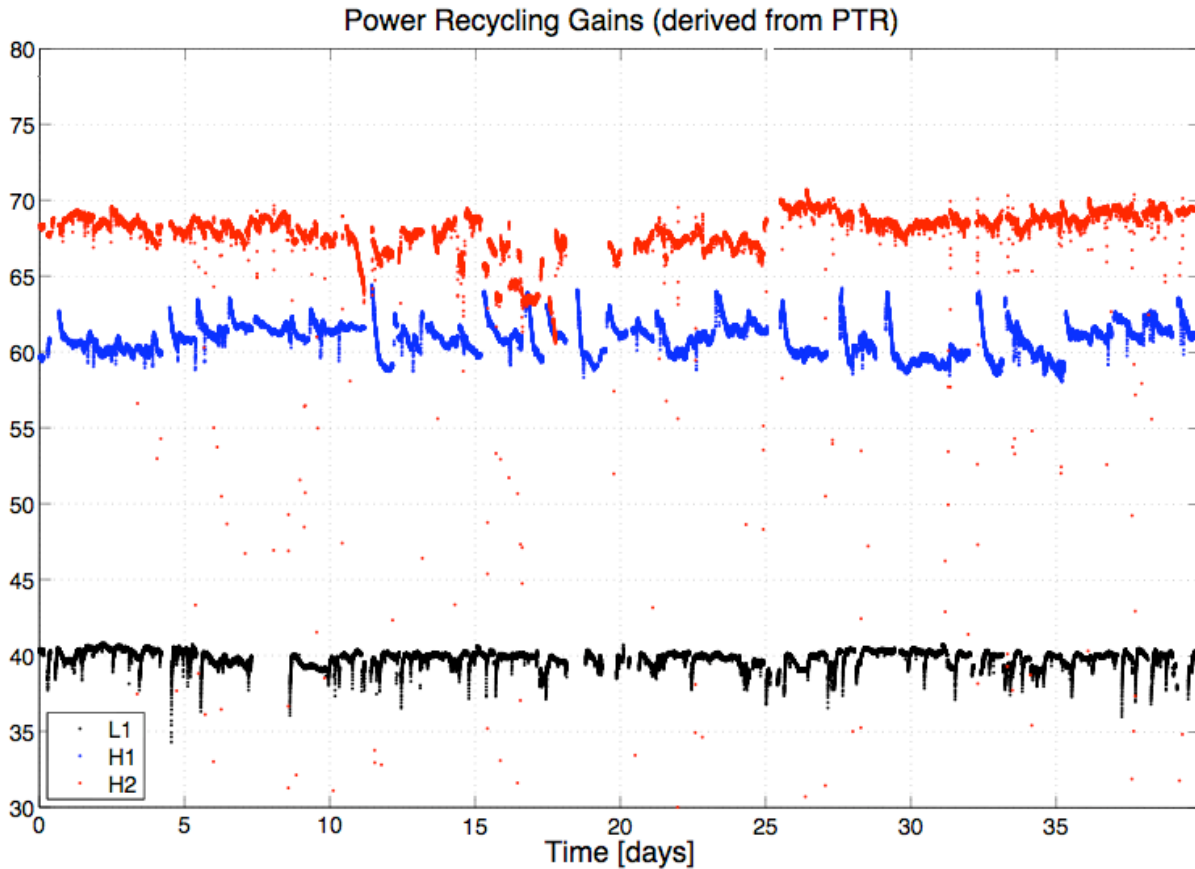
where C is the slope of the PSD and is 1.45 and  $\sigma$  is the rms calculated using 1D PSD.



**Figure 1.** PSD function of LIGO I ITM. This is Fig.10 in Ref.[11]. The green dotted line is a fit of all LIGO I mirrors,  $A f^C$ ,  $A=7\times 10^{-19}$  and  $C=1.45$  and the red dotted line is a fit with  $A=7.4\times 10^{-21}$  and  $C=1$ . Blue circles are measurements by B.Kells [4] and red squares are recent measurements by V.Frolov [6]. X means the data for ITMX and Y means the data for ITMY. The vertical size of the square box corresponds to the ambiguity of the conversion from BRDF to PSD due to the slope ambiguity,  $C = 1\sim 1.45$ .

Second is the intermediate wavelength region,  $\lambda_s = 6\text{mm}\sim 1\text{mm}$ , bumps between  $1\sim 10\text{ cm}^{-1}$  in Figure 1. This is due to an aberration by the CSIRO polishing. The FFT simulation did not include the loss due to this region, and it was assumed that the scattering loss due to this region would be included in the microroughness contribution. There is a preliminary estimation that this region can cause loss of the order of a 10ppm [10]. It is necessary to analysis this region more carefully to have a quantitative estimation of the loss from this region.

Another improvement is the increase of the power recycling gain as the LIGO I operation goes on. The reason of improvement is not clear. It is possible that the tuning of the interferometer got better or mirrors with lossy surface were replaced. Both H1 and H2 had one of their ITM mirrors replaced, while L1 had no mirror replaced. Figure 2 shows three power recycling gains for H1, H2 and L1 measured using transmitted powers [7].

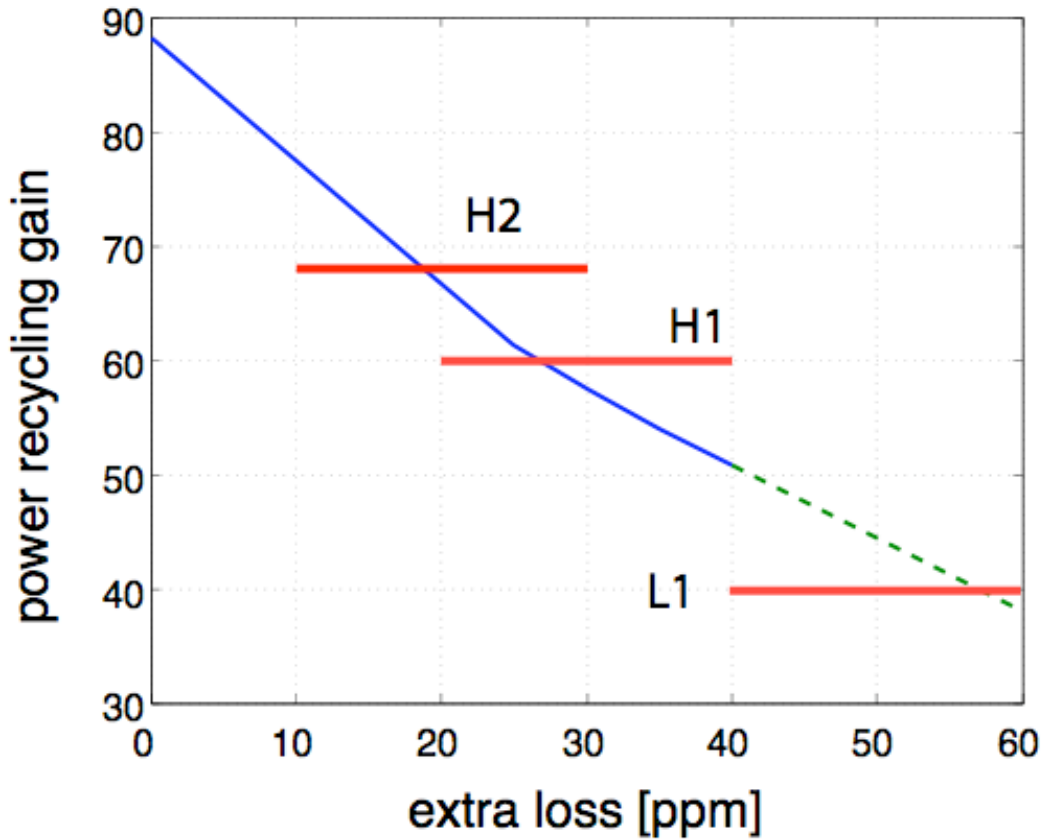


**Figure 2.** Power recycling gain measured from June to July in 2007 during the S5 run.

Figure 3 shows the recycling gain of H1 as a function of the extra loss (see appendix about the definition of “extra loss”). As can be seen from this figure, the recycling gain has a strong correlation with the loss. For the old value of the recycling gain below 50, an extra loss of 40 ppm or larger had to be used. But, with the current gain of 60, the extra loss needed is around 25 ppm.

As is explained in the appendix, the known loss due to long wavelength component is calculated to be  $\sim 20\text{-}30\text{ppm}$  per mirror using the measured phasemaps. The phasemap data sets have systematic errors, due to e.g., air disturbance, temperature dependence and interference patterns, and these effects tend to overestimate the loss of long wavelength scattering. Because of this systematic bias, the extra loss estimated can be larger by  $\sim 10\text{ppm}$  per mirror to compensate this overestimation of the “know loss”.

If we estimate the extra loss per mirror is 25~35ppm per mirror, and subtract 19 ppm as the known loss due to micro roughness, the extra loss comes out to be 6 ~ 16ppm.



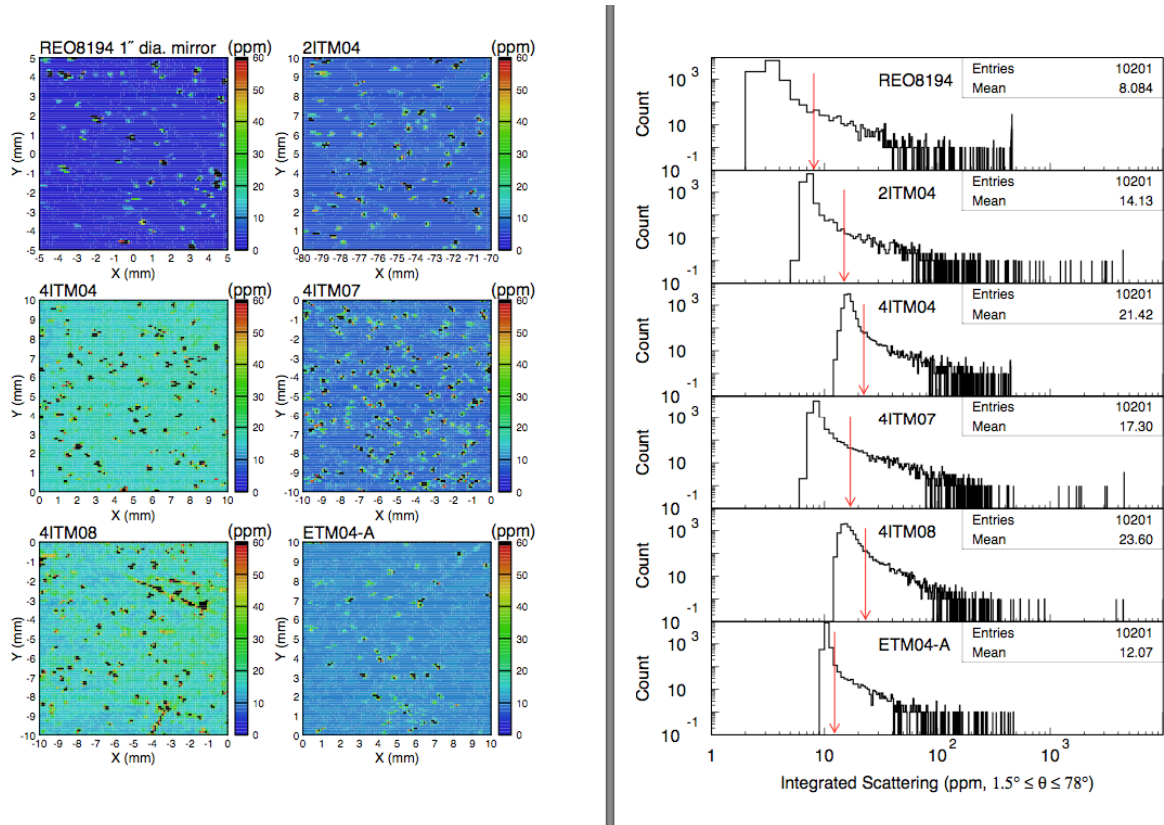
**Figure 3.** Power recycling gain as a function of extra loss. The blue solid line is calculated using H1 mirror phase maps. Three red lines are power recycling gains of three IFOs. Lines for H2 and L1 are for reference and the predictions may be different from the blue line.

### 3 Estimation of the point scattering loss based on OTF RTS data

Scattering by LIGO I mirrors were measured at OTF by using an integrating sphere. The device integrates the reflected light with angle larger than  $1.5^\circ$ , which corresponds to  $\lambda s < 41 \mu\text{m}$  or  $f_s > 244 \text{ cm}^{-1}$ . As can be seen in the scatter plot on the left hand side in Figure 4, there are points which scatter off powers larger than the average areas. The populations of the integrated power is shown in the right hand side.

If one assumes that the surface is uniform following certain spectrum, the scattered energy distribution becomes a poisson distribution. There is a threshold structure and some bias due to the actual measurement setup, but all data sets show long tails which correspond to points with large scattered power.

The contribution of these points with large scattering was estimated in the following way. For each mirror data set, define the threshold to characterize the start of high scattering points by adding the width of the peak to the average value. For 2ITM04 case, the threshold is 20 ppm is chosen as the threshold, and there are 362 points with scattering larger than 20 ppm out of 10k measured points . The average value of these high scattering points is 193 ppm. But these high scattering points are only 3.6% of the entire measure points, and the average contribution of these points in the entire set is 7.0 ppm. In other words, out of the 16 ppm average loss of the entire sample, 7 ppm comes from the high scattering points and the rest comes from the low scattering points.



**Figure 4.** Scattering measurements using an integrating sphere with angle  $1.5^\circ \sim 78^\circ$ . 1cm x 1cm region is scanned at 100x100 points using a laser with 200  $\mu\text{m}$  beam size. Red arrows mark the average value of the scattered power.

	Threshold (ppm)	Number of high scattering points	Average loss of all points (ppm)	Average loss of high scattering points (ppm)	Average loss of high scattering among all data points (ppm)
2ITM04	20	362	16	193	7.0
4ITM04	30	613	23	95	5.7
4ITM07	20	882	18	105	9.1
4ITM08	30	936	25	91	8.3
ETM04A	15	356	12	53	2.0

**Table 1** Summary of the characterization of high scattering points.

The result is summarized in Table 1. This is a rather artificial way to quantify the contribution of estimating the effect of the high scattering points. Although there are points which has high scattering values, the average contribution is less than 10ppm for all mirrors.

The size of the scattering source is rather small,  $\sim 1 \mu\text{m}$ , and the scattering angular pattern will be uniform in angle, rather than a typical structure with a sharp peak at the normal direction. Because of this, the correction due to the missing angular region,  $\theta < 1.5^\circ$ , will be negligible.

The average of the last column of Table 1 is 6.4ppm. This is a rather crude estimation, but the contribution from points with larger scattering loss seems to be  $< 10$  ppm.

The contribution from the smooth roughness is not quite consistent between this measurement and the CSIRO data. If the slope of 1.45 is used (green dotted line in Figure 1), the total scattering in the integrating sphere is estimated to be 2.4ppm, and if the slope of 1 is used (red dotted line in Figure 1), the total scattering is estimated to be 3.8ppm. Because of the clear threshold structure in Figure 4, the contribution from any source should be larger than that threshold value, and all mirrors have thresholds a few times larger than this estimation from the PSD. One possibility is that the calibration of the background introduces offset of the scattered light of the order of 5ppm. The estimation of the loss due to high scattering power points is not affected much by this offset, if there were any.

#### 4 Direct measurement of larger angle scattering

There was a measurement done by B.Kells at LHO 4k IFO [4]. Recently, a new measurement was done by V.Frolov at LLO 4k IFO [6]. The measured BRDF values were converted to the 1 dimensional PSD using the following formula [1];

$$PSD_1(f) = \frac{f}{D} \left( \frac{\lambda^2}{4\pi} \right)^2 BRDF(\theta = f \cdot \lambda) \quad (2)$$

D is defined in Eq.(1), and it varies from  $D=1$  for  $C=1$  to  $D=1.3$  for  $C=1.45$ .

These data points by V.Frolov are compared with the PSD measured by CSIRO [8] and with the old B.Kells measurement. The vertical extent of the red box corresponds to the variation of D when the slope C changes from 1 to 1.45. The quote measurement error is around the same size.

The difference between the old measurement and the new measurement needs to be understood. But the new data at LLO seem to be consistent with the CSIRO measurement of the substrate surface spectrum.

The scattering at the largest angle,  $45^\circ$ , seems to be larger than the CSIRO data. If this scattering loss is dominated by the point scattering, the angular distribution of BRDF will be uniform. For non contiguous scattering source, the conversion of BRDF to PSD is meaningless, because PSD is a quantify assuming that the statistical surface roughness structure is uniform over the surface. Point scattering data are analogous to glitch anomalies in time series data. Glitch anomalies popping up now and then cannot be well represented and analyzed by a Fourier series.

The angular integral of this value,  $BRDF = 8 \times 10^{-6} / \pi$ , over half hemisphere, gives an estimation of the loss to be 16 ppm. The scattering estimated using the CSIRO PSD spectrum is 2.4 ppm by

integrating the red line in Figure 1 from  $1000\text{cm}^{-1}$  to  $10000\text{cm}^{-1}$ . This large angle scattering measurement indicates an excess loss of  $\sim 15\text{ppm}$ , on top of the scattering loss due to smooth roughness.

## 5 Summary

Three kinds of estimation were done to understand the excess loss which have not been understood well. The indirect estimation using the power recycling gain predicts a loss around  $6\sim 16\text{ppm}$ , and the uncertainty of the upper limit is dominated by the mirror surface phasemap.

The second analysis uses an integrating sphere measurement at OTF. This clearly shows non contiguous high scattering points. Using the losses of those high scattering points, the loss due to those point scattering was estimated to be  $6\text{ppm}$ . This measurement has some systematic to be understood to have a consistent picture regarding the smooth roughness.

Third data was based on the recent direct measurement of larger angle scattering. The measurement at  $1^\circ$  is consistent with the PSD measured by CSIRO, but the measured power at  $45^\circ$  predicts  $\sim 14\text{ppm}$  loss due to non smooth roughness. There is an old measurement by B.Kells which indicates larger PSD values, and this discrepancy needs to be understood.

With these results in hand, it is preferable to make an Advanced LIGO optical system design which is tolerant about this size of extra loss.

## 6 Appendix : FFT simulation

In this appendix, the FFT simulation is explained to clarify what are inputs and what are predicted outputs, especially what “know loss” and “extra loss” mean.

The FFT-based simulation program [2] calculates stationary fields in an interferometer using measured mirror surface phasemaps [3]. The tilt and curvature are subtracted from the measured data, and these phasemap data are used together with the measured curvature of each mirror. The phasemap data are provided in grid points with finite size. Most of the calculations for the initial LIGO simulation were done using a grid size of  $2.73\text{mm}$  ( $=35\text{cm}/128$ ) over a square window of  $35\text{cm}$ . The actual mirror diameter is  $25\text{cm}$ , and the null region outside of the mirror is used to suppress unavoidable alias effect. In each pixel, the mirror surface is assumed to be flat. In other words, there is no scattering loss with spatial frequency shorter than  $2x$  grid size (or  $5.76\text{mm}$  for the above mentioned case).

When the field is calculated, effects of the following elements are included as inputs or as a result of simulation:

- surface figures with long wavelengths ( $> 2x$  grid size) of mirror surfaces and radius of curvatures
- finite aperture size and thickness of optics
- profile of resonating fields in the cavity

The resonating beam profile is a result of the simulation using optics parameters, and the diffractive loss and the regions on mirrors affecting beam profiles are dependent on the beam profile. The diffractive loss and loss due to the long wavelength components are calculated based on the input parameters, and these are counted as known losses.

The simulation has a parameter to represent the extra loss of the mirror, which cannot be included in the simulation. The large angle scattering loss due to the surface variation with short spatial wavelength is one major component of this loss parameter. The “extra loss” means this parameter for each mirror, which represents all unknown losses. A field losses by this amount of energy at the interaction with a mirror in addition to all mechanisms implemented in the simulation, including the small angle scattering and the diffractive loss.

The diffractive loss of LIGO I arm is 0.4ppm per arm determined by the mirror curvatures, apertures and the arm length, and the loss due to the mirror surface figure is around 20~30ppm .

In this wavelength region, there is one thing to note about. Some of the LIGO I test mass mirror phasemaps have a characteristic ring pattern with radius around 3.5cm, which is produced by the polishing technology used by CSIRO. The beam size on ITM is 3.6cm and that on ETM is 4.6cm. Due to this difference, the effect of this ring structure is different for ITM and for ETM, i.e., the beam on ETM is affected more by the ring than the beam on ITM. The effect was quantified by calculating the loss of a FP with  $ROC(ITM) = 14.2km$  and  $ROC(ETM) = 7.4km$ . When the phasemap of ETM01 is placed on ETM pairing with ITM with smooth surface, the loss due to this aberration was 42ppm. When the same loss is calculated using the same ETM01 phasemap for ITM with ROC of 14.2km pairing with ETM with smooth surface, the loss comes out to be 9.2ppm.

This shows a good example that the loss due to the long wavelength component depends on the beam size on the mirror and on the relative orientation with respect to other mirrors. Because of this, strictly speaking, the effect of the surface figure of a mirror can be discussed only in a given IFO setup.

## 7 References

1. H. Yamamoto, “LIGO I mirror scattering loss by microroughness”, LIGO-T070082-03 (2007);
2. B. Bachner, “Modelling the Performance of Interferometric Gravitational-Wave Detectors with Realistically Imperfect Optics”, LIGO-P980004-00-R, MIT PhD Thesis (1998); Hiro Yamamoto, “New FFT for AdvLIGO”, LIGO-G060572 (2006)
3. G.Billingsley, <http://www.ligo.caltech.edu/~gari/LIGO1/>
4. Bill Kelles, “Initial LIGO COC Loss investigation : Summary”, LIGO-T070051-00-D (2007)
5. Liyuan Zhang, private communication, see also [4]
6. Valery Frolov, [http://ilog.ligo-la.caltech.edu/ilog/pub/ilog.cgi?group=detector&date\\_to\\_view=07/19/2007&anchor\\_to\\_scroll\\_to=2007:07:19:21:40:03-valera](http://ilog.ligo-la.caltech.edu/ilog/pub/ilog.cgi?group=detector&date_to_view=07/19/2007&anchor_to_scroll_to=2007:07:19:21:40:03-valera)
7. Rana Adhikari, private communication.
8. “Method used by CSIRO to combine data from TOPO 2.5x and 40x heads to certify initial LIGO microroughness to a requirement of < 0.2nm rms”, LIGO-C971407-00-D (1997)
9. “ETM01”, LIGO-T990140-00-D (1999)
10. H. Yamamoto, “Scattering loss @ CIT COC Meeting on April 11, 2007”, LIGO-G070240 (2007)

11. C.J.Walsh, A.J.Leistner and B.F.Oreb, “Power spectral density analysis of optical substrates for gravitational-wave interferometry”, *Appl.Opt.* 38, 4790-4801 (1999)