

**Subject:** Re: [Aligo\_sus] Wire bending points  
**From:** Phil Willems <willems@ligo.caltech.edu>  
**Date:** Wed, 17 Dec 2003 07:21:51 -0800 (PST)  
**To:** "Greenhalgh, RJS (Justin)" <J.Greenhalgh@rl.ac.uk>  
**CC:** "Aligo\_Sus@Ligo. Caltech. Edu (E-mail)" <aligo\_sus@ligo.caltech.edu>

Dear Justin;

This quantity is easily derived from the differential equation for a stiff beam:

$$EI X'''' - TX'' + \rho w^2 X = 0$$

where  $X(z)$  is the wire displacement,  $\rho$  the linear mass density, and  $w$  the frequency. This has solutions in sines and cosines, and in hyperbolic sines and cosines. Inspect the term  $k$  in the solution  $X = \cosh(kz)$  in the limit  $w \rightarrow 0$ , this tells you how the wire bends at its endpoints.

Sincerely,  
 Phil

On Wed, 17 Dec 2003, Greenhalgh, RJS (Justin) wrote:

All,

Something I meant to ask for on the agenda for yesterday's SUS meeting, but forgot:

I took an action at the summit in Glasgow to document the method for calculating the difference between breakoff point and effective pivot point for a wire (this affects the "d" distance). Geppo explained to me offline that this is a standard problem and the answer is

$$\lambda = \sqrt{EI/T}$$

where

$\lambda$  is the distance from the breakoff point to the effective hinge point,

where  $E$  = Young's mod,

$I$  = second moment of area

$T$  is wire tension

Before applying the formula to the advanced LIGO suspensions I'd like to have some idea how it was derived, whether it has been experimentally verified, etc.

I had a good look through Calum's thesis but I didn't see it there. It doesn't appear in my standard solid mechanics texts. Has there been any work written up in this area?

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Aligo\_sus mailing list

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