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Calculations of Fields in Optical Cavities
(Summation Approximation)

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1 Complex Fields

The laser beam is represented by an almost monochromatic wave

$$\mathcal{E}(t,z) = a(t,z)\ e^{i\phi(t,z)} e^{i(\omega t - kz)}, \tag{1}$$

where a and ϕ are slowly changing functions of time.

The phase modulated light can be represented as a multiplet of sidebands

$$s = \begin{cases} 0, & \text{for carrier} \\ \pm 1, \pm 2, \ldots, & \text{for sidebands} \end{cases} \tag{2}$$

Then the sideband frequencies and the wavenumbers are

$$\omega = \omega_0 + \Omega, \tag{3}$$
$$k = k_0 + K, \tag{4}$$

where Ω and K are the frequency and the wavenumber relative to the carrier. These are defined by the modulation

$$\Omega = s\ \Omega_{mod}, \tag{5}$$
$$K = s\ K_{mod}, \tag{6}$$

where Ω_{mod} is the modulation frequency in rad/sec and K_{mod} is the modulation wavenumber.

Then the sideband fields are given by

$$\mathcal{E}(t,z) = \left[a(t,z)\ e^{i\phi(t,z)}\right] e^{i(\omega_0 t - k_0 z)}\ e^{i(\Omega t - Kz)}, \tag{7}$$

where the last term represents the phase of the sideband relative to the carrier.

It is convenient to omit the exponential factor $e^{i\omega t}$ in the definition of the electric field. Also it is convenient to take into account the relative phases e^{-ikz} through the propagators. In this approach, the electro-magnetic wave is represented by the following complex field:

$$E(t,z) = \left[a(t,z)\ e^{i\phi(t,z)}\right]. \tag{8}$$

The propagator is

$$P_{ab} = e^{-iW_{ab}}, \tag{9}$$

where W_{ab} is the propagation phase

$$W_{ab} = kL_{ab}. \tag{10}$$

Such a phase is usually a large number. The multiples of 2π can be omtted.

$$W_{ab} = kL_{ab}, \tag{11}$$
$$= k_0 L_{ab} + KL_{ab}, \tag{12}$$
$$\equiv V_{ab} + KL_{ab}. \tag{13}$$

where

$$V_{ab} = k_0 L_{ab} \pmod{2\pi}. \tag{14}$$

The amplitude transmissivity and reflectivity is

$$t_a = \sqrt{T_a}, \tag{15}$$

$$r_a^{(0)} = \sqrt{R_a} = \sqrt{1 - T_a - \mathcal{L}_a}. \tag{16}$$

where a is the mirror label.

2 Fabry-Perot Cavity

The amplitude reflectivities are:

$r_a^{(0)}$ - reflectivity of Input Mirror,
$r_b^{(0)}$ - reflectivity of End Mirror.

Displacements are

$x_a(t)$ - displacement of Input Mirror,
$x_b(t)$ - displacement of End Mirror.

In the expressions below the reflectivities always enter with the corresponding phases. These are complex reflectivities which depend on the displacements

$$r_a(t) = r_a^{(0)} \, e^{-2ikx_a(t)}, \tag{17}$$

$$r_b(t) = r_b^{(0)} \, e^{-2ikx_b(t)} \tag{18}$$

2.1 Selfconsistent equation

The forward propagating field measured at the front mirror inside the cavity satisfies the selfconsistent equation

$$E(t) = U(t) \, E(t - 2T) + t_a E_{in}(t). \tag{19}$$

Here $U(t)$ is the round-trip operator

$$U(t) = r_a^*(t) \, r_b(t - T) e^{-2i\Phi}, \tag{20}$$

and asterisk (*) stands for complex conjugation. The phase is

$$\Phi = \Omega T = KL, \tag{21}$$

where T is the one-way propagation time the cavity and L its length:

$$T = \frac{L}{c}. \tag{22}$$

2.2 Adiabatic approximation

Neglecting the delay due to finite propagation times,

$$E(t-2T) \approx E(t). \tag{23}$$

we obtain the adiabatic approximation.

In the adiabatic approximation, the field satisfies the equation

$$E(t) = U(t)\ E(t) + t_a E_{in}(t). \tag{24}$$

Its solution is

$$E(t) = \frac{t_a E_{in}(t)}{1-U(t)}. \tag{25}$$

Also in the adiabatic approximation we can assume that

$$r_b(t-T) \approx r_b(t), \tag{26}$$

and therefore the round-trip operator is given by

$$U(t) \approx r_a^*(t)\ r_b(t) e^{-2i\Phi}. \tag{27}$$

2.3 Iterations

Summation approximation can be obtained as follows. After one round trip

$$E(t) = U(t)\ E(t-2T) + t_a\ E_{in}(t). \tag{28}$$

After two round trips

$$\begin{aligned} E(t) &= U(t)\ U(t-2T)\ E(t-4T) + && (29)\\ & t_a\ E_{in}(t) + && (30)\\ & t_a\ U(t)\ E_{in}(t-2T), && (31) \end{aligned}$$

and so on. After n round trips

$$\begin{aligned} E(t) &= U(t)\,U(t-2T)\ldots U(t+2T-2nT)\ E(t-2nT) + && (32)\\ & t_a\ E_{in}(t) + && (33)\\ & t_a\ U(t)\ E_{in}(t-2T) + \ldots + && (34)\\ & t_a\ U(t)\ U(t-2T)\ldots U(t+4T-2nT)\ E_{in}(t+2T-2nT). && (35) \end{aligned}$$

This equation can be written in terms of products as

$$\begin{aligned} E(t) &= E(t-2nT) \prod_{m=0}^{n-1} U(t-2mT) + && (36)\\ & t_a \sum_{p=0}^{n-1} E_{in}(t-2pT) \prod_{m=0}^{p-1} U(t-2mT). && (37) \end{aligned}$$

for $n = 1, 2, \ldots$.

2.4 n-Round Trip Approximation

Assume that the input fields do not change significantly over time of n round trips

$$E_{in}(t) \approx E_{in}(t-2T) \approx \ldots \approx E_{in}(t-2t-2nT). \tag{38}$$

Assume also that the round-trip cavity operators do not change significantly over time of n round trips

$$U(t) \approx U(t-2T) \approx \ldots \approx U(t-2t-2nT). \tag{39}$$

Then the selfconsistent equation becomes

$$E(t) = U(t)^n E(t-2nT) + S_n(t) t_a \; E_{in}(t). \tag{40}$$

where

$$S_n(t) = 1 + U(t) + \ldots + U(t)^{n-1}. \tag{41}$$

The summation can be done exactly

$$S_n(t) = \frac{1-U(t)^n}{1-U(t)}. \tag{42}$$

The limit of infinite number of round trips is

$$n \;\; \rightarrow \;\; \infty, \tag{43}$$

$$S_n(t) \;\; \rightarrow \;\; \frac{1}{1-U(t)}, \tag{44}$$

and the n-round trip approximation becomes the adiabatic approximation.

3 Dual Recycling Cavity

3.1 Notations and Conventions

3.1.1 Field Notations

The fields are

E_{in}	- incoming fields ,
E_{out}	- outgoing fields .
E	- intertal fields (moving towards BS),
E'	- intertal fields (moving away from BS).

Assume that the incoming fields E_{in} are known. Then the internal fields, E, E', and the outgoing fields, E_{out}, are calculated for given mirror motions.

Individual fields are denoted

$$E = \{A, B, C, D\}, \tag{45}$$

corresponding to the mirror or port as shown in Figure 1.

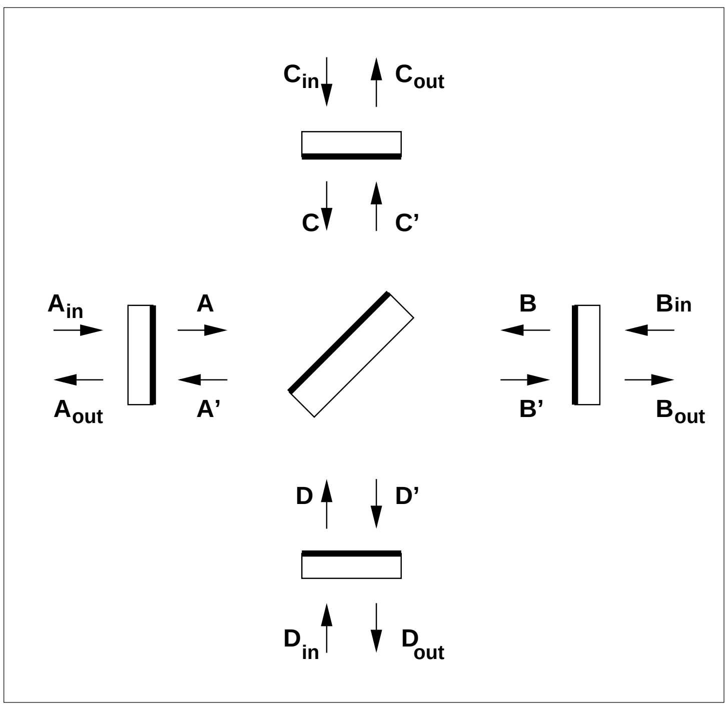


Figure 1: Fields in Dual Recycling Cavity

3.1.2 Mirror Positions

The displacements of each mirror relative to their reference planes are

x_a	- displacement of Mirror "a" ,
x_b	- displacement of Mirror "b" ,
x_c	- displacement of Mirror "c" ,
x_d	- displacement of Mirror "d" ,
x_0	- displacement of Beam Splitter .

3.1.3 Mirror Reflectivities

Nominal (amplitude) reflectivities are

$r_a^{(0)}$	- reflectivity of Mirror "a" ,
$r_b^{(0)}$	- reflectivity of Mirror "b" ,
$r_c^{(0)}$	- reflectivity of Mirror "c" ,
$r_d^{(0)}$	- reflectivity of Mirror "d" ,
$r_0^{(0)}$	- reflectivity of Beam Splitter .

The extra phases which occur in the reflections off moving mirrors are "hidden" in the corresponding reflectivities:

$$r_a = \sigma_a \, r_a^{(0)} \, e^{2ikx_a}, \tag{46}$$

$$r_b = \sigma_b \, r_b^{(0)} \, e^{2ikx_b}, \tag{47}$$

$$r_c = \sigma_c \, r_c^{(0)} \, e^{2ikx_c}, \tag{48}$$

$$r_d = \sigma_d \, r_d^{(0)} \, e^{2ikx_d}, \tag{49}$$

$$r_0 = \sigma_0 \, r_0^{(0)} \, e^{\sqrt{2}ikx_0}. \tag{50}$$

The sign convention is

$\sigma_j = +1$	- if the orientation of mirror as in Fig.1,
$\sigma_j = -1$	- if the orientation of mirror is opposite

3.2 Propagators and Phases

3.2.1 Detuning Phases

Define detuning phases for each length as

$$U_j = k_0 L_j, \tag{51}$$

where k_0 is the carrier wavenumber and L_j are the lengths. In the program these detuning phases must be specified explicitly. (Their default values are zero.)

Then the phase acquired in one-way trips from mirror to mirror are

$$\begin{aligned} V_{aob} &= U_{ao} + U_{bo}, & (52) \\ V_{aoc} &= U_{ao} + U_{co}, & (53) \\ V_{dob} &= U_{do} + U_{bo}, & (54) \\ V_{doc} &= U_{do} + U_{co}, & (55) \end{aligned}$$

3.2.2 Propagation Phases

The phases associated with each one-way trip in the dual recycling cavity are

$$\begin{aligned} W_{aob} &= V_{aob} + KL_{aob}, & (56) \\ W_{aoc} &= V_{aob} + KL_{aoc}, & (57) \\ W_{dob} &= V_{aob} + KL_{dob}, & (58) \\ W_{doc} &= V_{aob} + KL_{doc}, & (59) \end{aligned}$$

where K is the sideband wavenumber relative to the carrier:

$$K = sK_{mod}. \tag{60}$$

3.2.3 Propagators

The one-way propagators for fields in the dual-recycling cavity are

$$\begin{aligned} P_{aob} &= e^{-iW_{aob}}, & (61) \\ P_{aoc} &= e^{-iW_{aoc}}, & (62) \\ P_{dob} &= e^{-iW_{dob}}, & (63) \\ P_{doc} &= e^{-iW_{doc}}, & (64) \end{aligned}$$

3.3 Equations for Fields

The internal fields satisfy the equations

$$A(t) = t_a\ A_{in}(t) - r_a\ A'(t), \tag{65}$$

$$B(t) = t_b\ B_{in}(t) - r_b\ B'(t), \tag{66}$$

$$C(t) = t_c\ C_{in}(t) - r_c\ C'(t), \tag{67}$$

$$D(t) = t_d\ D_{in}(t) - r_d\ D'(t), \tag{68}$$

and also

$$A'(t) = t_0\ P_{aob}\ B(t - T_{aob}) - r_0^*\ P_{aoc}\ C(t - T_{aoc}), \tag{69}$$

$$B'(t) = t_0\ P_{aob}\ A(t - T_{aob}) + r_0\ P_{dob}\ D(t - T_{dob}), \tag{70}$$

$$C'(t) = t_0\ P_{doc}\ D(t - T_{doc}) - r_0^*\ P_{aoc}\ A(t - T_{aoc}), \tag{71}$$

$$D'(t) = t_0\ P_{doc}\ C(t - T_{doc}) + r_0\ P_{dob}\ B(t - T_{dob}). \tag{72}$$

Then the equations for outgoing fields are

$$A_{out}(t) = r_a^*\ A_{in}(t) + t_a\ A'(t), \tag{73}$$

$$B_{out}(t) = r_b^*\ B_{in}(t) + t_b\ B'(t), \tag{74}$$

$$C_{out}(t) = r_c^*\ C_{in}(t) + t_c\ C'(t), \tag{75}$$

$$D_{out}(t) = r_d^*\ D_{in}(t) + t_d\ D'(t). \tag{76}$$

3.4 4×4 Matrix Representation

Eliminate the primed fields A', B', C', D'. The result is

$$A(t) = -t_0 r_a \; P_{aob} \; B(t - T_{aob}) + r_0^* r_a \; P_{aoc} \; C(t - T_{aoc}) + t_a A_{in}(t), \quad (77)$$

$$B(t) = -t_0 r_b \; P_{aob} \; A(t - T_{aob}) - r_0 r_b \; P_{dob} \; D(t - T_{dob}) + t_b B_{in}(t), \quad (78)$$

$$C(t) = -t_0 r_c \; P_{doc} \; D(t - T_{doc}) + r_0^* r_c \; P_{aoc} \; A(t - T_{aoc}) + t_c C_{in}(t), \quad (79)$$

$$D(t) = -t_0 r_d \; P_{doc} \; C(t - T_{doc}) - r_0 r_d \; P_{dob} \; B(t - T_{dob}) + t_d D_{in}(t). \quad (80)$$

These equations can be written as

$$X(t) = M(t) \; X(t - 2T) + F(t), \quad (81)$$

where

$$X(t) = \begin{bmatrix} A(t) \\ B(t) \\ C(t) \\ D(t) \end{bmatrix}, \qquad F(t) = \begin{bmatrix} t_a A_{in}(t) \\ t_b B_{in}(t) \\ t_c C_{in}(t) \\ t_d D_{in}(t) \end{bmatrix}, \quad (82)$$

and $M(t)$ is the 4×4 matrix

$$M = \begin{pmatrix} 0 & -t_0 r_a P_{aob} & r_0^* r_a P_{aoc} & 0 \\ -t_0 r_b P_{aob} & 0 & 0 & -r_0 r_b P_{dob} \\ r_0^* r_c P_{aoc} & 0 & 0 & -t_0 r_c P_{doc} \\ 0 & -r_0 r_d P_{dob} & -t_0 r_d P_{doc} & 0 \end{pmatrix}. \quad (83)$$

The result of n iterations is given by

$$X(t) = M(t)^n \; X(t - 2nT) + S_n(t) \; F(t), \quad (84)$$

where

$$S_n(t) = [1 - M(t)]^{-1} [1 - M^n(t)]. \quad (85)$$

3.5 2×2-Block Matrix Representation

An alternative approach is to reduce the 4×4 matrix into two 2×2 matrices. This reduction is based on the fact that the square of the matrix M is block diagonal.

3.5.1 Second Iteration

$$
\begin{aligned}
A(t) \quad = \quad & + A(t - T_{aob} - T_{boa})P^2 \; t_0 \; r_b \; t_0 \; r_a && (86)\\
& + A(t - T_{aoc} - T_{coa})P^2 \; r_0^* \; r_c \; r_0^* \; r_a && (87)\\
& + D(t - T_{dob} - T_{boa})P^2 \; r_0 \; r_b \; t_0 \; r_a && (88)\\
& - D(t - T_{doc} - T_{coa})P^2 \; t_0 \; r_c \; r_0^* \; r_a && (89)\\
& - B_{in}(t - T_{boa})P \; t_b \; t_0 \; r_a && (90)\\
& + C_{in}(t - T_{coa})P \; t_c \; r_0^* \; r_a && (91)\\
& + A_{in}(t) \; t_a \; , && (92)\\
& && (93)\\
B(t) \quad = \quad & + B(t - T_{boa} - T_{aob})P^2 \; t_0 \; r_a \; t_0 \; r_b && (94)\\
& + B(t - T_{bod} - T_{dob})P^2 \; r_0 \; r_d \; r_0 \; r_b && (95)\\
& - C(t - T_{coa} - T_{aob})P^2 \; r_0^* \; r_a \; t_0 \; r_b && (96)\\
& + C(t - T_{cod} - T_{dob})P^2 \; t_0 \; r_d \; r_0 \; r_b && (97)\\
& - A_{in}(t - T_{aob})P \; t_a \; t_0 \; r_b && (98)\\
& - D_{in}(t - T_{dob})P \; t_d \; r_0 \; r_b && (99)\\
& + B_{in}(t) \; t_b \; , && (100)\\
& && (101)\\
C(t) \quad = \quad & + B(t - T_{bod} - T_{doc})P^2 \; r_0 \; r_d \; t_0 \; r_c && (102)\\
& - B(t - T_{boa} - T_{aoc})P^2 \; t_0 \; r_a \; r_0^* \; r_c && (103)\\
& - C(t - T_{cod} - T_{doc})P^2 \; t_0 \; r_d \; t_0 \; r_c && (104)\\
& + C(t - T_{coa} - T_{aoc})P^2 \; r_0^* \; r_a \; r_0^* \; r_c && (105)\\
& - D_{in}(t - T_{doc})P \; t_d \; t_0 \; r_c && (106)\\
& + A_{in}(t - T_{aoc})P \; t_a \; r_0^* \; r_c && (107)\\
& + C_{in}(t) \; t_c \; , && (108)\\
& && (109)\\
D(t) \quad = \quad & - A(t - T_{aoc} - T_{cod})P^2 \; r_0^* \; r_c \; t_0 \; r_d && (110)\\
& + A(t - T_{aob} - T_{bod})P^2 \; t_0 \; r_b \; r_0 \; r_d && (111)\\
& + D(t - T_{doc} - T_{cod})P^2 \; t_0 \; r_c \; t_0 \; r_d && (112)\\
& + D(t - T_{dob} - T_{bod})P^2 \; r_0 \; r_b \; r_0 \; r_d && (113)\\
& - C_{in}(t - T_{cod})P \; t_c \; t_0 \; r_d && (114)\\
& - B_{in}(t - T_{bod})P \; t_b \; r_0 \; r_d && (115)\\
& + D_{in}(t) \; t_d \; . && (116)
\end{aligned}
$$

3.5.2 Factorization

Equations for A and D

$$
\begin{aligned}
A(t) \quad = \quad & +A(t-2T)P^2\ (t_0\ r_b\ t_0 + r_0^*\ r_c\ r_0^*)\ r_a & (117)\\
& +D(t-2T)P^2\ (-t_0\ r_c\ r_0^* + r_0\ r_b\ t_0)\ r_a & (118)\\
& -B_{in}(t-T)P\ t_b\ t_0\ r_a & (119)\\
& +C_{in}(t-T)P\ t_c\ r_0^*\ r_a & (120)\\
& +A_{in}(t)\ t_a & (121)\\
& & (122)\\
D(t) \quad = \quad & +A(t-2T)P^2\ (t_0\ r_b\ r_0 - r_0^*\ r_c\ t_0)\ r_d & (123)\\
& +D(t-2T)P^2\ (t_0\ r_c\ t_0 + r_0\ r_b\ r_0)\ r_d & (124)\\
& -C_{in}(t-T)P\ t_c\ t_0\ r_d & (125)\\
& -B_{in}(t-T)P\ t_b\ r_0\ r_d & (126)\\
& +D_{in}(t)\ t_d. & (127)
\end{aligned}
$$

Equations for B and C

$$
\begin{aligned}
B(t) \quad = \quad & +B(t-2T)P^2\ (t_0\ r_a\ t_0 + r_0\ r_d\ r_0)\ r_b & (128)\\
& +C(t-2T)P^2\ (t_0\ r_d\ r_0 - r_0^*\ r_a\ t_0)\ r_b & (129)\\
& -A_{in}(t-T)P\ t_a\ t_0\ r_b & (130)\\
& -D_{in}(t-T)P\ t_d\ r_0\ r_b & (131)\\
& +B_{in}(t)\ t_b, & (132)\\
& & (133)\\
C(t) \quad = \quad & +B(t-2T)P^2\ (-t_0\ r_a\ r_0^* + r_0\ r_d\ t_0)\ r_c & (134)\\
& +C(t-2T)P^2\ (-t_0\ r_d\ t_0 + r_0^*\ r_a\ r_0^*)\ r_c & (135)\\
& -D_{in}(t-T)P\ t_d\ t_0\ r_c & (136)\\
& +A_{in}(t-T)P\ t_a\ r_0^*\ r_c & (137)\\
& +C_{in}(t)\ t_c. & (138)
\end{aligned}
$$

3.5.3 2-Vectors

Introduce $F_{1,2}$ and $G_{1,2}$ as follows

$$
\begin{aligned}
F_1(t) &= -B_{in}(t-T)P\; t_b\; t_0\; r_a + C_{in}(t-T)P\; t_c\; r_0^*\; r_a + A_{in}(t)\; t_a\;, && (139)\\
& && (140)\\
F_2(t) &= -C_{in}(t-T)P\; t_c\; t_0\; r_d - B_{in}(t-T)P\; t_b\; r_0\; r_d + D_{in}(t)\; t_d\;, && (141)\\
& && (142)\\
G_1(t) &= -A_{in}(t-T)P\; t_a\; t_0\; r_b - D_{in}(t-T)P\; t_d\; r_0\; r_b + B_{in}(t)\; t_b\;, && (143)\\
& && (144)\\
G_2(t) &= -D_{in}(t-T)P\; t_d\; t_0\; r_c + A_{in}(t-T)P\; t_a\; r_0^*\; r_c + C_{in}(t)\; t_c\;. && (145)
\end{aligned}
$$

3.5.4 Matrix Equations

Then combine these into complex 2-vectors

$$
F(t) = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix}, \qquad G(t) = \begin{bmatrix} G_1(t) \\ G_2(t) \end{bmatrix}. \tag{146}
$$

Also form

$$
X(t) = \begin{bmatrix} A(t) \\ D(t) \end{bmatrix}, \qquad Y(t) = \begin{bmatrix} B(t) \\ C(t) \end{bmatrix}. \tag{147}
$$

Then the iteration equations can be written in the matrix form:

$$
\begin{aligned}
X(t) &= M\; X(t-2T) + F(t), && (148)\\
Y(t) &= N\; Y(t-2T) + G(t). && (149)
\end{aligned}
$$

where M and N are complex 2×2 matrices

$$
M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \qquad N = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}. \tag{150}
$$

Their coefficients are

$$
\begin{aligned}
m_{11} &= (t_0\; r_b\; t_0 + r_0^*\; r_c\; r_0^*)\; r_a P^2, && (151)\\
m_{12} &= (-t_0\; r_c\; r_0^* + r_0\; r_b\; t_0)\; r_a P^2, && (152)\\
m_{21} &= (t_0\; r_b\; r_0 - r_0^*\; r_c\; t_0)\; r_d P^2, && (153)\\
m_{22} &= (t_0\; r_c\; t_0 + r_0\; r_b\; r_0)\; r_d P^2. && (154)
\end{aligned}
$$

$$
\begin{aligned}
n_{11} &= (t_0\; r_a\; t_0 + r_0\; r_d\; r_0)\; r_b P^2, && (155)\\
n_{12} &= (t_0\; r_d\; r_0 - r_0^*\; r_a\; t_0)\; r_b P^2, && (156)\\
n_{21} &= (-t_0\; r_a\; r_0^* + r_0\; r_d\; t_0)\; r_c P^2, && (157)\\
n_{22} &= (-t_0\; r_d\; t_0 + r_0^*\; r_a\; r_0^*)\; r_c P^2. && (158)
\end{aligned}
$$

3.5.5 Summation Approximation

The result of n iterations is given by

$$X(t) = M^n \; X(t - 2nT) + S_n \; F(t) \tag{159}$$

where

$$S_n = (1 - M)^{-1}(1 - M^n) \tag{160}$$

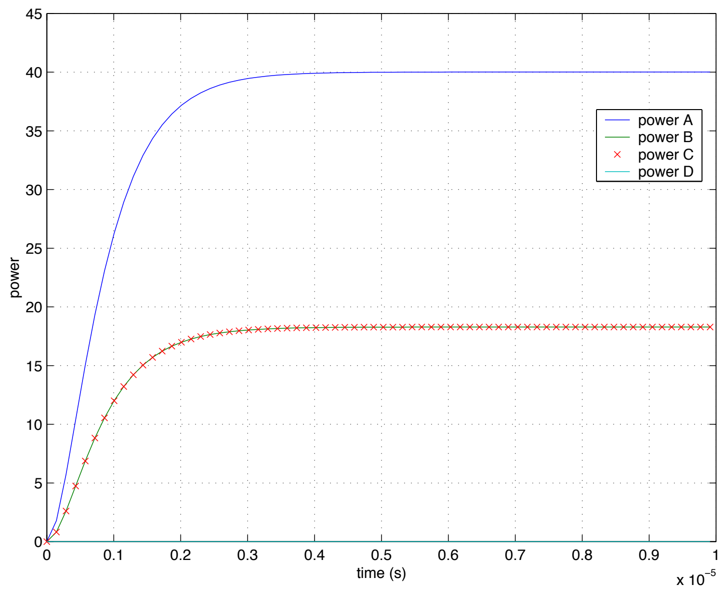
A similar result can be obtained for Y-vector:

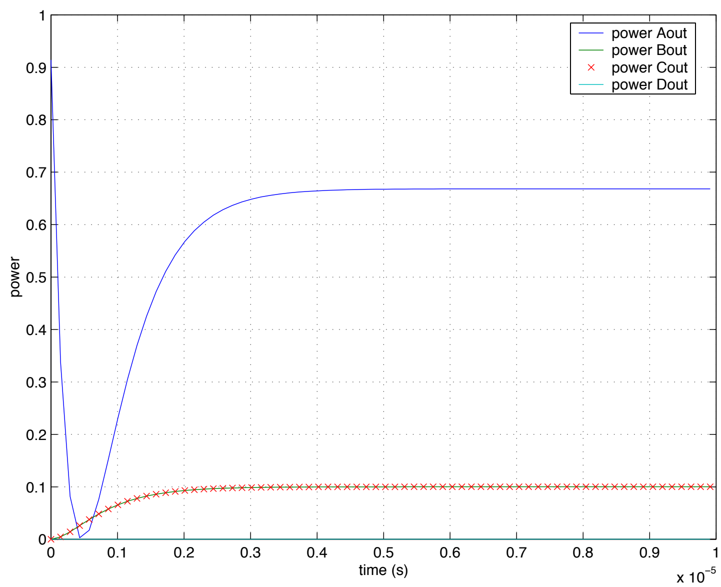
$$Y(t) = N^n \; Y(t - 2nT) + R_n \; G(t) \tag{161}$$

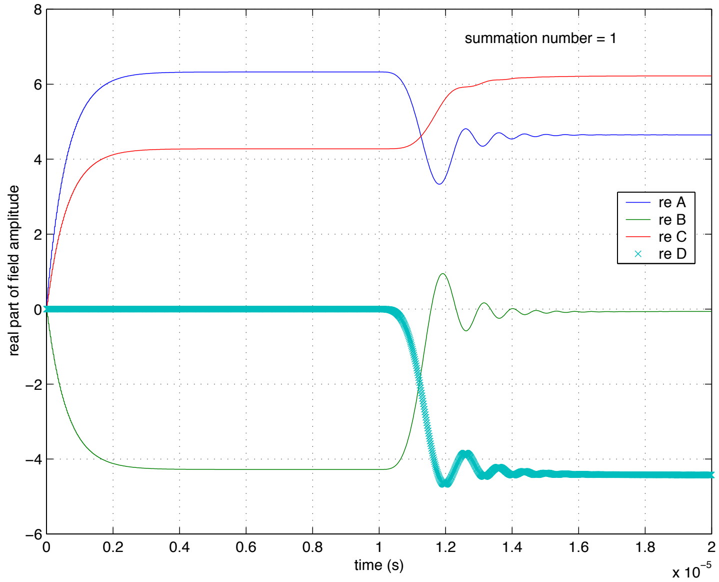
where

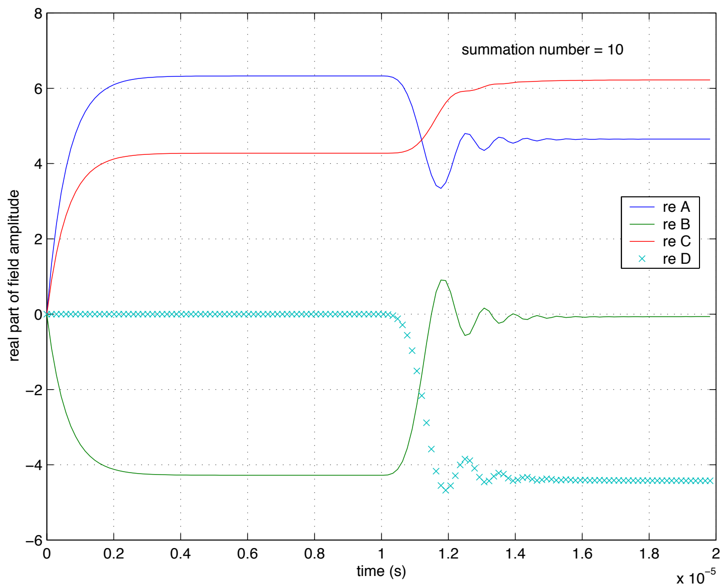
$$R_n = (1 - N)^{-1}(1 - N^n) \tag{162}$$

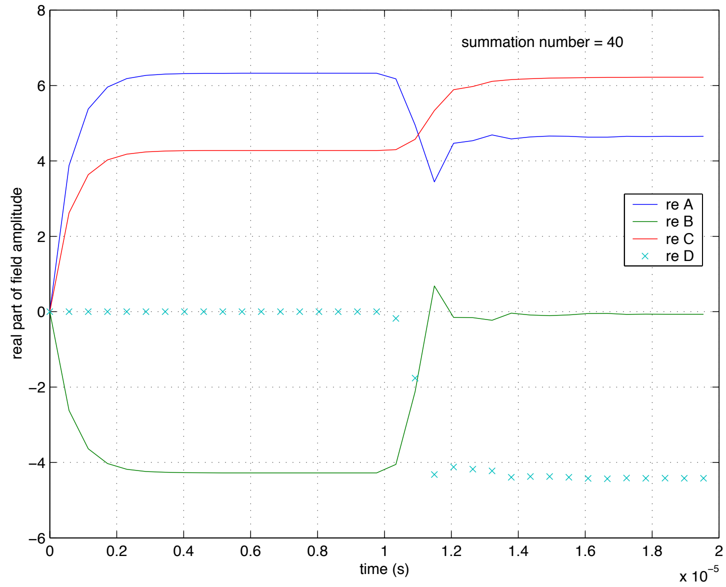
4 Numerical Simulation

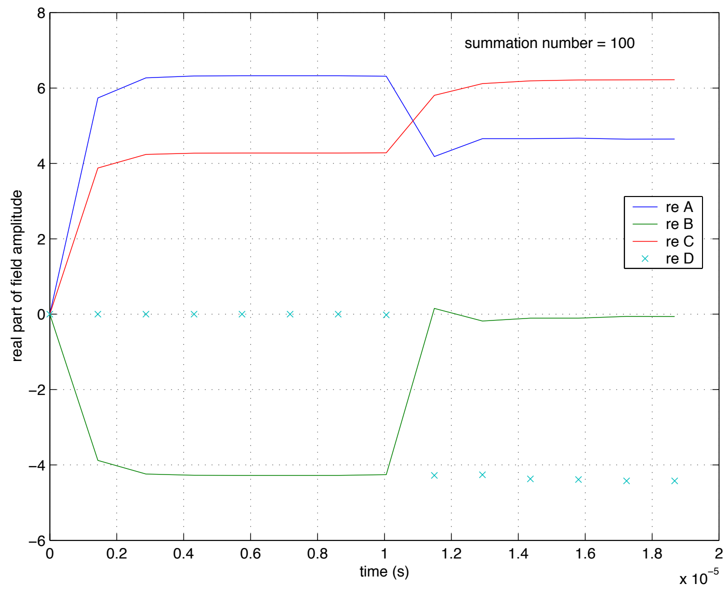


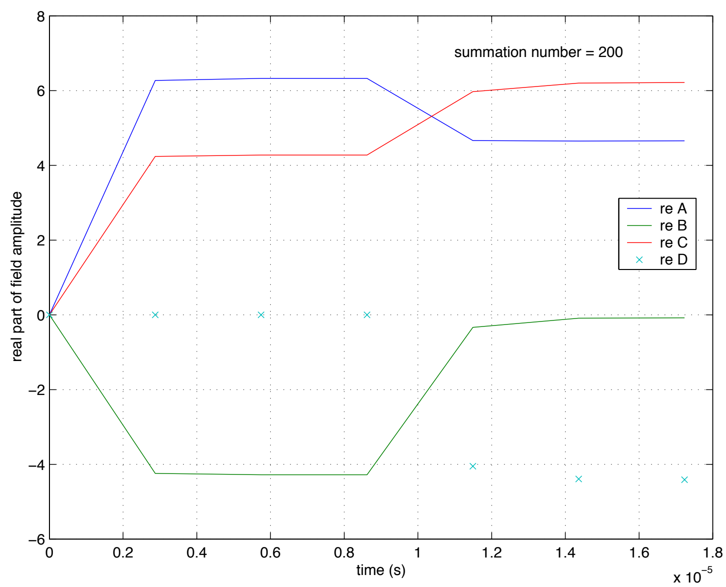



summation number = 1
re A
re B
re C
re D
real part of field amplitude
time (s)
x 10^-5


summation number = 10
re A
re B
re C
re D
real part of field amplitude
time (s)
x 10^-5


summation number = 40
re A
re B
re C
re D
real part of field amplitude
time (s)
x 10⁻⁵
8
6
4
2
0
−2
−4
−6
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


summation number = 100
re A
re B
re C
re D
real part of field amplitude
time (s)
x 10^−5


summation number = 200
re A
re B
re C
re D
real part of field amplitude
time (s)
x 10^−5