LIGO 1 Suspension: Threedimensional Model for E2E

Program Restart

Lagrangian

Kinetic Energy

$$T := \frac{1}{2} M (X_v^{\,2} + Y_v^{\,2} + Z_v^{\,2}) + \frac{1}{2} II_2 \Theta_{v2}^{\,2} + \frac{1}{2} II_1 \Phi_{v2}^{\,2} + \frac{1}{2} II_3 \Sigma_{v1}^{\,2}$$

Potential Energy

$$U := \frac{1}{2} k_a (\Xi_1^{\,2} + \Xi_2^{\,2}) + \frac{1}{2} M g \left(\frac{1}{4} \frac{d\, \mathrm{D0}\, \Sigma_1^{\,2}}{H} + H \Theta_1^{\,2} + h \Theta_2^{\,2} + H \Phi_1^{\,2} + h \Phi_2^{\,2} \right)$$

$$\Xi_1 := \frac{\frac{1}{2} \mathrm{D0}\, \Phi_2 + y_1 - y_0}{\alpha}$$

$$\Xi_2 := \frac{-\frac{1}{2} \mathrm{D0}\, \Phi_2 + y_1 - y_0}{\alpha}$$

$$\Theta_2 := \theta_2$$

$$\Phi_2 := \phi_2$$

$$\Theta_1 := \frac{z_1 - z_0}{H} - \frac{h \theta_2}{H}$$

$$\Phi_1 := \frac{x_1 - x_0}{H} - \frac{h \phi_2}{H} + \phi_0$$

$$\Sigma_1 := \sigma_1 - \sigma_0$$

$$Lagr := \frac{1}{2} M (X_v^{\,2} + Y_v^{\,2} + Z_v^{\,2}) + \frac{1}{2} II_2 \Theta_{v2}^{\,2} + \frac{1}{2} II_1 \Phi_{v2}^{\,2} + \frac{1}{2} II_3 \Sigma_{v1}^{\,2} - \frac{1}{2} k_a \left(\frac{\left(\frac{1}{2} \mathrm{D0}\, \phi_2 + y_1 - y_0\right)^2}{\alpha^2} + \frac{\left(-\frac{1}{2} \mathrm{D0}\, \phi_2 + y_1 - y_0\right)^2}{\alpha^2} \right)$$

$$-\frac{1}{2} M g \left(\frac{1}{4} \frac{d\, \mathrm{D0}\, (\sigma_1 - \sigma_0)^2}{H} + H \left(\frac{z_1 - z_0}{H} - \frac{h\, \theta_2}{H} \right)^2 + h\, \theta_2^{\ 2} + H \left(\frac{x_1 - x_0}{H} - \frac{h\, \phi_2}{H} + \phi_0 \right)^2 + h\, \phi_2^{\ 2} \right)$$

Stiff and Mass Matrices

$$p0 := [x_0, y_0, z_0, 0, \phi_0, \sigma_0]$$

$$p1 := [x_1, y_1, z_1, \theta_2, \phi_2, \sigma_1]$$

$$vp1 := [X_v, Y_v, Z_v, \Theta_{v2}, \Phi_{v2}, \Sigma_{v1}]$$

$$UBase := \begin{bmatrix} \frac{M g}{H} & 0 & 0 & 0 & -\frac{M g h}{H} & 0 \\ 0 & 2 \frac{k_a}{\alpha^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{M g}{H} & -\frac{M g h}{H} & 0 & 0 \\ 0 & 0 & -\frac{M g h}{H} & \frac{M g h (h + H)}{H} & 0 & 0 \\ -\frac{M g h}{H} & 0 & 0 & 0 & \frac{1}{2} \frac{2 M g h^2 \alpha^2 + 2 M g h \alpha^2 H + k_a \mathrm{D0}^2 H}{\alpha^2 H} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \frac{M g d\, \mathrm{D0}}{H} \end{bmatrix}$$

$$U0 := UBase$$

$$\Omega_3 := \Omega_1$$

$$\begin{bmatrix} \Omega_1^{\,2} M & 0 & 0 & 0 & -h\,\Omega_1^{\,2} M & 0 \\ 0 & \Omega_2^{\,2} M & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_1^{\,2} M & -h\,\Omega_1^{\,2} M & 0 & 0 \\ 0 & 0 & -h\,\Omega_1^{\,2} M & \Omega_4^{\,2} II_2 & 0 & 0 \\ -h\,\Omega_1^{\,2} M & 0 & 0 & 0 & \Omega_5^{\,2} II_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Omega_6^{\,2} II_3 \end{bmatrix}$$

$$\begin{bmatrix} M & 0 & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & 0 & 0 & II_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & II_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & II_3 \end{bmatrix}$$

Suspension Point Matrices

$$SusCoord := [x_0, y_0, z_0, dummy, \phi_0, \sigma_0]$$

$$SU0 := \begin{bmatrix} -\frac{M\,g}{H} & 0 & 0 & 0 & M\,g & 0 \\ 0 & -2\,\frac{k_a}{\alpha^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{M\,g}{H} & 0 & 0 & 0 \\ 0 & 0 & \frac{M\,g\,h}{H} & 0 & 0 & 0 \\ \frac{M\,g\,h}{H} & 0 & 0 & 0 & -M\,g\,h & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}\,\frac{M\,g\,d\,\mathrm{D0}}{H} \end{bmatrix}$$

$$\begin{bmatrix} -\Omega_1^{\;2}\,M & 0 & 0 & 0 & M\,g & 0 \\ 0 & -\Omega_2^{\;2}\,M & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Omega_1^{\;2}\,M & 0 & 0 & 0 \\ 0 & 0 & h\,\Omega_1^{\;2}\,M & 0 & 0 & 0 \\ h\,\Omega_1^{\;2}\,M & 0 & 0 & 0 & -M\,g\,h & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}\,d\,\mathrm{D0}\,\Omega_1^{\;2}\,M \end{bmatrix}$$

Equation of motion (code ad hoc)

$$-\omega^2 x_1 = \Omega_1{}^2 x_1 - h\,\Omega_1{}^2 \phi_2 + \Omega_1{}^2 x_0 - g\,\phi_0$$

$$-\omega^2 y_1 = \Omega_2{}^2 y_1 + \Omega_2{}^2 y_0$$

$$-\omega^2 z_1 = \Omega_1{}^2 z_1 - h\,\Omega_1{}^2 \theta_2 + \Omega_1{}^2 z_0$$

$$-\omega^2 \theta_2 = -\frac{h\,\Omega_1{}^2 M z_1}{II_2} + \Omega_4{}^2 \theta_2 - \frac{h\,\Omega_1{}^2 M z_0}{II_2}$$

$$-\omega^2 \phi_2 = -\frac{h\,\Omega_1{}^2 M x_1}{II_1} + \Omega_5{}^2 \phi_2 - \frac{h\,\Omega_1{}^2 M x_0}{II_1} + \frac{M g h \phi_0}{II_1}$$

$$-\omega^2 \sigma_1 = \Omega_6{}^2 \sigma_1 + \frac{\frac{1}{4} d\,\mathrm{D0}\,\Omega_1{}^2 M \sigma_0}{II_3}$$