

# Advanced LIGO Suspension System Conceptual Design

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## **1. Introduction**

*This document has been updated in August/September 2003 to reflect the current status of designs following changes in requirements and developments in design since the last version was produced. These updates are given in italics throughout the document. The update referred to on Feb 5<sup>th</sup> 2002 has been superseded.*

*Reference should be made to “Cavity Optics Suspension Subsystem Design Requirements Document” DCC # LIGO-T010007-01 or subsequent revised version for full details of requirements.*

*The major updates to this document are to sections 3.1 and 3.2 giving the projected performance for current parameters of quadruple and triple suspensions. Those sections have been completely rewritten. For reference, the old versions of these sections are now included as an appendix – appendix D.*

The suspension system design for Advanced LIGO is based on the triple pendulum design developed for GEO 600 – the German/UK detector. This differs quite radically from the existing LIGO suspension design which has test masses hung as single pendulums on wire slings, with actuation for both damping the pendulum modes and global control of the interferometer being applied directly to the test masses via coil and magnet systems, with the magnets attached to the masses. The main features in the GEO design are as follows:

- The fused silica mirrors (6 kg) form the lowest stage of a triple pendulum, and are suspended on 4 vertical fused silica fibres of circular cross-section to reduce suspension thermal noise.
- The penultimate mass is also made of fused silica identical in size to the mirror.
- The fibres are welded to fused silica “ears” or prisms which are silicate bonded to the flat sides of the penultimate mass and the mirror below.
- Included in the triple pendulum are two stages of cantilever springs made of maraging steel to enhance the vertical seismic isolation.
- The damping of all of the low frequency modes of the triple pendulum is achieved by using 6 co-located sensors and actuators at the highest mass of the triple pendulum. To achieve adequate damping the design of the triple pendulum has to be such that all the modes couple well to motion of the top mass.

- DC alignment of mirror yaw and pitch is done by applying forces to the actuators at the highest mass. This requires that the penultimate mass and the mirror are each suspended by four wires, two on each side, so that the system behaves like a marionette from the highest mass downwards.
- Global control forces, including auto-alignment forces, are applied via a triple reaction pendulum, essentially identical in mechanical design to the main triple pendulum, but with wires replacing the silica fibres, and a metal penultimate mass.
- The global control is carried out using a split feedback system, with large low frequency motions applied magnetically between the penultimate masses, and small higher frequency signals applied electrostatically between the mirror and the corresponding lowest reaction mass which is made of silica with a patterned gold coating.

These techniques have been extended to meet the more stringent noise levels aimed at in Advanced LIGO. The key suspension areas in which improvements are required are the suspension thermal noise level, the overall isolation, and the isolation of electronic noise associated with the local control systems. The target sensitivity corresponds to a displacement sensitivity of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz at each of the main mirrors (ETM/ITM), *and falling off above 10 Hz as  $1/f^2$  approximately. To be more precise – the requirements call for the longitudinal thermal noise from the pendulum motion and the residual longitudinal seismic noise each to be at or below this noise level. Any technical noise sources such as for example residual sensor noise should be 1/10 of this figure.* In addition consideration of internal thermal noise has led to the decision to change the mirror material from fused silica. *This decision has not been made. A downselect between sapphire and silica is planned for first quarter 2004.* Taking all these factors into account the Advanced LIGO baseline design requires a quadruple pendulum with a final stage consisting of a 40 kg sapphire mass (*sapphire is the baseline*), suspended on fused silica ribbons or fibres for the main mirrors (ETM, ITM). A further requirement is that all pendulum modes which couple directly into the sensed direction should lie below 10 Hz. *This requirement has since been modified – reference “Low-Frequency Cutoff for Advanced LIGO”, P Fritschel et al (T020034-00-D). The recommendations of this paper with regard to the suspension design can be summarized as follows*

- *highest vertical mode frequency of 12 Hz or lower*
- *violin mode fundamental frequency of 400 Hz or higher*
- *horizontal thermal noise specified at  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  or lower at 10 Hz, per test mass*
- *technical noise sources (including local damping) at level to allow observations down to 10Hz.*

Other optics which may require a quadruple pendulum, are the beamsplitter and folding mirror (BS, FM). These optics however will be made of silica rather than sapphire, due to decreased sensitivity requirements of approx. factor of 100.

For the modecleaner mirrors and the recycling mirrors (MC1, MC2, MC3, PRM, SRM), the target sensitivity is taken as  $3 \times 10^{-17}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz, and this sensitivity should be achievable using a triple pendulum suspension. In fact the requirement for the recycling mirrors is relaxed from this figure.

*To clarify the above, for the modecleaner, the longitudinal displacement noise summed over all sources should meet the requirement  $3 \times 10^{-17}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz, and the corresponding number for the recycling mirror is  $4 \times 10^{-16}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz. Full details of requirements in other degrees of freedom can be found in LIGO T010007-01 or revision thereof.*

It should be noted that for achieving the required seismic isolation in Advanced LIGO, we assume that the isolation system will be such that the motion on its final stage is no greater than  $2 \times 10^{-13}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz in both horizontal and vertical directions, as specified in the Advanced LIGO Seismic Isolation Subsystem Design Requirements Document (LIGO-E990303-03-D). The overall isolation is thus taken to be the product of this figure and that achieved by the suspension system.

A full list of the sensitive mirrors and the current design parameters of mass, size, type of suspension, and reaction chain required, is presented as an appendix (Appendix A) to this document. Each of these different mirrors will require a detailed design to be worked out. However the purpose of this paper is to present conceptual ideas. Thus what we will discuss more fully below in section 2 are general design criteria which will apply to a quadruple and triple suspension. In section 3 we will present expected performance graphs covering the key areas of thermal noise, isolation and damping. These will be given for two examples, firstly a 40 kg sapphire quadruple suspension for E/ITM and secondly a modecleaner triple suspension. *Thermal noise graphs for a recycling mirror suspension are now also included.* We argue that if these can meet their target sensitivities, then slight modifications can be made to design all the other suspensions required. This same applies to the situation of the fallback option of silica E/ITM mirrors, where the main difference will be the larger diameter (34 cm vs. 31.4 cm).

Work on an all-metal prototype quadruple suspension is already well underway, *now largely finished*, the parts having been designed and procured in Glasgow, and sent out and assembled in the lab at MIT. Some details of this work are presented in Appendix B, with further references.

This document complements and adds to the original baseline design document produced by the GEO suspension team in Jan 2000 (LIGO-T000012-00).

## **2. Design Description**

There are several factors which the design has to address. Firstly there is the key issue of thermal noise performance. Secondly there are the issues of isolation and local damping. Thirdly there is the provision of suitable method(s) of actuating for global control. We make use of two main tools for this work:

- Thermal noise design (*Maple code, reference G Cagnoli, Glasgow, for further information*)
- Mechanical design and performance simulation (MATLAB code, now encapsulated in SIMULINK models, C Torrie and K Strain) *Examples of these codes can currently be found via C Torrie's web page*

<http://www.ligo.caltech.edu/~ctorrie/>. A new SUS modelling toolkit structure which separates parameter-setting code from calculation code is being developed by Mark Barton and in the future this will be the repository for the various MATLAB/SIMULINK models.

## 2.1 Suspension Thermal Noise Issues

The thermal noise performance of the suspension is the paramount design driver. The main contribution comes from the dissipation in the fused silica fibres used to suspend the mirror, giving a direct horizontal noise component. To minimise this noise, the baseline design for the E/ITM mirrors incorporates ribbons rather than fibres of circular cross-section, so that the dilution factor, by which the pendulum loss factor is reduced from the value of the intrinsic loss factor of the suspension material, is increased. Moreover moving to ribbons of the same cross-section also has the advantage of pushing up the thermoelastic peak in frequency, which has the effect of reducing the loss at the critical 10 Hz region. *See also possibility of using “dumbbell fibres” section 2.2*

Another strong contributor to the thermal noise spectrum arises from the flexing of the lowest set of blade springs, giving a vertical noise component which will couple into horizontal motion. In general thermal noise arising further up the pendulum chain is filtered by the stages below. However the vertical frequency of the final stage is necessarily higher than the horizontal frequency, since no blades are included at that stage, and thus there is less vertical filtering than horizontal filtering in the final stage. (The use of blades at the last stage was considered, but no solution has yet been identified that would give the required performance with acceptable technical risk.)

The baseline design for Advanced LIGO at present calls for a noise level at each of the test mirrors of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz, falling off above this. To achieve such a target requires that the highest vertical mode of the multiple pendulum should be kept below 10 Hz – otherwise a peak in the spectrum will occur in the operational frequency band of the detector. It should however be noted that the position of this mode is an outstanding issue being considered by members of the Advanced LIGO systems design team. *NB: This has since been reconsidered in the Fritschel et al paper referenced in section 1 above.*

The highest mode essentially corresponds to relative vertical motion of the mirror with respect to the penultimate mass. To push this frequency down, we use a combination of several factors:

- a) the fibre length is chosen as long as practicable consistent with ease of production, (but not so long that the violin modes are too low). 60 cm is the current design target.
- b) the fibre cross-section is chosen to be as small as practicable, consistent with working at least a factor of 3 away from the breaking strain seen for typical circular cross section fibres, assumed for this design exercise to be applicable to ribbons.
- c) the penultimate mass is chosen to be as heavy as possible, consistent with the overall design characteristics of the multiple pendulum. In the baseline design we have chosen to make this mass approximately double the mass of the mirror, limited by the availability of suitable materials from which to fabricate

*it. This is now superseded, given the revised requirements on cut-off – latest design has the penultimate mass the same mass as the test mass.*

To achieve c), a possible material for use is heavy glass – i.e. glass doped with lead oxide or bismuth oxide. Such glasses can have densities up to  $\sim 7200 \text{ kg m}^{-3}$ , (c.f. sapphire - density  $3980 \text{ kgm}^{-3}$ .) Work on investigating bonding of silica to such heavy glasses is underway. *Heavy glass, but not of such high density as  $7200 \text{ kg m}^{-3}$  is still under consideration. For example SF4 with density  $4800 \text{ kg m}^{-3}$  is a potential material for the penultimate mass in the ETM/ITM chains.*

## **2.2 Ribbons versus Fibres**

There are several potential advantages of using ribbons rather than fibres. Not only can the dilution factor be made larger for ribbons, but also reducing the thickness of the flexing element raises the frequency at which the maximum loss due to thermoelastic damping occurs. Thus, if by suitable choice of the ribbon dimensions this frequency is significantly further away from the crucial 10 Hz region than for cylindrical fibres, this can lead to a lower overall level of noise in this region.

However there are several other factors which need to be considered. Firstly, the recent work by Cagnoli and Willems has shown that there is a significant thermoelastic effect not previously considered, basically due to the variation of Young's modulus with temperature. This effect, in combination with the more familiar coefficient of thermal expansion, gives rise to an effective coefficient of thermal expansion which can be zero for a particular static stress. Hence under those conditions the thermoelastic damping goes to zero. The null condition can in principal be achieved by increasing the cross-section of the silica suspension over that which has been previously indicated as optimum from other design considerations. However increasing the cross-section to null the thermoelastic effect has two adverse consequences. Firstly a larger cross-section causes the highest vertical pendulum mode to be above 10 Hz. Secondly it pushes down the violin mode frequencies, thus increasing the number of these resonances which appear below 1 kHz. Thus for example working at the null for fibres one can achieve the target noise level at 10 Hz, but with a vertical mode above 10 Hz. Ribbons can still in principle achieve the target noise level at 10 Hz and above with a cross-section not optimised for the thermoelastic effect, whereas circular fibres cannot.

*An alternative possibility is to use “dumbbell fibres - circular cross-section fibres of varying cross-section , thicker near the ends and thinner in the middle section (ref Willems LIGO T020003). Such fibres could give good thermal noise performance; e.g. in this paper a model with 60 cm fibres, in which the middle 40 cm has diam. 380 micron and the ends have diam. 767 micron, gives a similar thermal noise curve to that shown in Figure 1 below.*

Another consideration is the breaking stress of ribbons versus fibres, and the ease with which they can be made. Measurements on fibres have shown that they can be as strong as high tensile steel, typical fibres produced for GEO having test strengths of  $\sim 3.5 \text{ GPa}$ . Ribbons of high breaking stress have yet to be developed, and this is an active area of research. *One early sample showed a strength of 3.75 GPa. However the latest data shows average breaking strength 1.18 GPa and maximum value 1.53 GPa. It has been found that shear stress due to shape imperfections is crucial in*

*limiting breaking strengths.* The development of techniques to pull ribbons of suitable cross-section and length, and to produce twists in them to avoid buckling effects as the mirror swings is another active area of research. *Current thinking is that twists may not be necessary.*

For this conceptual design we will show thermal noise curves for ribbons and fibre suspensions. *Fibres not shown in updated curves.* The final choice will depend on the results of further research work, and/or decisions about the desired low frequency target sensitivity and the level of the other limiting low frequency noise sources. Ribbons are currently the baseline, with fibres a low-risk fallback. *To clarify, ribbons are currently the baseline, dumbbell fibres are also being investigated as an alternative, and simple circular cross-section fibres are a third option with lower risk but poorer expected performance than ribbons or dumbbells.*

### **2.3 Thermal Noise Estimation for Quadruple Pendulum Suspension**

The thermal noise modelling which has been used for this estimation is carried out in the following way. The pendulum dynamics are simulated by four point-like masses linked by springs for both horizontal and vertical degrees of freedom, with no coupling between them. Suitable values to be used as input for the masses and other necessary parameters to calculate spring constants have previously been established using a MATLAB model of the quadruple pendulum, discussed more fully in the next section. The first three spring stages consist of maraging steel blades in series with steel wires, and the final (lowest) stage consists of silica fibres. The horizontal and vertical transfer functions are calculated separately and then combined to get the effective overall horizontal function, assuming a cross-coupling of vertical into horizontal of 0.1%. This is a figure we have used in GEO as a conservative estimate for cross-coupling, and is larger than the purely geometric effect due the curvature of the earth over the 4 km arms of LIGO. It is also larger than the typical size of cross-coupling effects due to mechanical imperfections in construction that we have estimated from modelling pendulums (work by Husman et al, *ref M Husman PhD thesis, University of Glasgow, 2000 and Husman et al Rev. Sci. Instrum. 71, 2546-2551, 2000*). Dissipation in the pendulum is introduced via the imaginary part of the spring constants, and hence using the fluctuation dissipation theorem the resulting thermal noise at the mirror in the horizontal direction is obtained.

Spring constants of the steel stages have been treated differently from the silica stage. In particular the spring constants for the steel stages are given by

$$k*(1 + i*\phi*Dilution)$$

where  $k$  is the spring constant modulus,  $\phi$  is the material loss angle and Dilution is the usual dilution factor (equal to 1 for vertical displacements). For silica, the spring constants have been worked out using the beam equation in which case the imaginary part comes from Young's modulus written as  $E*(1 + i*\phi)$ . As a consequence, the programme calculates the violin modes of the silica stage, but not of the steel stages.

The vertical restoring force is almost totally due to the action of the blades, the horizontal restoring force is essentially due to gravity. Dilution factors were calculated for the pendulum motion of the steel stages by considering the wire bending at both ends. Values ranged from 35 to 75, which are fairly modest and should be achievable in practice.

Loss angles for the materials arise as the sum of three parts: bulk, surface and thermoelastic effects.

- i) Bulk. This loss is assumed to be due to structural damping and hence is constant with frequency. For silica this contribution is negligible compared to the following two. For maraging steel the bulk loss angle is taken to be  $10^{-4}$ , based on measurements made in Glasgow, and for steel wires, the loss angle is taken to be  $2 \times 10^{-4}$ .
- ii) Surface. An estimate of this loss follows work by Gretarsson et al, which indicated that there is an energy loss proportional to the surface to volume ratio for silica which dominates the bulk dissipation. We have taken a value for the loss angle to be  $3 \times 10^{-11}/t$  or  $3 \times 10^{-11}/r$  for ribbons of thickness  $t$  or fibres of radius  $r$  respectively. For steel however this effect has been taken as negligible compared to the bulk loss.
- iii) Thermoelastic. This loss term has been considered in the pendulum motion of all 4 stages and in the vertical motion of the three steel stages in which the restoring force dominantly arises from the bending of the blades. The new thermoelastic effect, referred to above (*see paragraph 2 in section 2.2*), is applied in all cases except for the blades, since in them the static stress is anti-symmetrical with respect to the neutral line and hence the new effect is considered negligible.

## 2.4 Thermal Noise Estimation for Triple Pendulum Suspension

The thermal noise estimation for a triple pendulum suspension follows the above with the reduction of one stage of suspension. In the baseline design produced in Jan 2000 fibres were used in the final stage of the suspension to bring the thermal noise down to the necessary level assumed at that time. However, as indicated in section 3, with the more relaxed displacement noise specification, a steel wire suspension can in principle meet the target of  $3 \times 10^{-17}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz.

It is noted from *LIGO-T010007-01* that the vertical to horizontal coupling for the modecleaners may be taken as that due to the suspension – and therefore we use the figure of  $10^{-3}$  as assumed for the quadruple suspension. However for the SRM and PRM suspension the coupling is larger ( $1.8 \times 10^{-3}$ ), and this must be noted and altered accordingly in running the Maple code for these suspensions.

## 2.5 Modelling for Isolation, Damping and Control

Modelling for investigation and optimisation of the mechanical design for a quadruple suspension, with particular reference to the isolation and damping properties, has been carried out using an extension of the MATLAB model developed for the GEO 600 triple suspension. (*as referenced at the beginning of section 2*). The key elements of the design are very similar, with the addition of another stage. The aim has once again been to develop a model whose resonant frequencies all lie within a band from approximately  $\sim 0.5$  to  $\sim 4$  Hz, with the exception of the highest vertical and roll modes which are associated the extension of the silica fibres in the lowest pendulum stage. (*current models have the low frequencies lying in a slightly wider band, typically  $\sim 0.4$  to  $\sim 4.5$  Hz*). In addition we aim for good coupling of all the low frequency modes, so that damping of all such modes (typically to  $Q \sim 5$  in GEO) can be carried out at the top mass in the chain.

## 2.6 Mechanical Design

The mass at the top (*top mass*) is suspended from 2 cantilever-mounted, approximately trapezoidal pre-curved spring blades and 2 spring steel wires. The blades lie horizontally when loaded. The mass below this is suspended from 2

cantilever blades and 2 steel wire loops. The top mass and mass 2 (*upper intermediate mass*) have a ‘sandwich-type’ construction with the blades fitting in between, so that the break-off points for wires going both upwards and downwards lie close to the centre of mass of these masses. See figure 12 in Appendix B. Mass 3 (*penultimate mass*), which may be made of heavy glass, (*or silica or sapphire*) is suspended from 2 cantilever blades and 2 steel wire loops from mass 2. Fused silica ears silicate bonded to flats on the side of this mass form the breakoff points at the mass. Similar ears are bonded to the mirror (mass 4, *test mass*), and the final suspension is made by welding fibres or ribbons between the ears of masses 3 and 4, two fibres on each side.

There are several key points which differ from the original GEO design. Firstly, in order to achieve a smaller footprint, the blades are angled with respect to each other and crossed (as shown in appendix B). Secondly, due to space considerations, there are two blades rather than 4 at masses 1 and 2 each blade supporting two wires from its end. (*This comment applies to quadruple suspensions. For triples, the number of blades is 2 at the top of the suspension and 4 at the top mass, as in GEO*). As stated earlier the overall choice of number of wires or fibres is such that orientation of the mirror can be carried out from mass 1 (*the top mass*).

The blades are made from Marval 18 (18% Ni) maraging steel. The spring blades are stressed to approximately 800 MPa i.e one half of the elastic limit, which is a conservative stress limit. *We may revisit this criterion if higher blade internal mode frequencies are desired*. The clamps fitted to the ends of the blades should be as light as practicable to limit the effective mass of the blade/clamp unit.

There should be strong coupling of all degrees of freedom to motion of sensors/actuators at the top mass. *in order to achieve damping of all modes via damping of the top mass*. To a first approximation this is satisfied by having approximately the same mass in each stage, approximately the same moments of inertia about equivalent axes, and by suitable choices of wire angles and connection points. Typically connection points are all either 1 mm above or 1 mm below the plane of the centre of mass (in the direction of increasing stability). Further thermal noise considerations have necessitated the use of a significantly heavier penultimate mass than the other masses in the chain. *This is no longer the case*.

The MATLAB model *used for these calculations* consists ~~at present~~ of 4 uncoupled sets of dynamical equations, corresponding to vertical motion, yaw, longitudinal and pitch (together) and transverse and roll (together). To first order these motions are uncoupled in the GEO design. With the crossed blades in the LIGO design, there will be some coupling between the longitudinal/pitch and transverse/roll modes. ~~As yet~~ The model *as used* does not incorporate this coupling. However it is not expected to significantly affect either the isolation or damping properties of the pendulum.

It should also be noted that the violin modes and the internal modes of the blades are not included in the MATLAB model *used here*. The frequencies of the violin modes of the final stage can be seen in the thermal noise curves. The expected frequencies of the internal modes of the blades can be calculated from the dimensions of the blades, and are specific to each design of blade. Examples of their typical values are given in section 3.

## 2.7 Local control

In GEO the active local control damping is applied at the top mass ensuring that the pendulum stages below filter any extra motion caused by electronic noise in the feedback system. However given the more ambitious target noise level for LIGO of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz, the GEO design needs some modification. In particular to sufficiently isolate the mirror from electronic noise, a quadruple suspension is required. *Plus further steps – see comment at end of this section*

Consider firstly the optical shadow sensors used in GEO as part of the local control system for damping the low frequency pendulum modes. Their noise level is  $\sim 10^{-10}$  m/ $\sqrt{\text{Hz}}$ , and at frequencies where there is gain, the sensor noise contributes to the overall noise level. In GEO the target sensitivity at 50 Hz at the mirror is achieved by a combination of the isolation of the two stages below the top mass, and the aggressive roll-off in the electronic gain above the highest frequency of the low frequency pendulum modes. By the target frequency of 10 Hz in LIGO, very little roll-off in gain can be achieved. Thus most of the sensor noise isolation comes from the pendulums themselves. It should be feasible to turn down the gain of the longitudinal damping loop once the overall global control of the interferometer is switched on, thus reducing or indeed removing the noise imposed by the sensors in that direction. However it is not feasible to turn off the gain for some other modes, notably the vertical mode, although it could be turned down, giving a higher residual Q. Even allowing for this, development of very much better sensors would be needed if one wanted to use a triple pendulum. Moving to a quadruple pendulum and an improvement in sensor noise level to say  $10^{-11}$  m/ $\sqrt{\text{Hz}}$ , and assuming the gain can be turned down by a factor of order 100, the target sensitivity should be achievable. See section 3 for more details.

Alternatively a quadruple suspension and eddy current damping in 6 degrees of freedom at the top mass to Qs of approx. 10 is a possible alternative. *Although possible in theory – i.e. thermal noise requirements can be met whilst using sufficient damping for such low Qs - it is not straightforward to see how such levels of damping could be achieved in practice given the amount of force necessary to apply. See also comments below.* If eddy current damping were used, there is still likely to be a requirement for diagnostics involving sensing and/or actuation which could logically be done with the type of sensors and actuators used for active local control. Thus a number of such sensors and actuators might be required anyway. The final decision on active control vs. eddy current damping is TBD.

In conclusion therefore a quadruple suspension is necessary to meet the target displacement sensitivity for Advanced LIGO unless compact *reliable affordable* and easily mountable sensors with noise performance of order  $10^{-13}$  m/ $\sqrt{\text{Hz}}$  can be developed.

*Since the above was written, more attention has been given to the question of damping and the requirements on the sensors. See in particular “Advanced LIGO suspensions general interpretation of requirements for sensors” K A Strain, May 2003, to be found at the ALUK web site*

<http://www.physics.gla.ac.uk/~caroline/ALUK%20Glasgow%20Webpage/ALUK%20Glasgow%20Webpage.html>

(Document number ALUKGLA0005aJUN03.)

## 2.8 Global Control

The GEO philosophy for global control was briefly described in the introduction. The general idea is to apply forces between the main pendulum chain and an essentially identical reaction chain (which does not include fibre suspensions). The reaction chain is itself locally damped in the same manner as the main chain. In LIGO however, not all the sensitive optics require wide bandwidth global control, and in those cases the reaction chain does not require to have as many stages. In addition, where wide bandwidth is required, the final stage wide bandwidth low signal feedback could be actuated using photon drive, rather than electrostatically as in GEO. In that case also the lowest stage of reaction chain is not required. The number of stages for each different type of sensitive mirror is TBD. A possible scenario, with comments, is presented in Appendix A.

Another issue which needs to be considered is the potential need to damp (actively or passively) the very high Q violin modes of the silica suspensions to allow the global feedback to remain stable. Any such damping has to be done in such a way as not to compromise the low frequency thermal noise performance of the suspensions. In GEO we have taken the approach of using small amounts of amorphous PTFE coating on the fibres, suitably placed to damp the first few violin modes to Qs of around  $10^6$ , without compromising the low frequency suspension noise. For GEO we use two coated regions each 5 mm long, one at the centre and one at 1/3 of the way down the fibre. The LIGO situation has to be considered fully once a control philosophy has been decided upon, and there will be some trade-off required between controllability and thermal noise associated with the violin modes.

It should be noted that each type of reaction pendulum, if not *essentially* identical to its main partner, will need its own design worked out, with suitable blades and possible adjustments to masses, wire spacings etc. to achieve coupling of modes as required for local control from the top mass to work adequately. However there is no technical risk associated with this redesign process as we now have experience of the necessary steps to take.

## 2.9 Other Design Issues

There are several other areas of design and construction which need to be addressed, and are briefly covered here, in no particular order.

### 2.9.1 Method of Mechanical Assembly

From our experience, we advocate that in every case, the full suspension is assembled in an all-metal version, with suitable steel or aluminium masses to mimic the mass, dimensions and approximate moments of inertia of the silica, heavy glass or sapphire pieces. Once such an assembly is done, the final two stages can be replaced by the mirror and the penultimate mass, with their fibre suspensions, all held together in a suitable support structure.

Wire lengths are pre-assembled with suitable jigs to get the correct length and even loading.

The sequence of loading of the blades to their final shape is an important issue, as is keeping them stressed while the final stages are replaced. An advantage of the “sandwich” design for the top two masses, first explored in the MIT prototype, is that the sandwich can act as a retainer to stop those blades springing up when the load is removed. *Since this was written, the development of a modular approach to the design has removed the necessity for using a particular sequence for assembly. The different sets of blades can separately be loaded with custom loads attached using temporary wire clamps, and then the blades are retained in position using a blade retainer at the topmost blades, and the sandwich design for the two lower sets. Subsequently the suspension is assembled and the final wire clamps are swapped in.*

One issue in the currently advocated LIGO design which differs from the GEO design is that at the penultimate mass, the wires looped round this mass and going to the mass above lie inside the silica fibres or ribbons. This at first sight might present problems in assembly, since the method employed at GEO – to slip the loops off the metal penultimate mass to the front and back, and then slip them on to the silica penultimate mass with the welded silica fibres in place – cannot be done. However what can be done is to remove the whole assembly of clamps and wires from the blades above, and loop the wires round the penultimate mass before welding. Then the welding of the fibres can be carried out, and finally the clamps are reattached to the blades above. Obviously such an assembly technique needs to be practiced.

### **2.9.2 Clamps**

Experience with assembling the MIT prototype highlighted the importance of good clamp design and correct torquing of the clamp to hold the wire from slipping. One wishes to define where the wire lies prior to assembly – and the best way to make a suitable smoothly indented groove was found to be by clamping the steel wire against a shim of (softer) steel included in the clamp.

### **2.9.3 Interfacing Issues and Support Structure**

Interfacing with the isolation table is a TBD. The interfacing will be different for the cases of BSC and HAM suspensions, with in one case the isolated table lying above and in the other lying below the suspension *respectively*.

The MIT prototype utilises “Bosch” extruded aluminium profile framework for supporting the pendulum and its associated catcher structure and supports for local control actuation. Whereas this proved very useful and straightforward to use for prototyping, and we advocate it for LASTI if possible, it will presumably not be suitable for final LIGO requirements, and an alternative structure will be required. *All-welded structures are now being pursued.*

### **2.9.4 Handling**

Assembling the all-metal MIT prototype quad, whose largest masses are 30 kg and are just possible to be carried by one person, was not easy. Handling larger and more delicate masses as will be used in Advanced LIGO will require automated systems capable of fine adjustments

### **2.9.5 Footprints**

With reference to the document LIGO-T010076-01-D “Optical layout for Advanced LIGO”, we note that there is one BSC tank in particular which involves a tight fit of ITMy2 and folding mirror FMy. Using the lengths of uppermost blades and overall size of support structure which we have in the MIT prototype quad, we would not be able to satisfy the desired footprint for this ITM of approx. 71 cm x 55 cm (where 71

lies parallel to face of mirror), in the direction parallel to the mirror face. However by redesign of blade and reduction in size of support structure, we believe this target footprint can be achieved. The example quad design given in section 3 has uppermost blades reduced in length from the MIT prototype design. *Note added – even allowing for achieving the above footprint for the ITM, the tank alluded to above is still very tight in space and we will need address the layout with a special design involving a common support structure for the ITM and folding mirror in order to fit these mirrors in.*

*In summary, the footprints which are currently being used, where footprint defines the maximum outer edges of the suspension including its support structure, are*

*ETM/ITM: 710 mm length x 550 mm width x 2105 mm height*

*MC: 400 mm x 220 mm x 887 mm*

*RM: 480 mm x 300 mm x 810 mm*

*The length is parallel to face of mirror and height is defined as distance from seismic table to the last point on the structure or suspension at opposite end.*

### **3. Performance**

In this section we present various graphs, showing expected overall thermal noise performance, horizontal and vertical isolation performance with and without damping, and transfer functions from which residual sensor noise may be estimated. Key parameters used in the models to generate these graphs are also given.

In some cases several curves are given, where there are possible different choices of parameters. *A full list of parameters used to generate all of the following graphs is now included in Appendix C.*

#### **3.1 ETM/ITM Quadruple Suspensions**

*The suspension design for the ETM and ITM has evolved through various versions in terms of the overall masses in the quads. A brief history was written for the summary of a suspensions telecon, July 8<sup>th</sup> 2003, and we append the paragraph here for completeness. The paper referred to in the text is “Comparison of possible quadruple suspension models for Advanced LIGO” N. A. Robertson, May 2002, which can be found on the ALUK site at*

<http://www.physics.gla.ac.uk/~caroline/ALUK%20Glasgow%20Webpage/Document%20Register%20Glasgow.html>

*(Document number ALUKGLA0011aMAY02)*

##### Quad History

Norna provided a brief history of the quad design. The original theory was to keep the masses the same. They tried not to deviate from this by more than a factor of 2. So, the MIT quad is 15kg, 15kg, 16kg and 30kg (top to bottom). The 16 and 30 kg were chosen to mimic a sapphire test mass and a fused silica penultimate mass of the same size. The design was then changed to take a 40kg test mass and to meet the 10 Hz requirement by using a heavier penultimate mass made of ultra-dense glass. This design, which is in the conceptual design document written in Sept 2001, has masses 36,36, 72, 40. Peter Fritschel then submitted the 10 Hz cutoff paper. So Norna looked at several different models in her paper that don't satisfy the 10 Hz requirement and that have a lighter penultimate mass. One of these was a scaled

up version of the MIT quad, namely 22,22,22,40, with sapphire test mass and silica penultimate mass. This was the lightest option, and so attention was focussed on it for keeping overall mass as low as practicable when loading of the seismic table became an issue. However if silica is used for the test mass, the logical penultimate material would be silica and thus the chain becomes 22, 22, 40, 40. Such a chain is possible for sapphire as well and so this set of masses is under investigation with the aim of having a design which could work for sapphire or fused silica as the test mass material, with only a change in shape as opposed to mass. SF4 heavy glass, which is easily obtained, is a potential material for the penultimate mass for a sapphire test mass. It is actually more dense than sapphire at 4.8 g/cc. So, for the same shape as sapphire, it would be 48 kg. The top and bottom could be cut off to make the mass 40kg. Alternatively the penultimate mass could be sapphire.

*Note added to above for completeness. Between the MIT quad design (masses 15,15,16,30 kg) and the conceptual design of Sept 2001 (masses 36,36,72,40 kg), there was a design for a 30kg test mass and a sub 10Hz vertical frequency (masses 30, 30, 58, 30kg) which was presented in the document “LIGO II suspension: Reference Designs”, (LIGO-T000012-00).*

***It should be noted that the quad design is evolving constantly at present as the design is being developed, and so the graphs which follow should only be considered as representative of the current status.***

*Some key parameters used for all the curves presented for the E/ITMs are as follows (except where otherwise indicated). See appendix C for full list.*

*Final mass = 40 kg sapphire, 31.4 cm x 13 cm*

*Penultimate mass = 40 kg (heavy glass or sapphire)*

*Upper masses = 22 kg, 22 kg*

*Overall length (from top blade to centre of mirror) = 1.7 m*

*Ribbon parameters: length = 60 cm, cross-section = 113  $\mu\text{m}$  x 1.13 mm*

*Stress in ribbon = 770 MPa*

### **3.1.1 Thermal noise performance.**

*In figure 1 we show the thermal noise estimation. Note that the highest of the low frequency peaks (the highest vertical mode of the suspension) lies just under 9 Hz. Given the proximity of this peak to 10 Hz, a noise level of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  is not reached until slightly above 10 Hz, at approximately 11.5 Hz. The first violin mode appears at approximately 490 Hz. The recommendations from the Fritschel et al paper on low frequency cut-off are met with this design.*

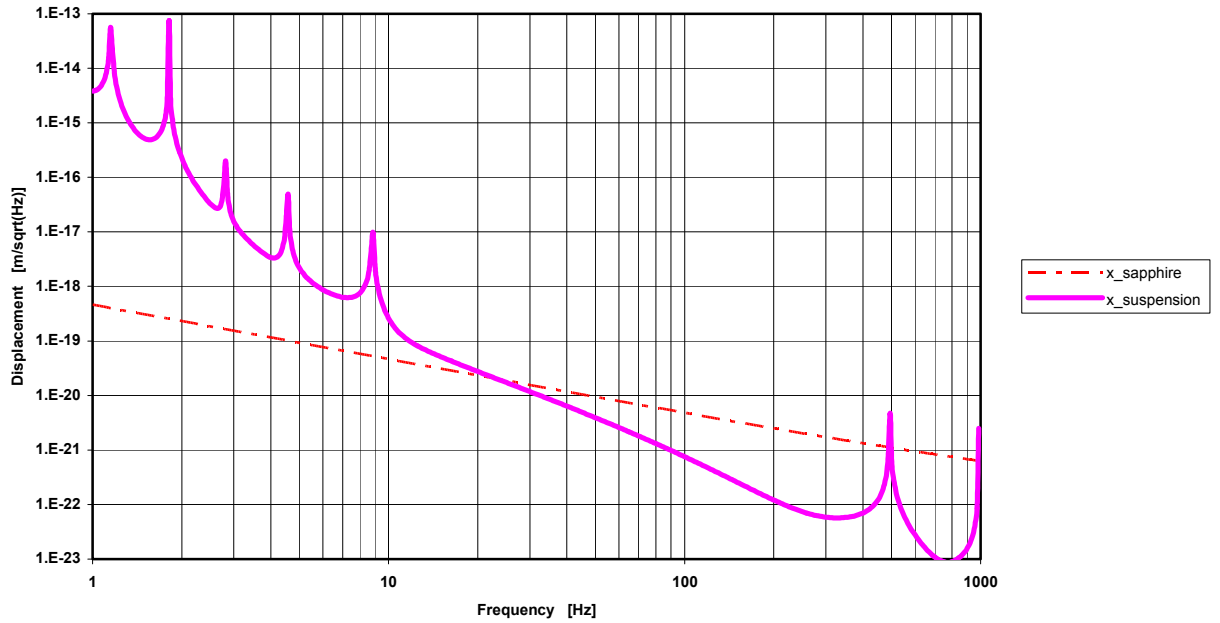


Figure 1. Suspension thermal noise for baseline 40kg quadruple pendulum. The solid magenta line shows the suspension thermal noise curve. For comparison we also show the calculated internal thermal noise curve for sapphire (red dot/dash line, using  $\phi = 5e-9$ ). Note that the internal thermal noise curve assumes no loss due to coatings, or due to bonding of ears for attaching the suspensions.

As explained in section 2.3 above, the suspension thermal noise curve is derived by combining the longitudinal and vertical thermal noise contributions. A full analysis involving all degrees of freedom has not yet been carried out. Thermal noise from angular motions should be considered, and in particular noise due to pitch motion, since for this motion there is no dilution factor in loss compared to yaw motion. An order of magnitude estimate was carried out for a previous design with 30 kg test mass which showed that pitch thermal noise would contribute less than  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz when the beam offset was less than 3 mm. (ref LSC talk by N A Robertson Hanford August 2000 LIGO document G000295, p 28). The requirement on beam offset is 1mm. Updating the pitch thermal noise estimation for the current design gives a noise level of  $3.8 \times 10^{-17}$  rad/ $\sqrt{\text{Hz}}$  which corresponds to  $3.8 \times 10^{-20}$  m/ $\sqrt{\text{Hz}}$  for a 1 mm offset. This is lower than the longitudinal suspension thermal noise requirement of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  but not by as much as a factor of 10, the number used in setting the pitch requirement (to be confirmed). A tighter specification on the beam offset would be needed to achieve that factor of 10.

### 3.1.2 Isolation performance

The overall mechanical isolation in Advanced LIGO will be achieved by a combination of a two-stage active isolation system and the isolation of the quadruple pendulum. The target noise level for the active system is  $2 \times 10^{-13}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz in both longitudinal and vertical directions. As can be seen from figure 2, the longitudinal isolation factor is  $\sim 3 \times 10^7$  at 10 Hz, which combined with the noise level quoted above, meets the requirement of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz. From figure 3, in the vertical the isolation factor is  $\sim 4 \times 10^4$  at 10 Hz which when combined with the noise level above gives  $8 \times 10^{-17}$  m/ $\sqrt{\text{Hz}}$ . This is larger than the quoted vertical requirement,

however it still gives a noise level  $10^{-19} \text{ m}/\sqrt{\text{Hz}}$  at 10 Hz assuming a cross-coupling of  $10^{-3}$ .

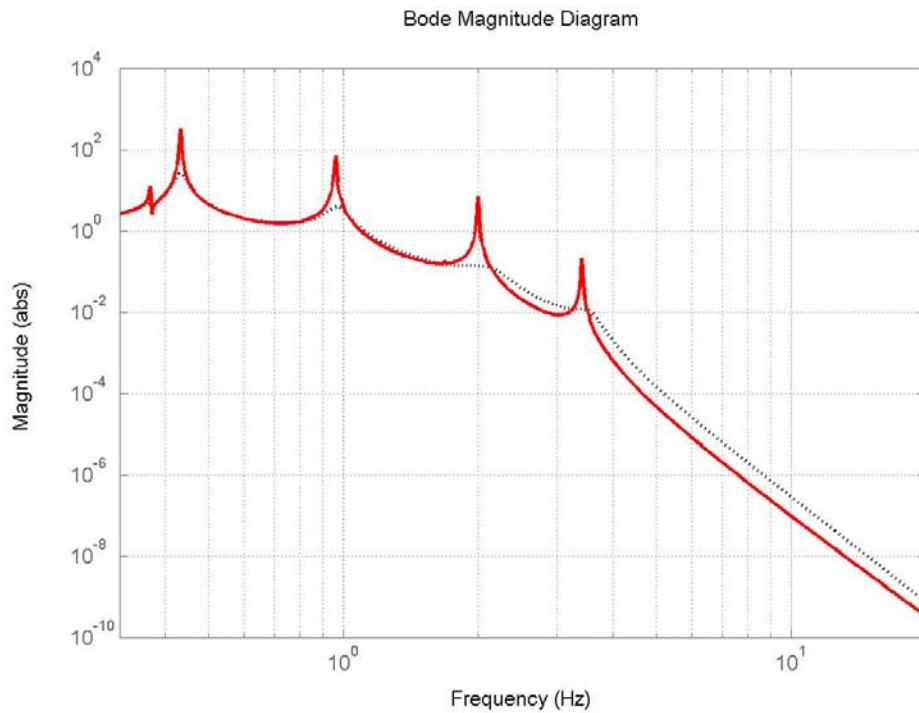


Figure 2. Longitudinal transfer function for quad pendulum, with (black, dotted) and without (red, solid) local control damping. Gain of local control is such that impulse decay time is 10 secs. Isolation factor at 10Hz (with damping) is  $2.8 \times 10^{-7}$ .

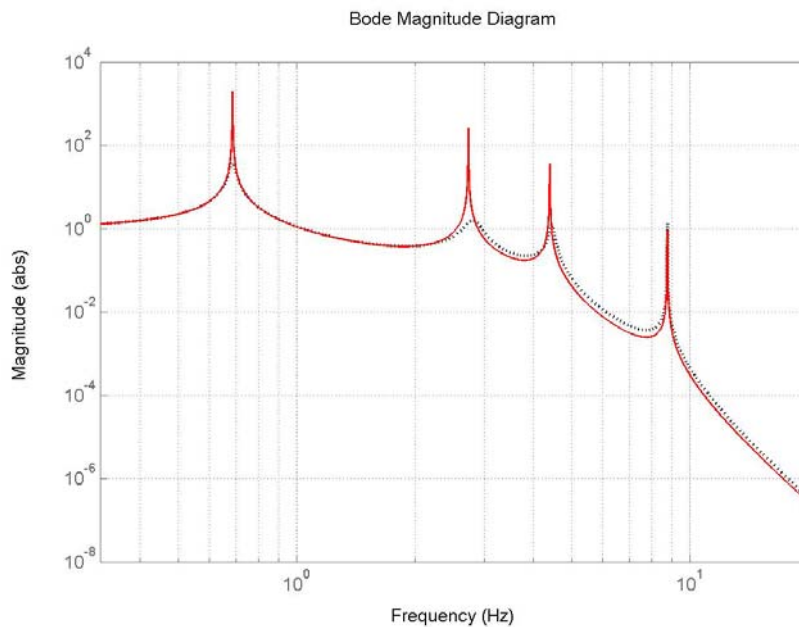


Figure 3. Vertical transfer function for quad pendulum, with (black, dotted) and without (red, solid) local control damping. Magnitude at 10 Hz (with damping) is  $4.1 \times 10^{-4}$ .

### 3.1.3 Sensor Noise Performance

Noise level at mirror can be calculated by taking the transfer functions as shown in

figures 4 and 5 for longitudinal and vertical directions respectively and multiplying by the sensor noise in  $m/\sqrt{\text{Hz}}$ . It can be seen that using a sensor with noise level of  $10^{-10} m/\sqrt{\text{Hz}}$  the residual sensor noise would be much larger than the technical noise requirement of  $10^{-20} m/\sqrt{\text{Hz}}$  at 10 Hz for both directions (assuming  $10^{-3}$  cross-coupling in vertical). An alternative strategy or strategies for operation in science mode (as opposed to acquisition mode) is required, as discussed in the document referred to in section 2.7 above.

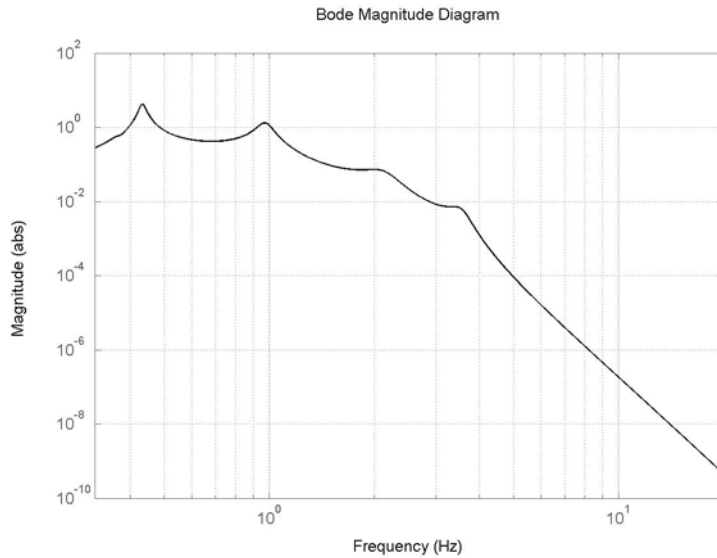


Figure 4. Longitudinal transfer function from sensor to mirror with local control damping at same level as in figure 2. Magnitude at 10 Hz is  $1.8 \times 10^{-7}$ .

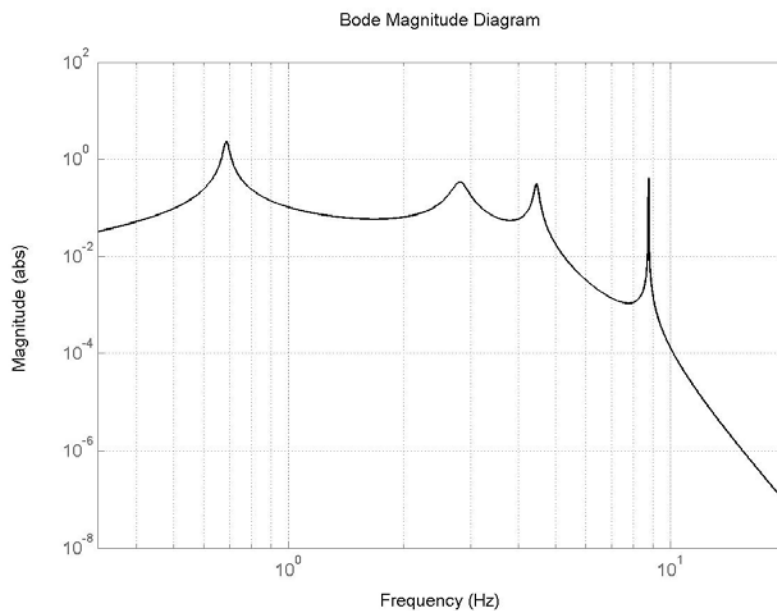


Figure 5. Vertical transfer function from sensor to mirror. Magnitude at 10 Hz is  $1.2 \times 10^{-4}$ .

### 3.2 Triple Suspensions

The modecleaner mirrors and the recycling mirrors have different displacement noise requirements as indicated in section 1. The more sensitive requirements are for the modecleaner and thus we consider it first.

### 3.2.1 Thermal Noise Performance for Modecleaner

The key parameters used in this design are as follows.  
 Lowest mass = 3.0 kg silica, 15 cm diameter by 7.5 cm thick  
 Top and second masses: 3.2 kg, 3.0 kg  
 Overall length (from top blade to centre of mirror) = 0.69 m

From consideration of figure 6 below, it can be seen that silica fibres meet the requirement figure of  $3 \times 10^{-17} \text{ m}/\sqrt{\text{Hz}}$  at 10 Hz with a large safety margin, except for a narrow peak at the highest vertical mode. Steel also just meets the requirement using a larger stress value. However it should be recalled that the requirement as quoted is for the sum of all noise sources. Thus for a conservative design we are advocating the use of silica in this suspension.

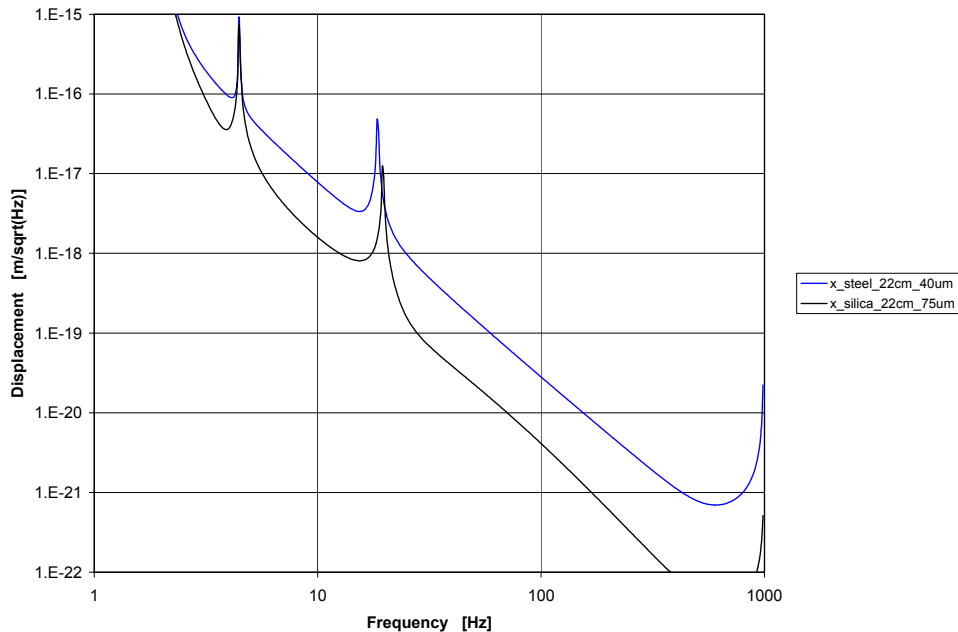


Figure 6. Suspension thermal noise for modecleaner triple pendulum suspension. Two suspension curves are shown, the lower curve using silica fibres with radius of 75 micron in the final stage, and the upper curve using steel of radius 40 micron.

### 3.2.2 Isolation Performance

The overall mechanical isolation is achieved by a combination of the two-stage active isolation system and the isolation of the triple pendulum. Recall the target noise level for the active system is  $2 \times 10^{-13} \text{ m}/\sqrt{\text{Hz}}$  at 10 Hz. By combining this number with the isolation factors given in Figures 7 and 8, the isolation is more than adequate to meet the design requirements in both longitudinal and vertical directions.

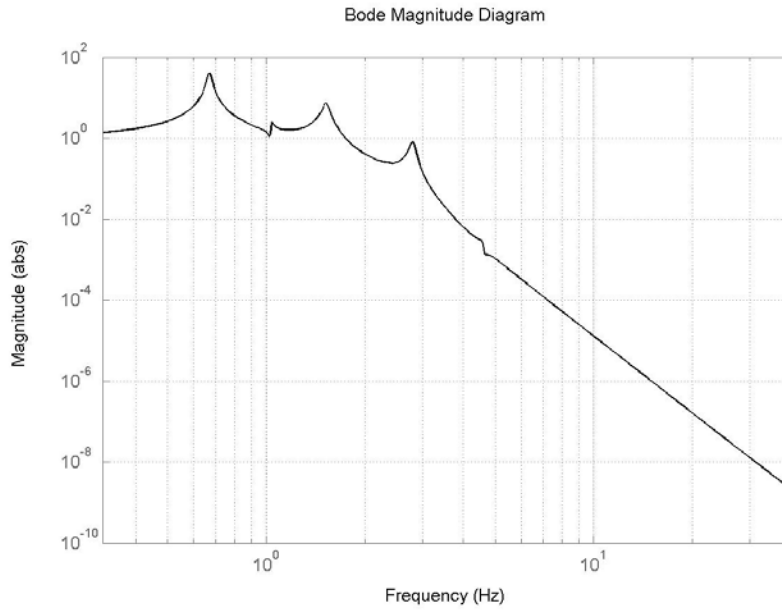


Figure 7. Longitudinal transfer function for modecleaner triple pendulum, with gain of local control damping to give impulse decay time approx. 10 secs. Magnitude at 10 Hz =  $1.3 \times 10^{-5}$ .

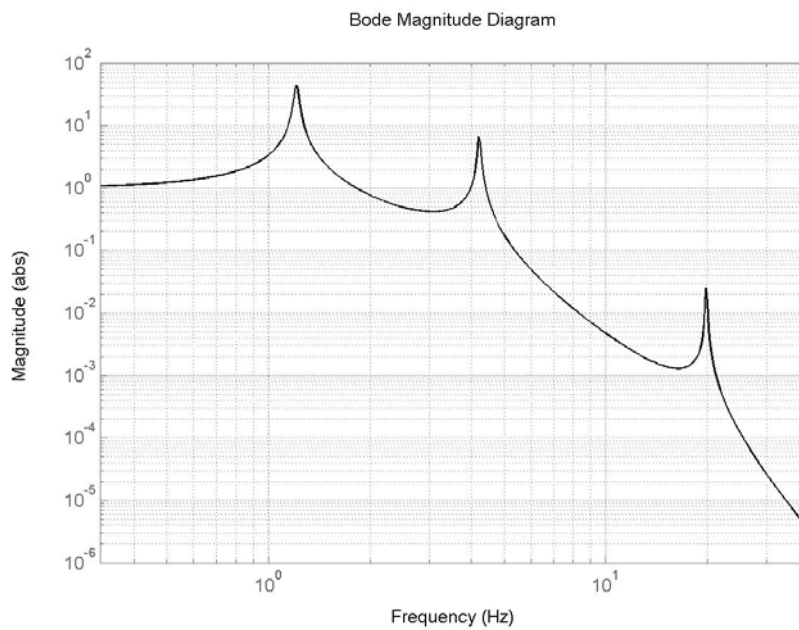


Figure 8. Vertical transfer function for triple pendulum, with gain of local control damping to give impulse decay time approx. 10 secs. Magnitude at 10 Hz =  $4.7 \times 10^{-3}$ .

### 3.2.3 Sensor Noise Performance

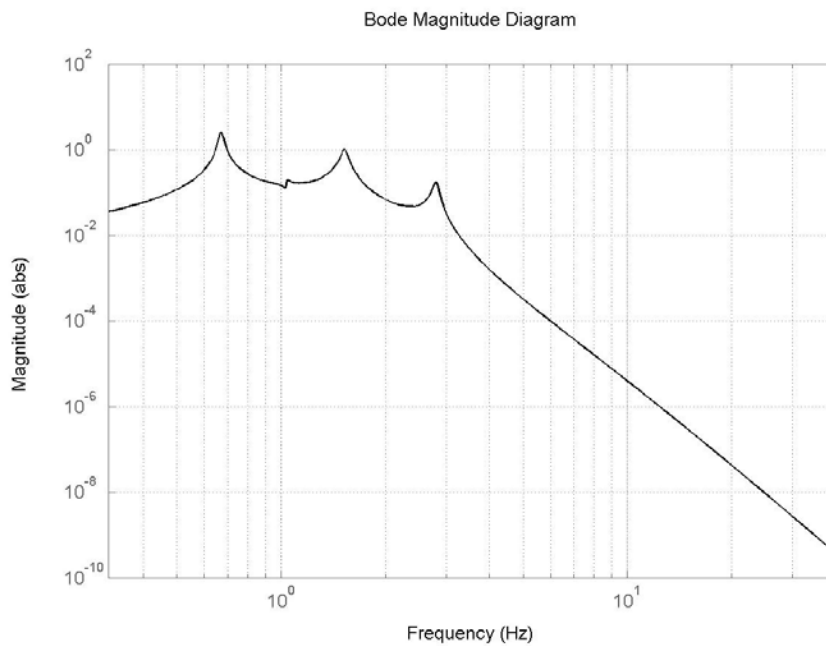


Figure 9. Longitudinal transfer function from sensor to mirror assuming damping as in figure 7. Magnitude at 10 Hz =  $3.9 \times 10^{-6}$ .

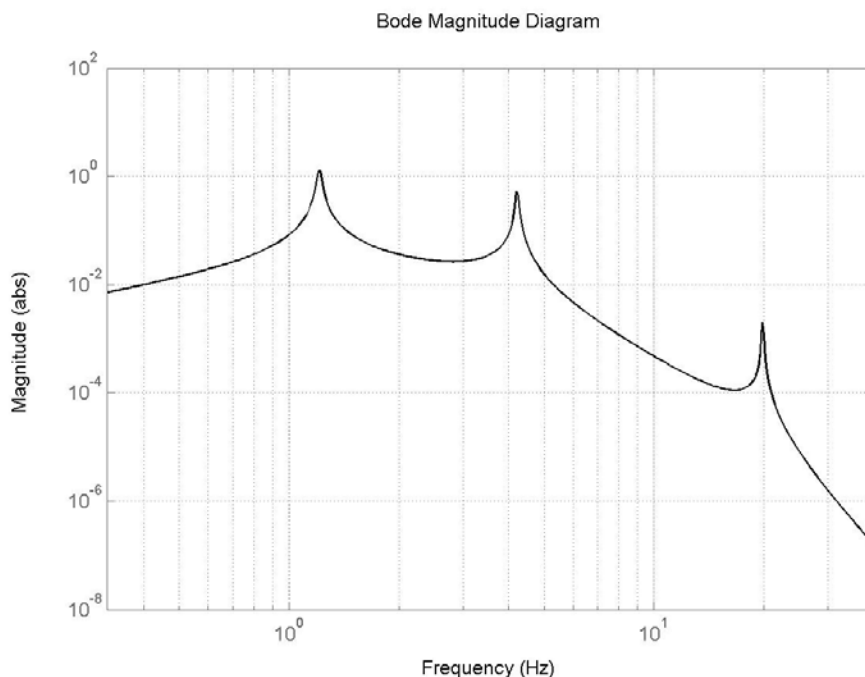


Figure 10. Vertical transfer function from sensor to mirror, assuming damping as in figure 8. Magnitude at 10 Hz =  $4.7 \times 10^{-4}$ .

It can be seen that using a sensor with noise level of  $10^{-10} \text{ m}/\sqrt{\text{Hz}}$  the sensor noise would exceed  $3 \times 10^{-17} \text{ m}/\sqrt{\text{Hz}}$  at 10 Hz in longitudinal and  $3 \times 10^{-14} \text{ m}/\sqrt{\text{Hz}}$  in vertical. As with the quadruple suspension, an alternative strategy or strategies for operation in science mode (as opposed to acquisition mode) is required, as discussed in the document referred to in section 2.7 above.

### 3.3 Recycling Mirror.

Since the recycling mirror has relaxed requirements compared to the modecleaner mirror, a similar design can be used with confidence. The sensor noise problem in particular is less challenging. In addition the thermal noise performance can comfortably be achieved using steel wires. See figure 11 below.

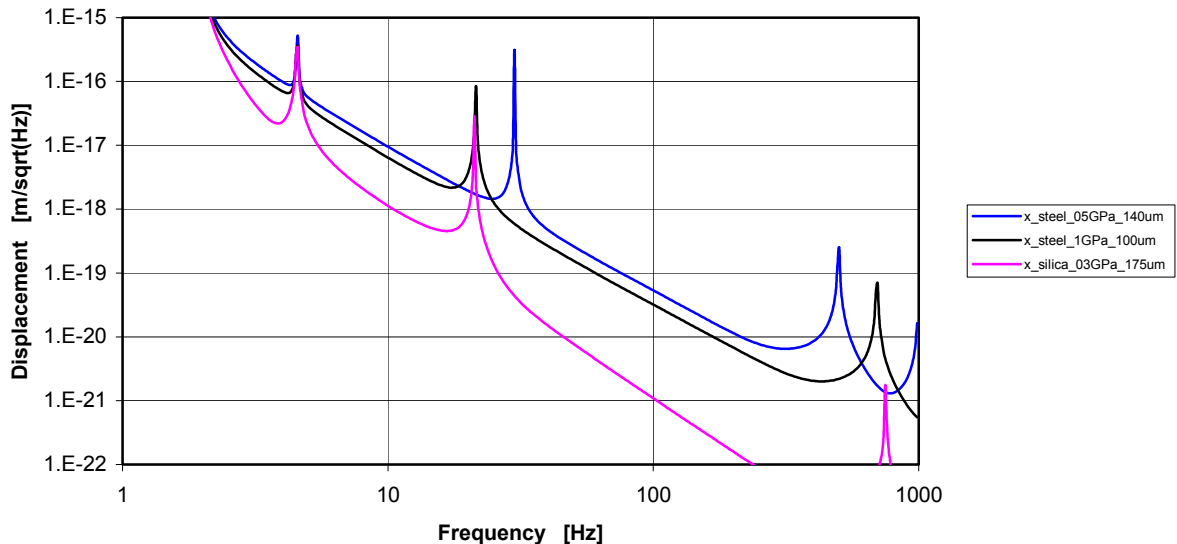


Figure 11. Suspension thermal noise for recycling mirror triple pendulum suspension. Three suspension curves are shown, the lowest curve using silica, the middle steel with radius of 100 micron in the final stage, and the upper curve using steel of radius 140 micron. This figure assumes coupling of 0.0018 from vert. to long.

### 3.4 Other Mirrors

The main type of mirror which we have not addressed here is the beamsplitter. Given that its displacement noise requirement is relaxed from those of the E/ITMs by a factor of around 100, we do not anticipate problems reaching the design requirement with a quadruple suspension. Indeed a triple may be enough. *This is under consideration at present.* Possibly the more challenging aspect will be due to its atypical aspect ratio (very much thinner for its diameter than other mirrors). *In addition the suspension for the folding mirror needs to be designed and a triple pendulum may also be suitable for this mirror. This will be addressed soon.* Finally the compensating plate suspension design must be addressed, and there has been a suggestion that this could form the final mass in the reaction chain of the ITM. *Again this needs attention.*

## 4. Conclusions and Development Strategy

The basic ideas have been presented for the suspensions of the sensitive mirrors in Advanced LIGO. There are several areas which are actively under research, and which are therefore TBD in the final design. These include

Ribbon *or dumbbell* manufacture, Q and strength  
Bonding of silica to sapphire and to heavy glass  
Final design of silica ears  
Bonding and welding tests with 40 kg masses  
Availability of sapphire with all the desired properties  
Active local control vs. eddy current damping

There are therefore key issues which are as yet undecided, and which impact the final design, such as:

Choice of material for mirrors (sapphire vs. silica)  
Low frequency cut-off (the 10 Hz vs. 13 Hz question) *Now settled*  
Control philosophy, and hence details of reaction chains required, degree (necessity) of violin mode damping required, and degree of residual local damping required  
Type of local damping

A suspensions development plan has been produced (LIGO-M000202-A-M), which summarises the main milestones and timescales for the various prototypes leading to final SUS designs. *Updated version??*

## **5. Appendix A: Summary of Suspension Parameters for Sensitive Mirrors**

Shown on the next two pages are two tables summarising some key points concerning the various different sensitive mirrors. Table 1 gives masses, sizes and some details of type of suspension. Table 2 summarises the different suspensions required, including reaction chains, and gives more details of the activity associated with each stage of the reaction chains. *These tables have been updated to reflect current plans. The original Table 2 is also included since it gives information on the stages at which various types of control will be applied. This is relevant even when a fixed mass rather than a reaction chain is used to actuate against.*

**Table 1.**

Summary of Suspension Parameters for Sensitive Mirrors for LIGO II								
updated August 2003								
Name	Description	Material	Mass(kg)	Diam(cm)	Thick(cm)	Type of Susp	Penultimate mass*	Reaction mass chain
ETM	end test mass	sapphire**	40	31.4	13	quad+silica ribbons	40kg heavy glass/sapphire same size as ETM	quad, lowest= 40kg heavy glass, same density as sapphire, gold plating on one face for electrostatic drive
ITM	inner test mass	sapphire**	40	31.4	13	quad+silica ribbons	40kg heavy glass/sapphire same size as ETM	triple
BS	beamsplitter	silica	12.7	35	6	quad+silica ribbons/fibres?	silica, same size as BS	down to penultimate mass
FM	folding mirror	silica	25	35	11.8	quad+silica ribbons/fibres?	silica, same size as FM	needs DC control
PRM	pwr recyc mirror	silica	12.1	26.5	10	triple steel wires	metal	no reaction chain
SRM	sig recyc mirror	silica	12.1	26.5	10	triple steel wires	metal	no reaction chain
MC1	modeclr 1 transmitter	silica	6.9	20	10	triple+silica fibres	silica, same size as MC1	no reaction chain
MC2	modeclr 2 non-transmitter	silica	6.9	20	10	triple+silica fibres	silica, same size as MC2	no reaction chain
MC3	modeclr 3 transmitter	silica	6.9	20	10	triple+silica fibres	silica, same size as MC3	no reaction chain
Notes	1) *this is the mass directly above the mirror 2) entries with ? need more input 3) ** ETM/ITM fallback							



## Table 2 (old version)

Table 2: Summary of Suspension Designs updated 12th Sept. 2001

	Type of Suspension	no. of stages	Purpose of reaction stages	Comments / TBDs
1	<b>ETM/ITM</b>	4		
2	Reaction pendulum ETM	3 ( poss. 4)	stage 1 = local control of reaction chain stage 2 = pitch and yaw bias stage 2 & 3 = pitch, yaw, long. global control	wide bandwidth glob. control 3 stages + photon drive or 4 w/ electrostat.
3	Reaction pendulum ITM	3	stage 1 = local control of reaction chain stage 2 = pitch and yaw bias stage 2 & 3 = pitch, yaw, long. global control	global bandwidth ~1Hz
4	<b>Beamsplitter</b>	4		possibly triple sufficient, TBD
5	Reaction pendulum for splitter	3	as for ITM if quad	as for ITM if quad
6	<b>Folding mirror</b>	4		possibly triple sufficient, TBD
7	Reaction pendulum for F mirror	2 or 3	stage 1 = local control of reaction chain stage 2 = pitch and yaw bias stage 3 if needed	TBD
8	<b>PRM/SRM</b>	3		
9	Reaction pendulum for RMs	2	stage 1 = local control of reaction chain stage 2 = pitch and yaw bias stage 2 = pitch, yaw, long. global control	global bandwidth ~1Hz
10	<b>MCs</b>	3		
11	Reaction pendulum for MC2	3	stage 1 = local control of reaction chain stage 2 = pitch and yaw bias stage 2 & 3 = pitch, yaw, long. global control	wide bandwidth glob. control
12	Reaction pendulum for MC1&3	2	stage 1 = local control of reaction chain stage 2 = pitch and yaw bias stage 2 = pitch, yaw, long. global control	global bandwidth ~1Hz
			Key to numbering of reaction stages 1 = top, increasing downwards	

## Appendix B: Prototype Quadruple Pendulum at MIT

We include here (Figure 12) examples of Autocad diagrams of the all metal prototype quadruple pendulum and reaction mass which was designed in Glasgow, and pictures of the actual pendulums hanging in the lab at MIT in the summer of 2001. This suspension mimics a 30 kg sapphire mirror with an identically sized silica penultimate mass, which was a previous baseline design, now superceded. More details and more pictures, can be found in the talk given by N Robertson at the LSC Hanford meeting, August 2001, at (<http://www.ligo.caltech.edu/docs/G/G010291-00/>)

Briefly, much experience was gained in assembly and handling, and lessons learnt on various design aspects such as clamp design, pitch adjustment, design of upper masses holding blades. Current and future work includes measuring mode frequencies, and investigating transfer functions, damping and global control

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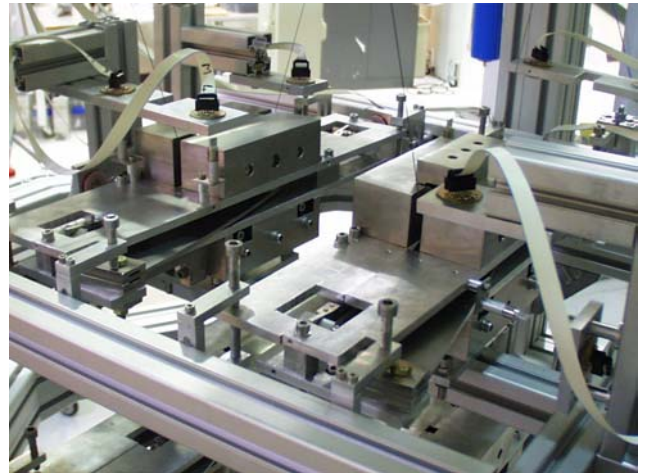
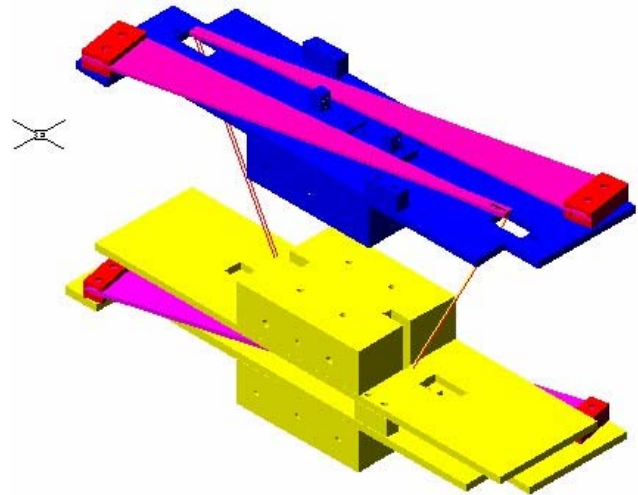
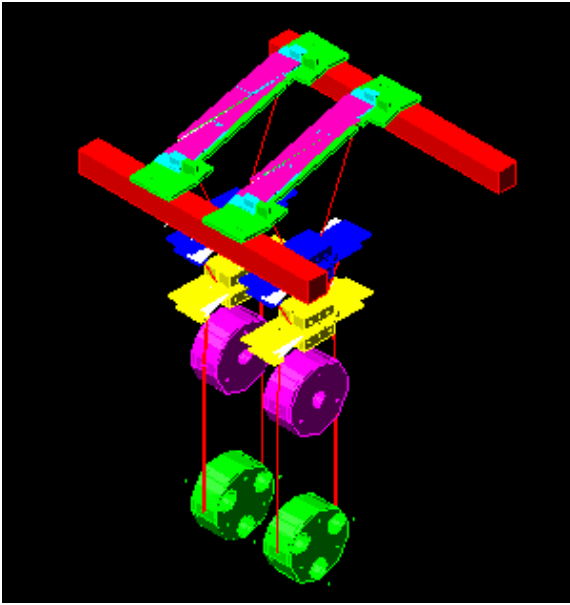


Figure 12. Autocad diagrams and pictures of prototype quadruple pendulum. Top left shows view of quadruple and reaction chain. Crossed blades at top can be seen. Top right shows first two masses in the chain, with the top of the upper mass removed to show where the crossed blades lie within the mass. Bottom left shows assembled pendulum sat MIT. Bottom right shows close up of top masses of both main and reaction chain, with some of local control electronics in place.



```

> sigma[Steel] := 0.3 ;
> sigma[Silica] := 0.17 ;
> sigma[Sapphire] := 0.23 ;

```

```

@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@@@@@

```

MAIN OPTICS SUSPENSION PARAMETERS

```

> m[1] := 21.9 ;# [kg]
> m[2] := 21.84 ;# [kg]
> m[3] := 40.066 ;# [kg]
> m[4] := 40.066 ;# [kg]
>
> L[1] := 0.54 ;# [m]
> L[2] := 0.304 ;# [m]
> L[3] := 0.302 ;# [m]
> L[4] := 0.6 ;# [m]
>
> N[1] := 2 ;
> N[2] := 4 ;
> N[3] := 4 ;
> N[4] := 4 ;
>
> r[1] := 700e-6 ;# [m]
> r[2] := 400e-6 ;# [m]
> r[3] := 350e-6 ;# [m]
> r[4] := 200e-6 ;# [m] not used because ribbons are used instead of
fibres
>
> t[1] := 4400e-6 ;# [m] maraging blade thickness
> t[2] := 4800e-6 ;# [m] maraging blade thickness
> t[3] := 4500e-6 ;# [m] maraging blade thickness
> t[4] := 113e-6 ;# [m] ribbon thickness
>
> W[4] := 1130e-6 ;# [m] ribbon width
>
> f_vert[1] := 2.41 ;# [Hz]
> f_vert[2] := 2.58 ;# [Hz]
> f_vert[3] := 2.1106 ;# [Hz]

```

```

@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@

```

SAPPHIRE MASS PARAMETERS  
(SOME OF THEM ALREADY LISTED BEFORE)

```

a = 0.157,      # m
H = 0.13,      # m
ro = 0.0422,   # m beam size (the same for near and far masses)
K = 33,        # J/K/m
alpha = 5.1e-6, # 1/K
sigma = 0.23,  #
rho = 3980,    # kg/m^3
C = 770,       # J/K/kg
Y = 4e11,      # Pa
Temp = 290,    # K
phi = 5e-9

```

```

@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@

```

SILICA MASS PARAMETERS  
(SOME OF THEM ALREADY LISTED BEFORE)

a = 0.19154, # m  
H = 0.1586, # m  
ro = 0.0515, # m  
K = 1.38, # J/K/m  
alpha = 5.1e-7, # 1/K  
sigma = 0.17, #  
rho = 2200, # kg/m^3  
C = 772, # J/K/kg  
Y = 7e10, # Pa  
Temp = 290, # K  
phi = 2e-8

@@

MODE CLEANER SUSPENSION PARAMETERS

> m[1] := 3.229 ;# [kg]  
> m[2] := 2.979 ;# [kg]  
> m[3] := 3.04 ;# [kg]  
>  
> L[1] := 0.295 ;# [m]  
> L[2] := 0.167 ;# [m]  
> L[3] := 0.22 ;# [m]  
>  
> N[1] := 2 ;  
> N[2] := 4 ;  
> N[3] := 4 ;  
>  
> r[1] := 180e-6 ;# [m]  
> r[2] := 100e-6 ;# [m]  
> r[3] := 75e-6 ;# [m]  
>  
> t[1] := 1500e-6 ;# [m]  
> t[2] := 1000e-6 ;# [m]  
>  
> f\_vert[1] := 2.3 ;# [Hz]  
> f\_vert[2] := 3.4 ;# [Hz]

@@

SIGNAL RECYCLING SUSPENSION PARAMETERS

> m[1] := 12.07 ;# [kg]  
> m[2] := 12.214 ;# [kg]  
> m[3] := 12.181 ;# [kg]  
>  
> L[1] := 0.2 ;# [m]  
> L[2] := 0.201 ;# [m]  
> L[3] := 0.253 ;# [m]  
>  
> N[1] := 2 ;  
> N[2] := 4 ;  
> N[3] := 4 ;  
>  
> r[1] := 300e-6 ;# [m]  
> r[2] := 200e-6 ;# [m]  
> r[3] := 150e-6 ;# [m]  
>

```

> t[1] := 2300e-6 ;# [m]
> t[2] := 1300e-6 ;# [m]
>
> f_vert[1] := 2.7 ;# [Hz]
> f_vert[2] := 3.18 ;# [Hz]

```

#####

## ***C.2 Parameters for transfer functions (generated by N Robertson, Stanford and Glasgow).***

*An explanation of the symbols used can be found within the appropriate MATLAB files as referenced in Section 2. In addition, diagrams for a quad pendulum explaining the various symbols for dimension, supplied by C Torrie, are included in C 2.3*

### ***C.2.1 Quadruple suspension parameters generated from NAR's current 22224040 MATLAB model***

```

g: 9.8100
  nx: 0.1300
  ny: 0.5000
  nz: 0.0840
denn: 4000
  mn: 21.9000
  Inx: 0.4740
  Iny: 0.0713
  Inz: 0.4900
  ux: 0.1300
  uy: 0.5000
  uz: 0.0840
den1: 4000
  m1: 21.8400
  I1x: 0.4678
  I1y: 0.0436
  I1z: 0.4858
  ix: 0.1300
  ir: 0.1570
den2: 3980
  m2: 40.0660
  I2x: 0.4938
  I2y: 0.3033
  I2z: 0.3033
  tx: 0.1300
  tr: 0.1570
den3: 3980
  m3: 40.0660
  I3x: 0.4938
  I3y: 0.3033
  I3z: 0.3033
  ln: 0.5400
  l1: 0.3040
  l2: 0.3020

```

*l3: 0.6000*  
*nwn: 2*  
*nw1: 4*  
*nw2: 4*  
*nw3: 4*  
*rn: 7.0000e-004*  
*r1: 4.0000e-004*  
*r2: 3.5000e-004*  
*r3: 2.0000e-004*  
*Yn: 2.2000e+011*  
*Y1: 2.2000e+011*  
*Y2: 2.2000e+011*  
*Y3: 7.0000e+010*  
*lnb: 0.4800*  
*anb: 0.0961*  
*hnb: 0.0045*  
*ufcn: 2.3596*  
*stn: 8.9910e+008*  
*intmode\_n: 73.5303*  
*l1b: 0.4200*  
*a1b: 0.0583*  
*h1b: 0.0049*  
*ufc1: 2.5555*  
*st1: 8.9994e+008*  
*intmode\_1: 104.5764*  
*l2b: 0.3400*  
*a2b: 0.0500*  
*h2b: 0.0045*  
*ufc2: 2.1106*  
*st2: 7.9192e+008*  
*intmode\_2: 146.5517*  
*dm: 0.0010*  
*dn: 0.0010*  
*d0: 0.0010*  
*d1: 0.0010*  
*d2: 0.0010*  
*d3: 0.0010*  
*d4: 0.0010*  
*twistlength: 0*  
*d3tr: 0.0010*  
*d4tr: 0.0010*  
*sn: 0*  
*su: 0.0030*  
*si: 0.0030*  
*sl: 0.0080*  
*nn0: 0.2500*  
*nn1: 0.0900*  
*n0: 0.2000*  
*n1: 0.0700*  
*n2: 0.1200*  
*n3: 0.1635*

*n4: 0.1585*  
*n5: 0.1585*  
*tln: 0.5168*  
*tl1: 0.2768*  
*tl2: 0.3009*  
*tl3: 0.6020*  
*l\_com: 1.6964 (tip of top blades to CoM of test mass)*  
*bd: 0*  
*longpitch1: [0.3698 0.4327 0.8520 0.9600]*  
*longpitch2: [1.6768 1.9957 3.4073 3.9375]*  
*yaw: [0.6457 1.3642 2.3789 3.1399]*  
*transroll1: [0.4407 0.8199 0.9760 2.0311]*  
*transroll2: [2.7208 3.5610 3.9241 12.4805]*  
*vertical: [0.6867 2.7317 4.4000 8.7578]*

*Note that the blade parameters listed above are those which are generated in this model, but these values get refined using better knowledge gained from testing of blades already manufactured and more detailed modeling. A note of revisions to the above for the top two stages of blades (from M Plissi, Glasgow) is given here for completeness.*

-----

*uppermost blades (Mn) a=9.5 cm, h=4.4 mm, uncoupled freq.=2.41 Hz, max. stress=951 MPa (shape factor used=1.36)*

*blades (M1) a=5.8 cm, h=4.8 mm, uncoupled freq.=2.58 Hz, max. stress=943 MPa (shape factor used=1.4)*

*The max. stress has gone up a bit in both cases but is still below 60% of the elastic limit, which is a good safety margin.*

-----

### ***C.2.2 Modecleaner triple suspension parameters generated from ALIGOMC\_may\_07\_2003 using noise prototype parameters***

*m1\_parameters: 'Calculated from SWorks Assembly 01/09/02'*

*material1: 'combination steel+alum'*

*m1: 3.2290e+000*

*I1x: 2.3800e-002*

*I1y: 2.4000e-003*

*I1z: 2.3800e-002*

*m2\_parameters: 'Noise P-type: Silica Mass without flats and ears'*

*material2: 'silica'*

*ix: 7.5000e-002*

*ir: 7.5000e-002*

*m2: 2.9184e+000*

*I2x: 8.2081e-003*

*I2y: 5.4721e-003*

*I2z: 5.4721e-003*

*m3\_parameters: 'Noise P-type: Silica Mass without flats and ears'*

*material3: 'silica'*

*tx: 7.5000e-002*

*tr: 7.5000e-002*

*m3: 2.9184e+000*

*I3x: 8.2081e-003*

*I3y: 5.4721e-003*

*I3z: 5.4721e-003*

*l1: 2.9500e-001*

*l2: 1.6700e-001*

*l3: 2.2000e-001*

*nw1: 2*

*nw2: 4*

*nw3: 4*

*r1: 1.8000e-004*

*r2: 1.0000e-004*

*r3\_parameters: 'Fused-Silica'*

*r3: 7.5000e-005*

*Y1: 2.2000e+011*

*Y2: 2.2000e+011*

*Y3\_parameters: 'Fused-Silica Fibres'*

*Y3: 7.0000e+010*

*ufc1: 2.2900e+000*

*ufc2: 3.2200e+000*

*d0: 5.0000e-003*

*d1: 4.0000e-003*

*d2: 1.0000e-003*

*d3: 1.0000e-003*

*d4: 2.5000e-003*

*su: 0*

*si: 2.8500e-002*

*sl: 5.0000e-003*

*n0: 7.7000e-002*

*n1: 1.0000e-001*

*n2: 4.0000e-002*

*n3: 7.6500e-002*

*n4: 7.6500e-002*

*n5: 7.6500e-002*

*tl1: 2.9410e-001*

*tl2: 1.6296e-001*

*tl3: 2.2000e-001*

*l\_total: 7.6556e-001*

*l\_com: 6.9056e-001*

*longpitch1: [6.7049e-001 1.0362e+000 1.5137e+000]*

*longpitch2: [2.4185e+000 2.7733e+000 4.5175e+000]*

*yaw: [1.0924e+000 1.9470e+000 3.5363e+000]*

*transroll1: [6.7809e-001 1.5151e+000 2.1572e+000]*

*transroll2: [2.7845e+000 3.8033e+000 2.8664e+001]*

*vertical: [1.2073e+000 4.1866e+000 1.9889e+001]*

*The cantilever blades have the following updated numbers: -*

*Upper Blades: thickness = 1.5mm; length = 250mm; width = 40mm; uncoupled mode frequency = 2.3 Hz; internal mode = 90 Hz;*

*stress = 745 MPa; shape factor = 1.32; deflection = 143mm; radius = 188mm; mass total / blade = 4.6kg.*

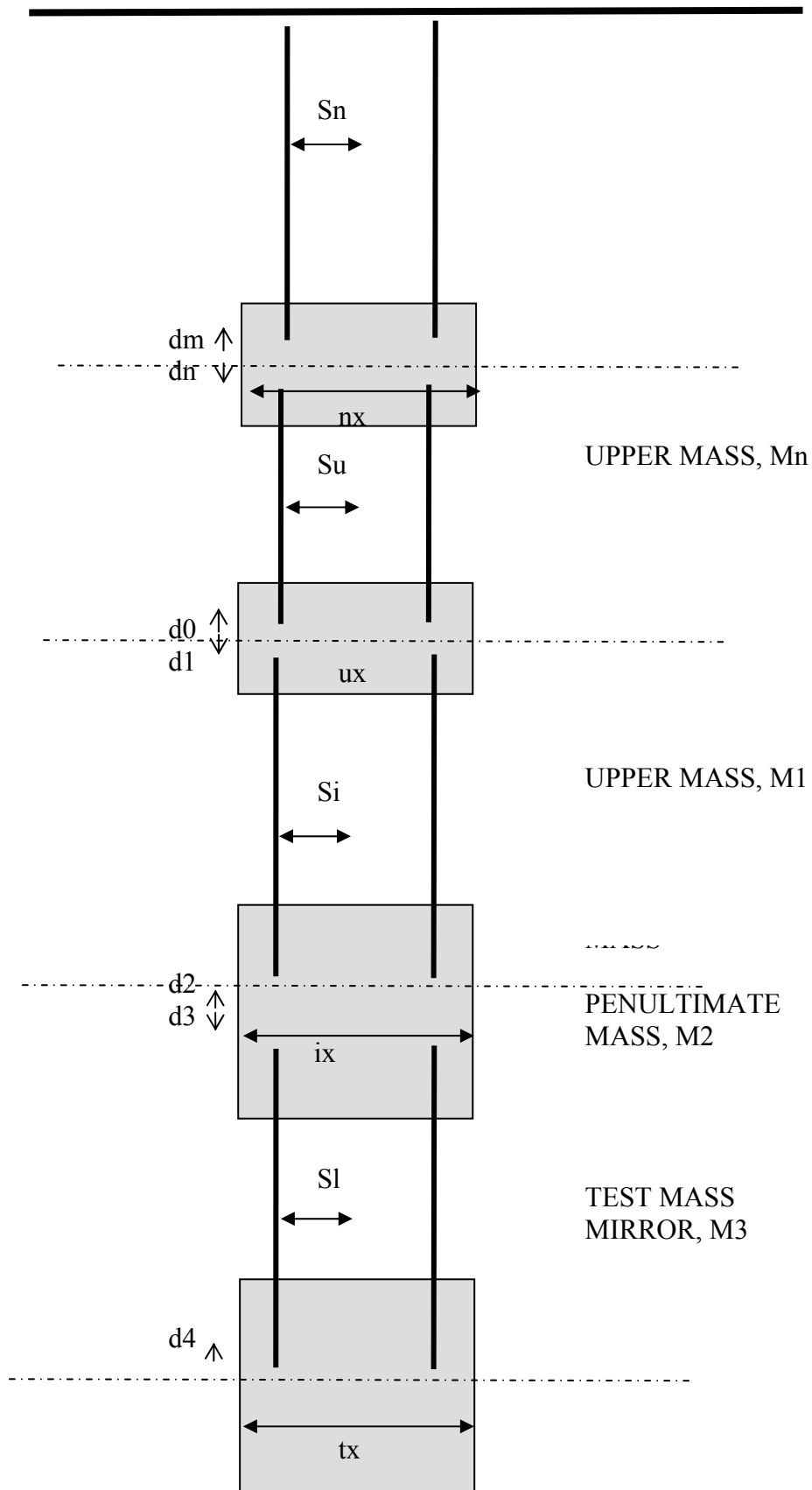
*(measured uncoupled mode = 2.28Hz, with 1.55kg; internal mode 86 Hz)*

*Lower blades: thickness = 1.0mm; length = 120mm; width = 18mm; uncoupled mode frequency = 3.4 Hz; internal mode = 261 Hz;*

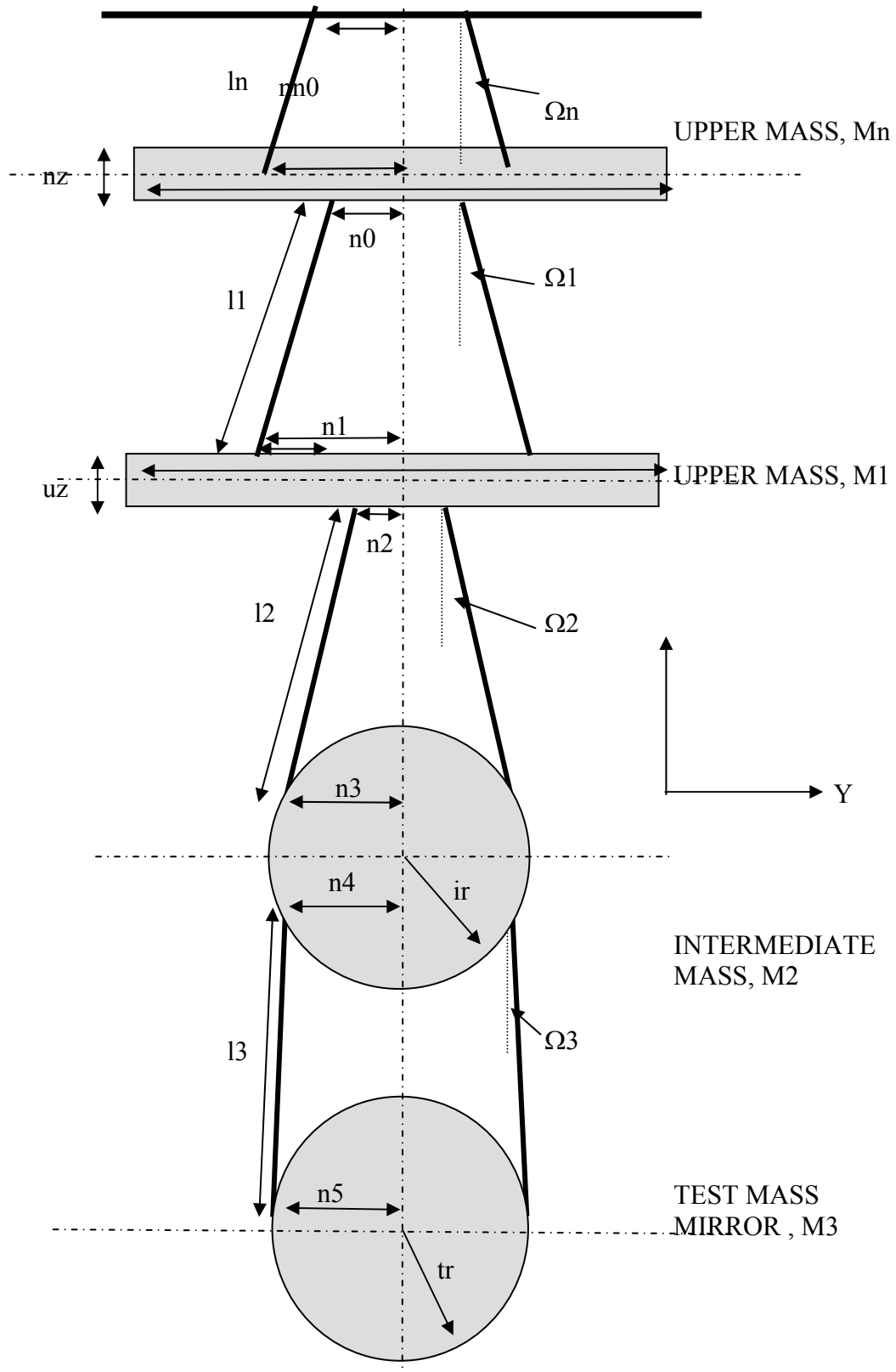
*stress = 797 MPa; shape factor = 1.55. deflection = 49mm; radius = 138mm; mass total / blade = 1.5kg.*

*(measured uncoupled mode = 3.31Hz, with 0.71kg; internal mode 226 Hz)*

C 2.3. The parameters of a quadruple pendulum (side on view)



The parameters for a quadruple pendulum (face on view)



## Appendix D: original version of sections 3.1 and 3.2

### 3.1 ETM/ITM Quadruple Suspensions

The key parameters used for all the curves presented in this section are as follows (except where otherwise indicated)

Final mass = 40 kg sapphire, 31.4 cm x 13 cm

Penultimate mass = 72 kg (heavy glass)

Upper masses = 36 kg, 36 kg

Overall length (from top blade to centre of mirror) = 1.7 m

Ribbon parameters: length = 60 cm, cross-section = 113  $\mu\text{m}$  x 1.13 mm

Stress in ribbon = 767 MPa

Lowest internal modes of blades (from top blades to bottom) = 75, 98, 118 Hz

#### 3.1.1 Thermal noise performance.

In figure 1 we present the baseline design. The target figure of  $10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz is essentially met (in fact the noise is slightly above the target at exactly 10 Hz). We also show what happens if the penultimate mass is made of silica rather than a heavy glass, raising the uppermost vertical mode frequency of the quadruple pendulum to above 10 Hz.

Various changes could be made to the baseline design. A marginal improvement to the performance at 10 Hz and above could be made if one lengthened the final stage to say 70 cm. Increasing the cross-section of the fibre could gain some improvement above 10 Hz at the expense of raising the vertical resonant frequency to be closer to 10 Hz, and lowering the violin mode frequencies. This improvement arises since the changing the cross-section changes the position of the thermoelastic peak. Using circular fibres loaded to the same stress as the baseline design (thus keeping the vertical mode frequency at the same value) raises the thermal noise in the 10 Hz region and above – as can be seen from figure 2.

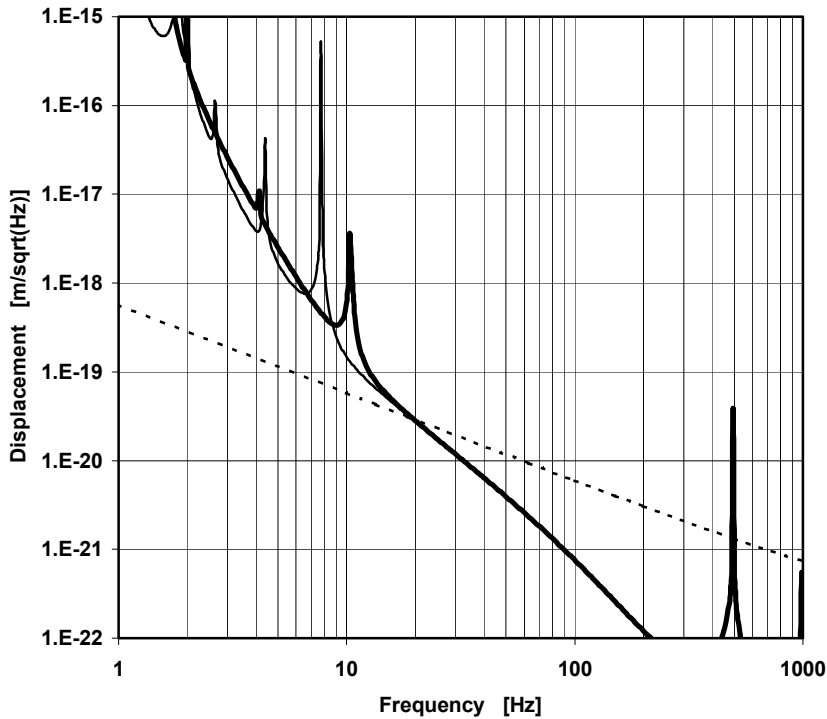


Figure 1. Suspension thermal noise for baseline 40kg quadruple pendulum. Two suspension curves are shown. The light solid line is the baseline design. The heavy solid line shows the effect of replacing the 72 kg heavy glass penultimate mass with a silica mass of same dimensions (weighing 22.1 kg). For comparison we also show the expected internal thermal noise curve for sapphire (dotted line). Note that the internal thermal noise curve assumes no loss due to coatings, or due to bonding of ears for attaching the suspensions.

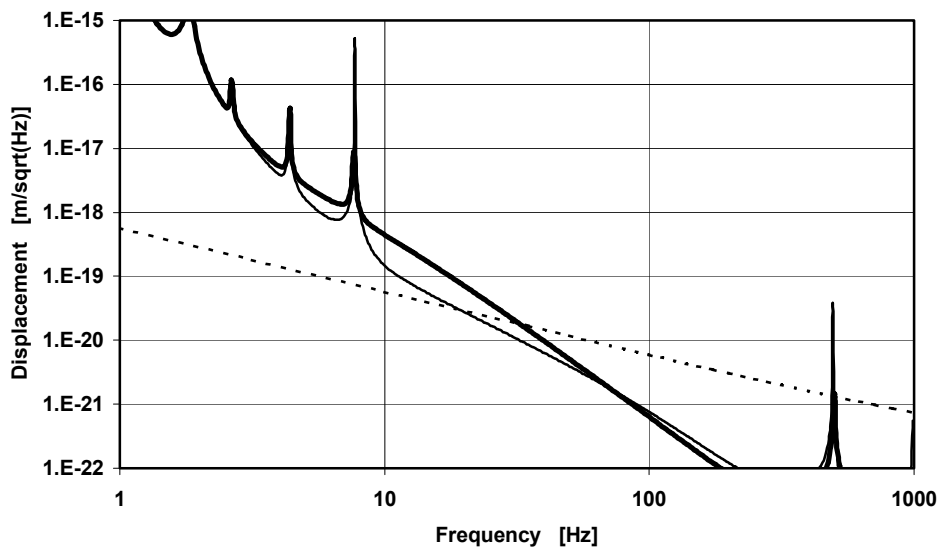


Figure 2. Heavy solid line is thermal noise for fibres of 200mm diameter, stressed to same value as baseline ribbon design. Light solid line is baseline, dotted line is internal thermal noise for sapphire.

### 3.1.2 Isolation performance

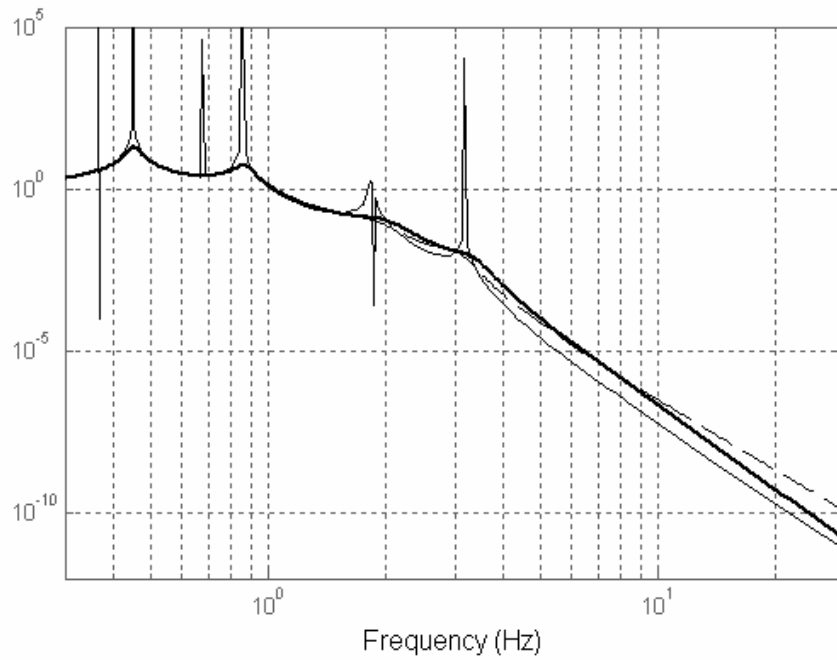


Figure 3. Longitudinal isolation for quad pendulum, with (heavy solid line) and without (light solid line) local controls on, and with eddy current damping (dashed line).

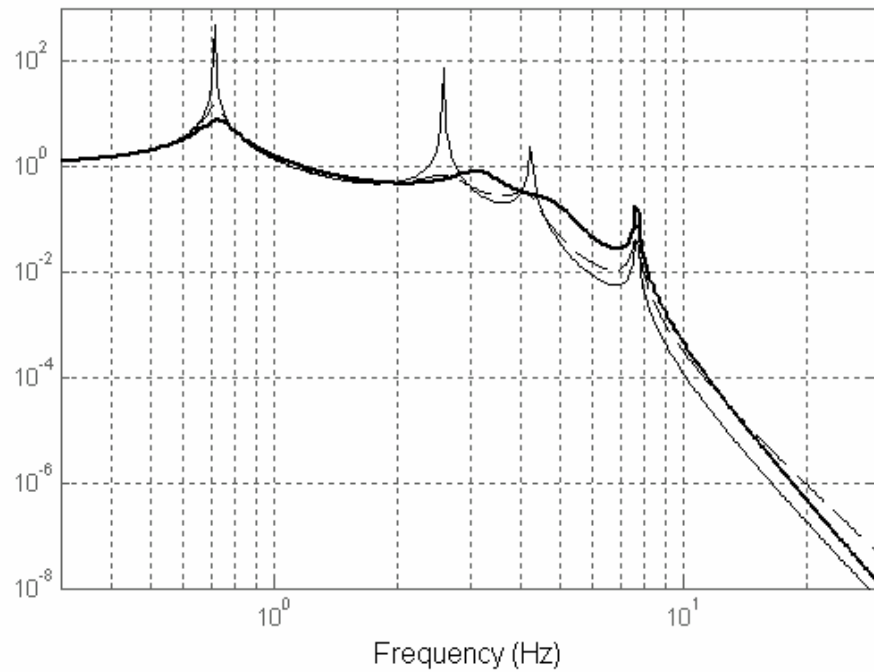


Figure 4. Vertical isolation for quad pendulum, with (heavy solid line) and without (light solid line) local controls on, and with eddy current damping (dashed line).

### 3.1.3 Sensor Noise Performance

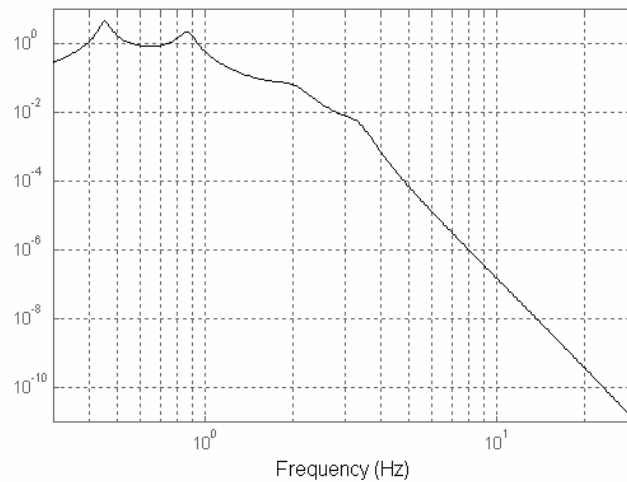


Figure 5. Longitudinal transfer function from sensor to mirror.

Noise level at mirror can be calculated by taking this transfer function and multiplying by the sensor noise in  $\text{m}/\sqrt{\text{Hz}}$  and the fraction of the full gain being used to damp the modes (at the level of damping shown in figure 2). For example, with a sensor noise of  $10^{-11} \text{ m}/\sqrt{\text{Hz}}$ , and turning down the gain by a factor of say 100 (corresponding for example to taking a damped Q of 10 to a Q of 1000) the overall noise at the mirror is  $10^{-11} \times 10^{-2} \times \text{TF value (m}/\sqrt{\text{Hz}})$

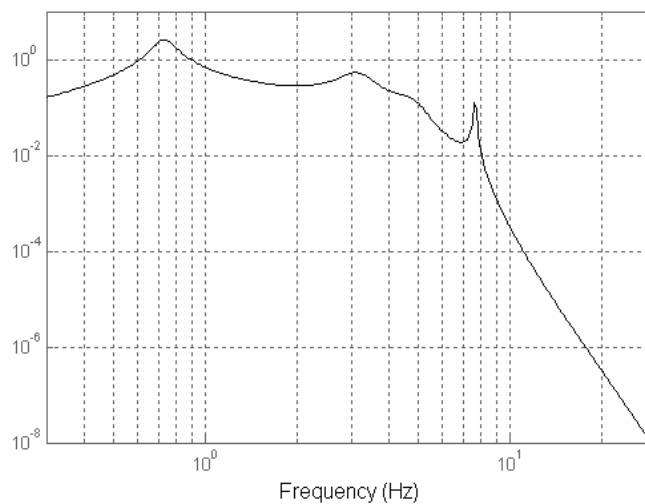


Figure 6. Vertical transfer function from sensor to mirror.

Longitudinal noise level at mirror can be calculated as above, with an extra factor, the cross-coupling factor, in the product, i.e. sensor noise  $\times$  gain fraction  $\times$  transfer function  $\times$  cross-coupling factor.

### 3.2 Triple Suspensions

The recycling mirrors and the modecleaner mirrors have different displacement noise requirement. The more sensitive requirement is for the modecleaners and thus we present the curves for it. If we can meet the target for those mirrors, the recycling mirrors should not be so difficult.

### 3.2.1 Thermal Noise Performance

As indicated in the introduction, in the baseline document presented in January 2000, the triple suspension design incorporated fused silica fibres. Since that time, the displacement spec. has been relaxed, and is set at  $3 \times 10^{-17}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz. Thus we have considered the possible use of steel rather than silica suspensions, as shown in Figure 7.

The key parameters used in this model are as follows.

Final mass = 2.9 kg silica, 15 cm diameter by 7.5 cm thick

Top and second masses: 3.2 kg, 2.9 kg

Overall length (from top blade to centre of mirror) =  $\sim 0.7$  m

Lowest internal modes of blades (from top to bottom) = 120, 260 Hz

**Note added 5<sup>th</sup> Feb 2002. After this document was written, we found a mistake in how the particular parameter file for this triple pendulum was input into the version of the MATLAB model used to produce the graphs shown below. Thus the graphs are not correct for the parameters as listed above. The mistake has been rectified, and we are currently updating the full parameter set to fit a tighter footprint, and to achieve comparable performance to that shown. With the new parameters, the vertical isolation performance should be essentially unchanged from that shown below, horizontal a factor of  $\sim 2$  worse than shown. Once the new design is finalised, updated versions of Figs. 7 - 11 will be posted.**

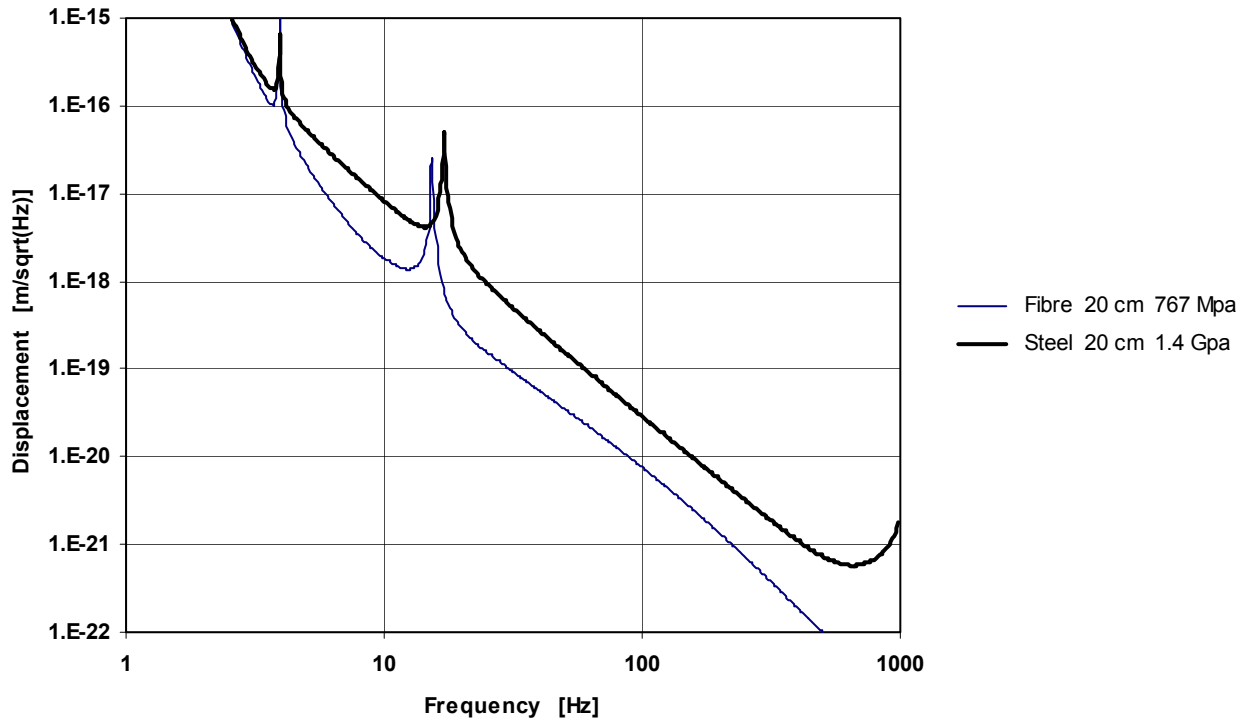


Figure 7. Suspension thermal noise for baseline modecleaner triple pendulum suspension. Two suspension curves are shown, one for silica, one for steel.

From consideration of the above curves, it can be seen that silica fibres meet the goal with a large safety margin, except for a narrow peak at the highest vertical mode. Steel also meets the requirement. A larger stress has been assumed. However the value taken gives a safety margin of at least two. The VIRGO collaboration has experience of using steel wires at this stress level, having first treated them to an annealing process. The dilution factor assumed in this curve for the final stage is  $\sim 930$ . This may be a more difficult target to achieve with steel wires due to possible losses at break-off bars. From measurements of violin mode Qs of  $\sim 10^6$  in the GEO modecleaner suspensions, which are on steel wires, we estimate we have achieved a dilution factor of  $\sim 200$ , in a system in which no significant effort was taken to maximise this. With a modest improvement in dilution factor over his value, the steel thermal noise curve would still meet the design requirement, and thus we are proposing that steel is seriously considered for use in these suspensions.

### 3.2.2 Isolation Performance

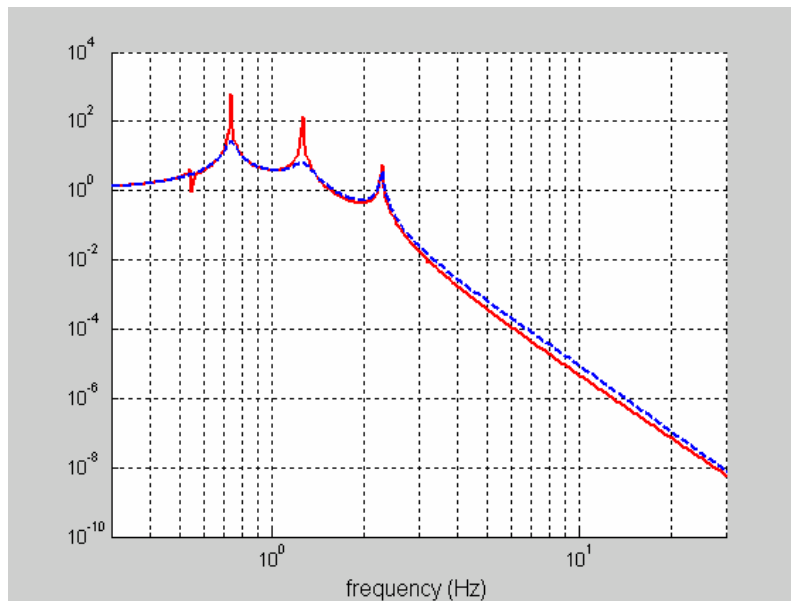


Figure 8. Longitudinal isolation for triple pendulum, without (red, solid line) and with (blue dotted line) local controls on.

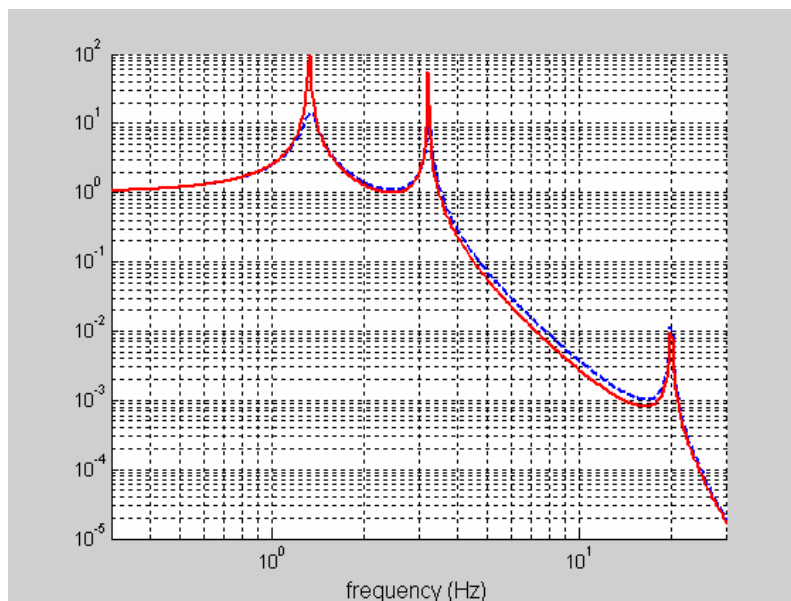


Figure 9. Vertical isolation for triple pendulum, without (red, solid line) and with (blue dotted line) local controls on.

### 3.2.3 Sensor Noise Performance

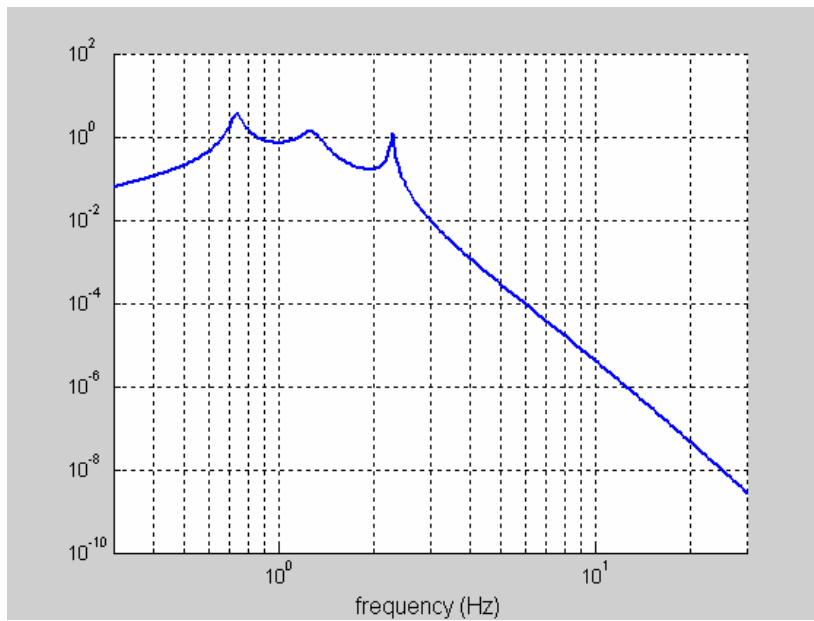


Figure 10. Longitudinal transfer function from sensor to mirror. (see caption for figure 5 for interpretation).

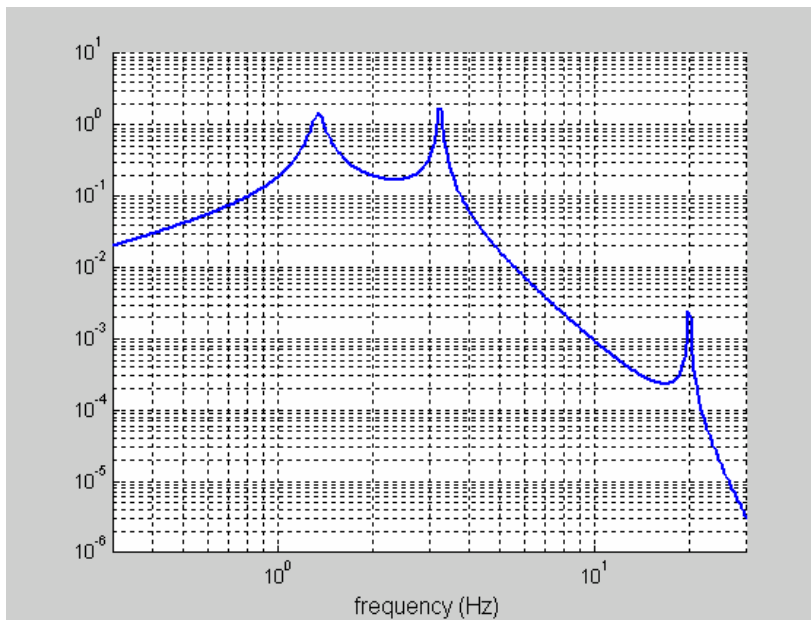


Figure 11. Vertical transfer function from sensor to mirror