HAM-SAS Mechanics
Status of modeling

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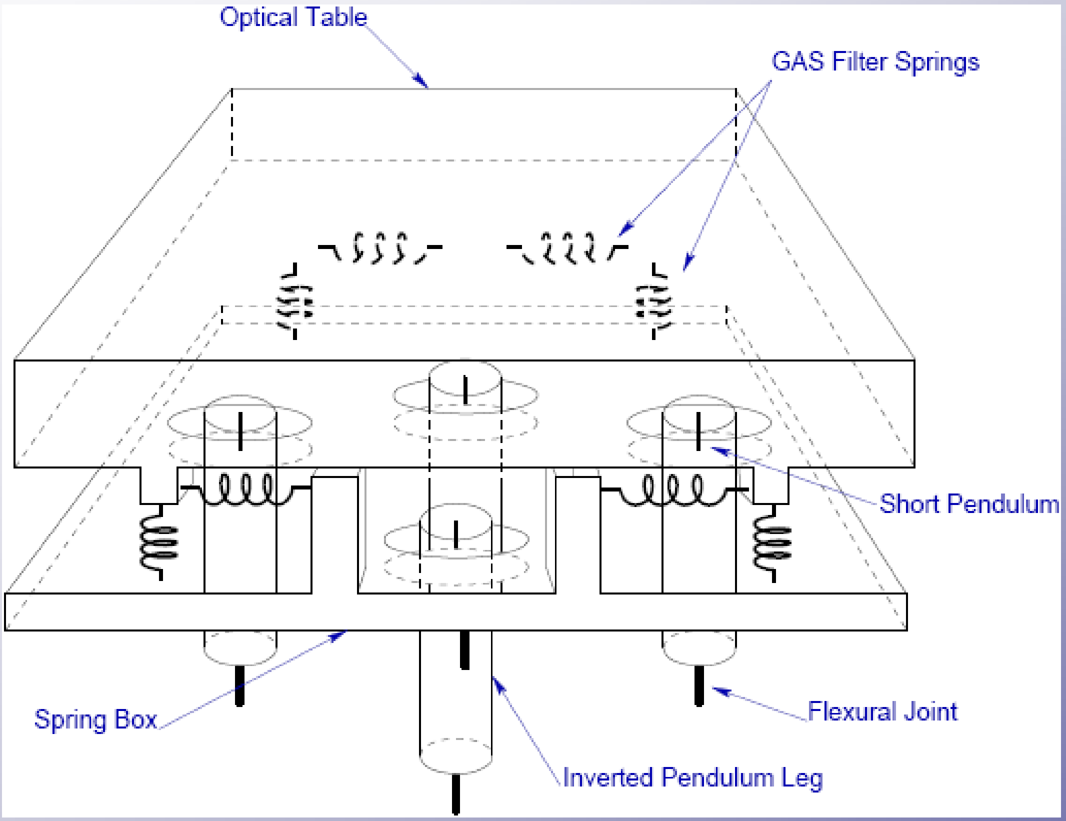
Introduction

HAM-SAS Attenuation Stages

HAM-SAS is a seismic attenuation system expressly designed to fit in the tight space of the LIGO HAM vacuum chamber.



Rigid Bodies

- 4 Inverted Pendula Legs (IPs)

- 4 MGAS Springs: Spring Box (SB)

- Optical Table (OT)

- Payload (mode cleaner suspensions, etc.)

Introduction

Modeling Approach

A state-space model of HAM-SAS mechanical structure have been developed using an Analytical approach.

Let’s summarize the approximations used in the model:

• Lumped system, i.e. rigid body approximation

• Elastic elements are approximated using quadratic potentials, i.e. small oscillation regime

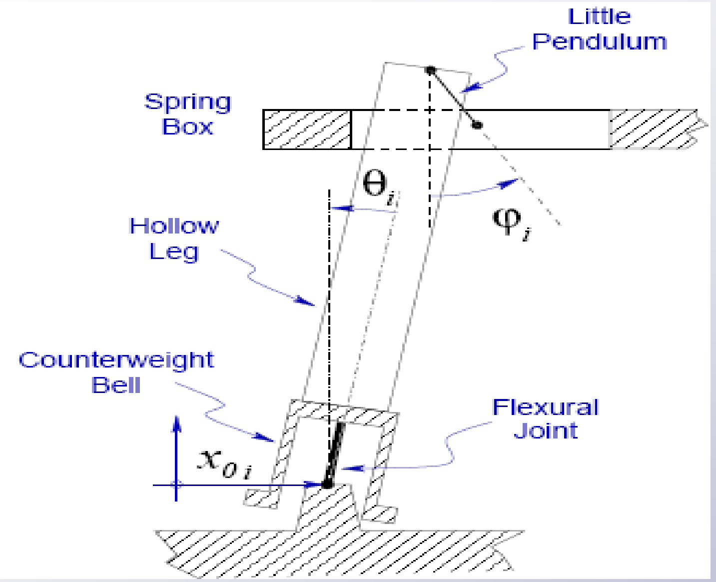
• Dissipation mechanisms are accounted using viscous damping which approximate structural/hysteretic damping in the small oscillation regime

•The system is considered symmetric enough to separate horizontal displacements x, y, and yaw from pitch, roll and vertical displacement z

•Internal modes of the mechanical structures are not accounted

Introduction

Modeling Approach

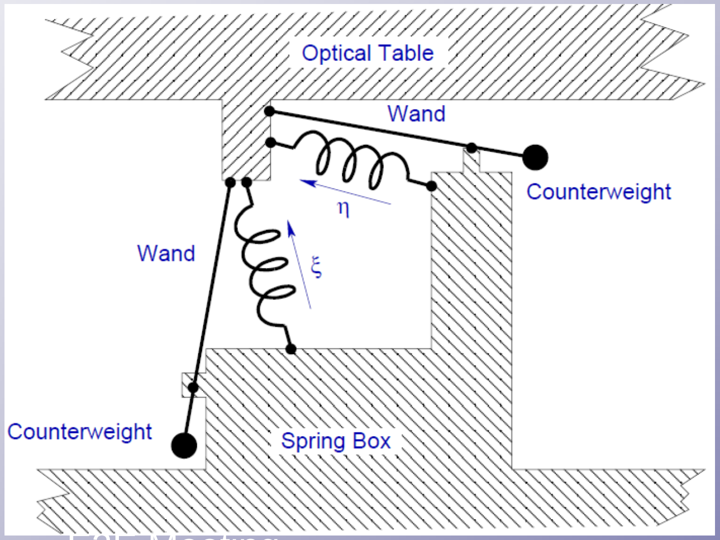


Inverted Pendulum

- Flexural Joint with Ideal pivot point about the attachment point.

-Leg, a rigid body

- Hysteretic/structural damping approximated with viscous damping.



GAS

- Blade stiffness modeled with simple Springs

- Hysteretic/structural damping approximated with viscous damping.

- Transmissibility saturation modeled using the "magic wand"

Maple scripts

Modular structure

The way that the code has been written is such that allows to progressively introduce new features to improve the accuracy and remove degrees of freedom to check the consistency of the simulation.

Lagrangian Generalized Coordinates

```
> q := vector([seq(SP_x [i],i=1..2),seq(theta1 [i] ,i=1..Pendula),seq(theta2 [i] ,i=1..Pendula),seq(phi   [i],i=1..Masses-Constraints)]);
  p := vector([seq(SP_xp[i],i=1..2),seq(theta1p[i] ,i=1..Pendula),seq(theta2p[i] ,i=1..Pendula),seq(phi_p[i],i=1..Masses-Constraints)]);
  u := vector([seq(Fx[i],i=1..Masses),seq(Ft[i],i=1..Masses),SP_Fx,SP_Ft]);
```

Lagrangian

Kinetic Energy Shaker

```
> SP_T := 1/2*SP_M*SP_Xp^2+ 1/2*SP_Iz*Theta_SP_zp^2;
```

Potential Energy Shaker

```
> SP_U := 1/2*SP_k*(SP_x[1]^2+SP_x[2]^2);
```

Kinetic Energy Masses

```
> MC_T := add(1/2 *MC_m[i] * MC_Xp[i]^2 + 1/2*MC_Iz[i]*Theta_MC_zp[i]^2+ 1/2*MC_Ix[i]*phi_p[i]^2,i=1..Masses);
```

Gravitational Potential Energy IPs

```
> MC_U :=   1/2 *g*add( MC_M[i]*(MC_H[i]*theta1[i]^2+MC_H[i]*theta2[i]^2)/2           ,i=1..Pendula )
          + 1/2 *g*add( MC_M[i]* MC_h[i]* phi[i]^2, i = 1..Masses);
```

External Forces

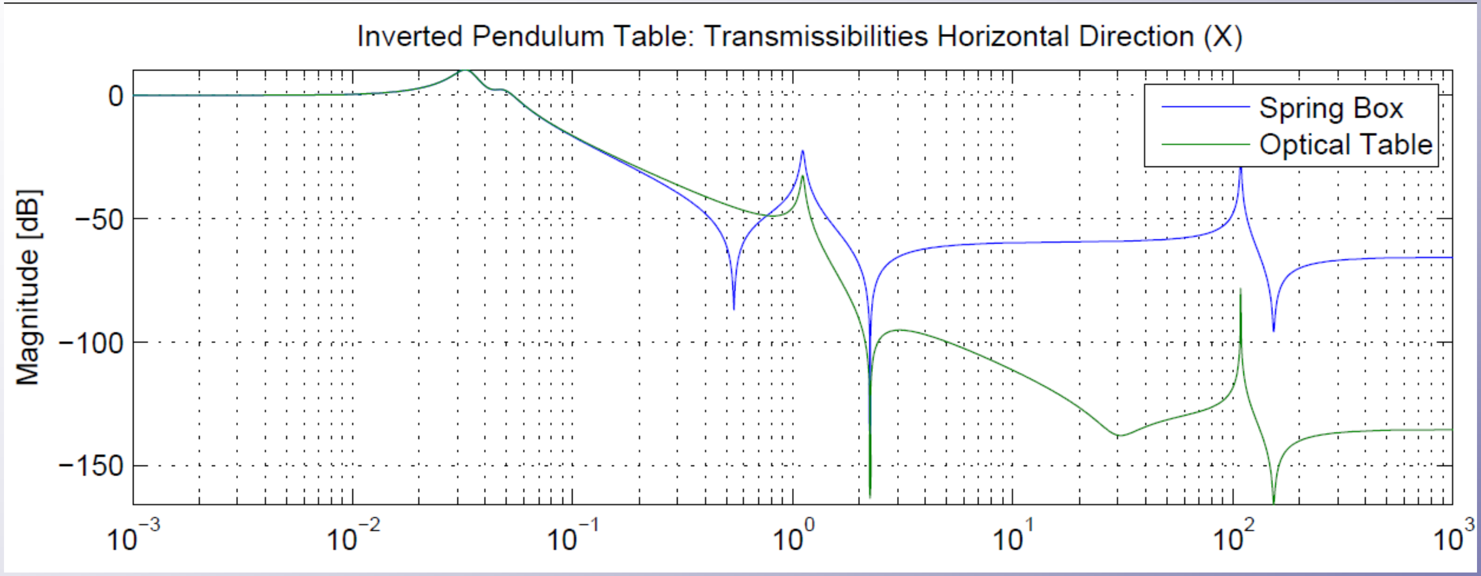
```
> MC_EF :=  add( Fx[i]* MC_h[i]* phi[i]                                    ,i=1..Masses)
         +add( Ft[i]*(MC_H[i]*theta1[i]-MC_H[i]*theta2[i])/(2*MC_Db[i]),i=1..Masses)
         + SP_Fx*SP_X
         + SP_Ft*Theta_SP_z;
```

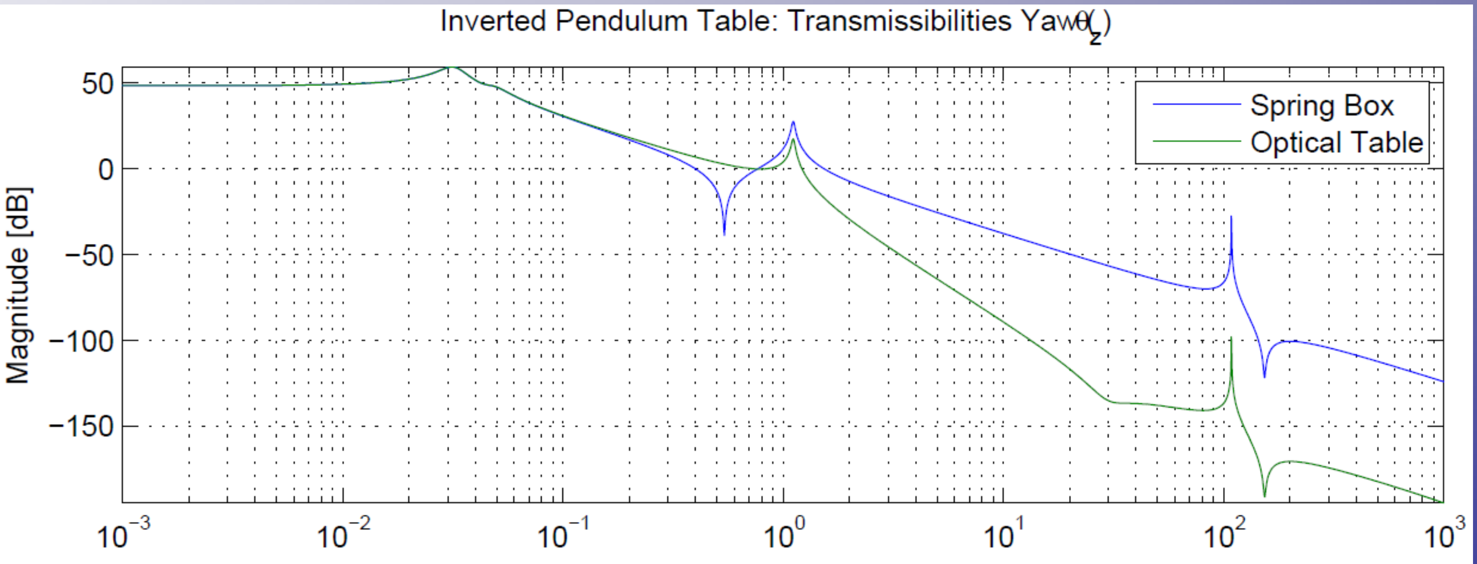
Lagrangian

```
> MC_Lagr:= SP_T+MC_T-SP_U-MC_U+ MC_EF:
```

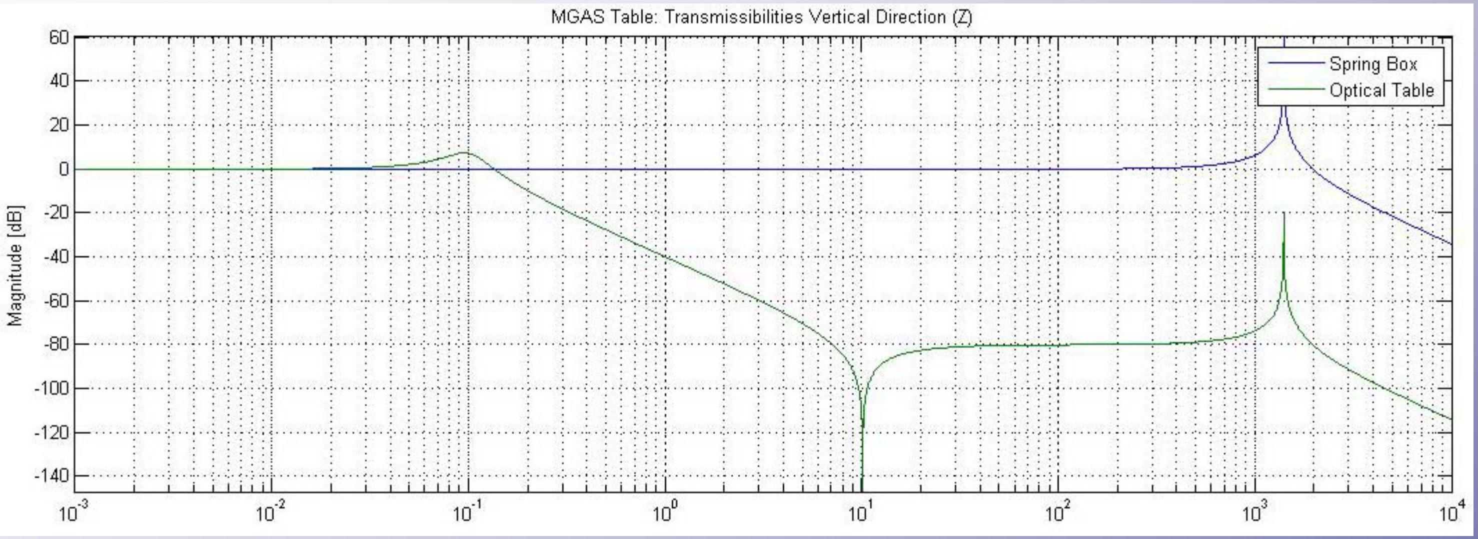
Results

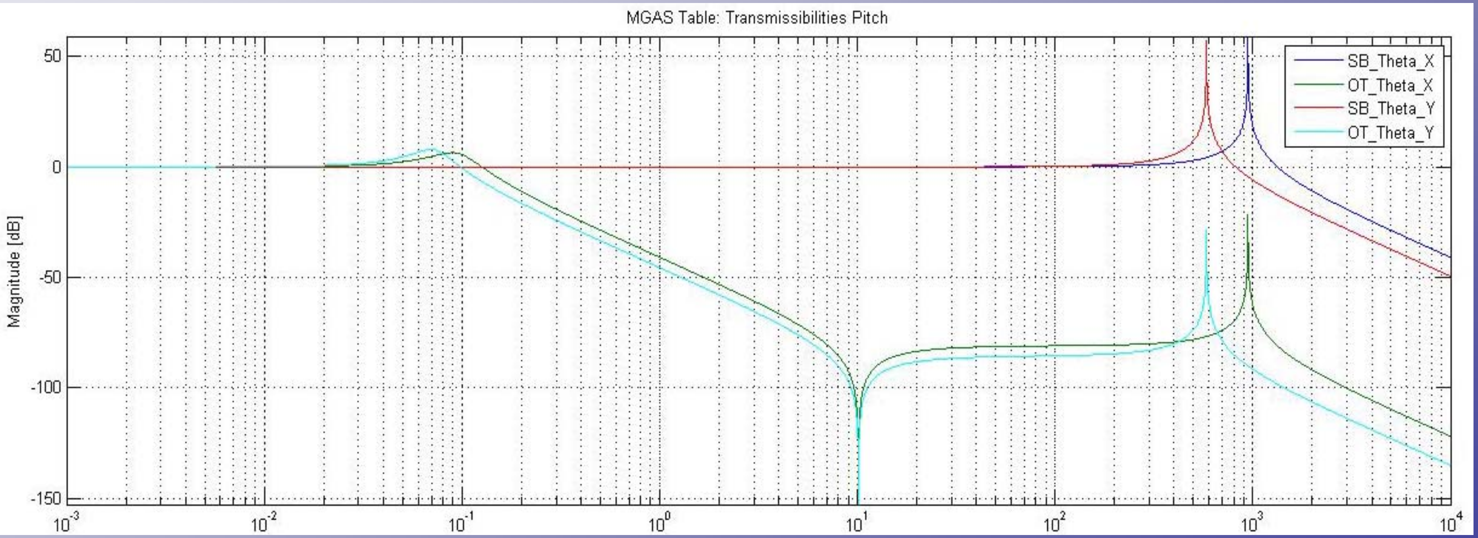
Horizontal Stage Model v5.0





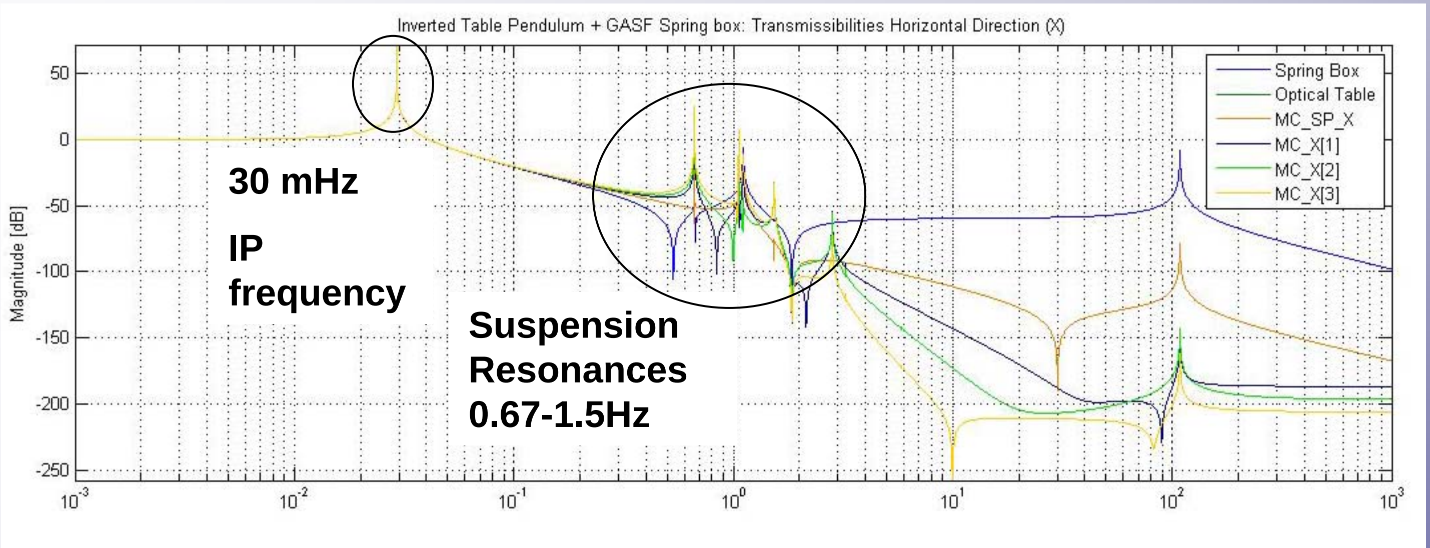
Results
Vertical Stage Model v3.3

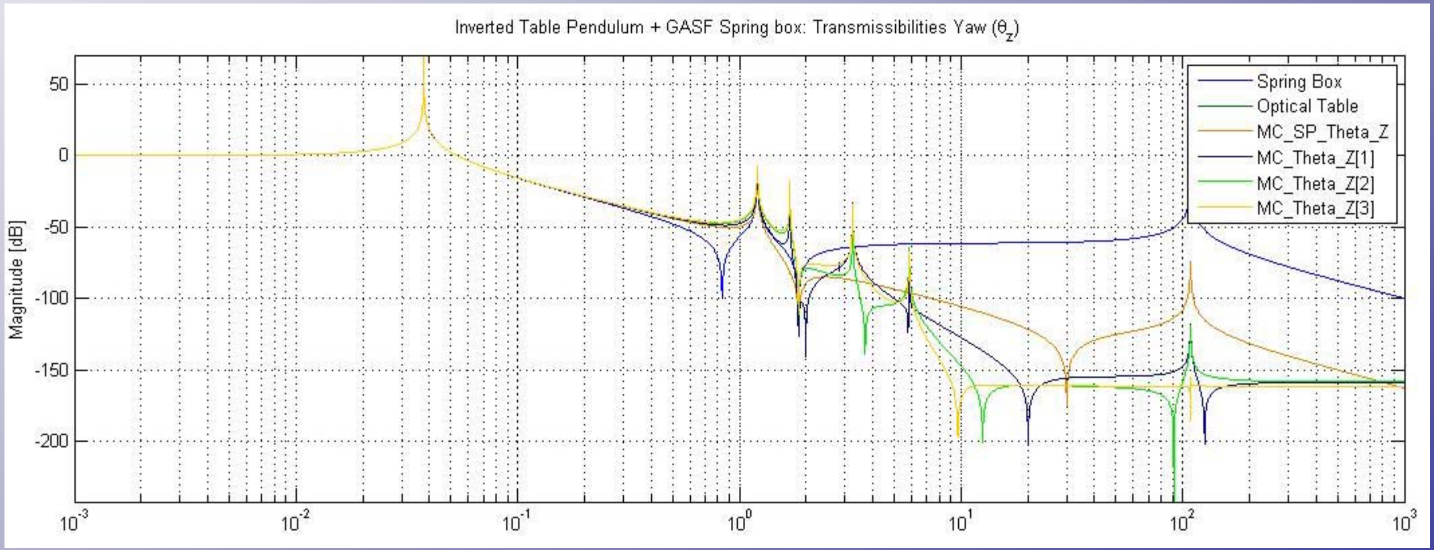




Results

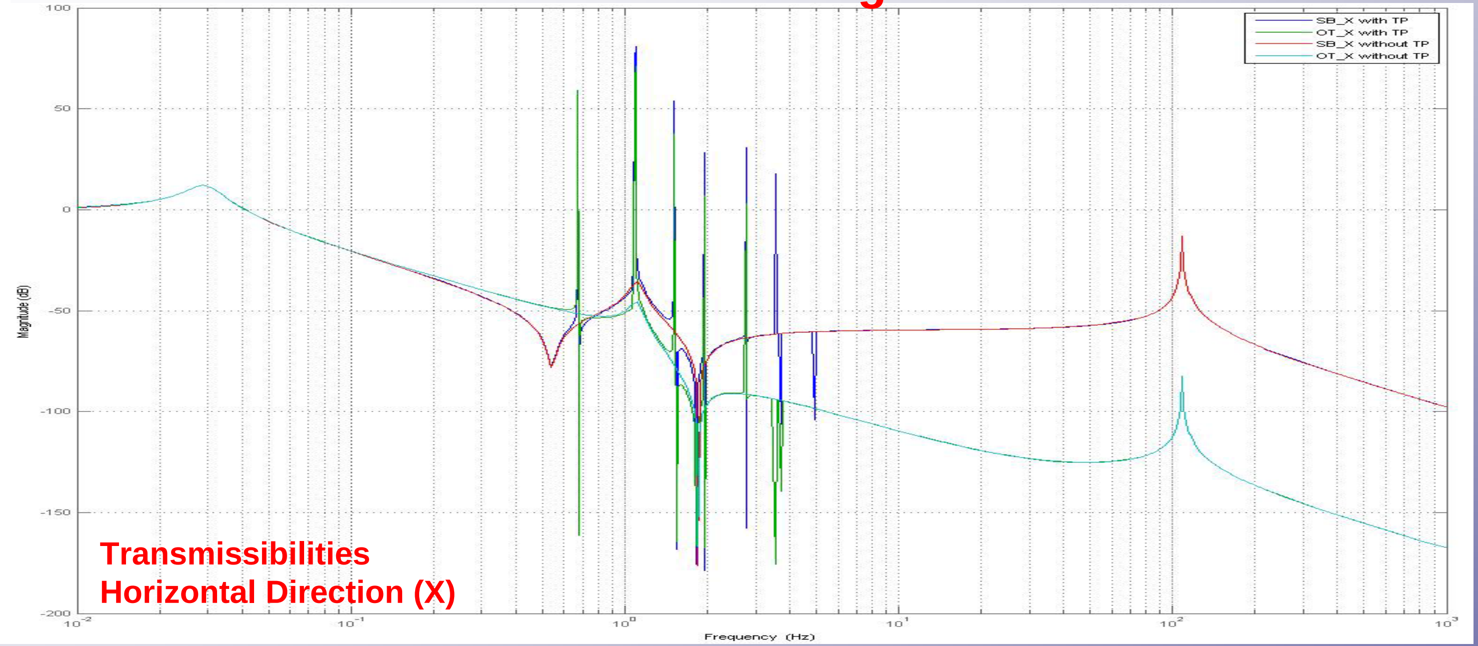
Triple Pendulum + Horizontal Stage Model

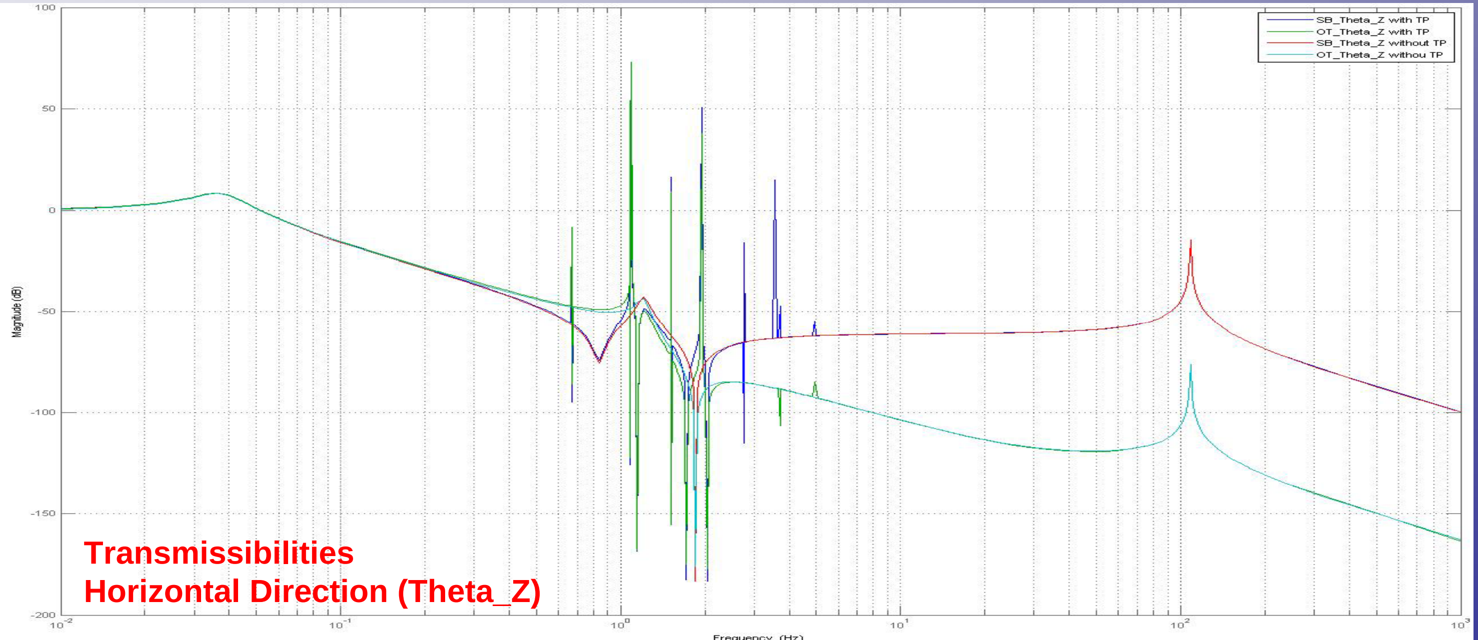




Results

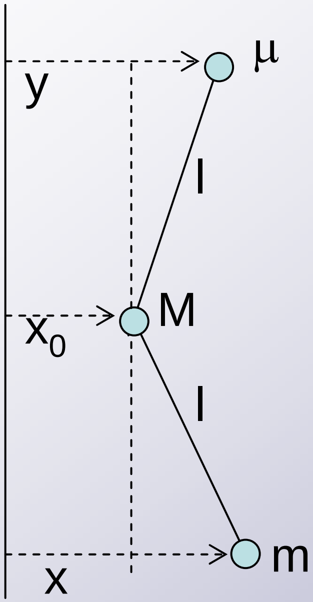
Barton's TP + Horizontal Stage Simulink Model





Problem

Backreaction Lagrangian Example #1



$$L_{tot} = L + L_{back}$$

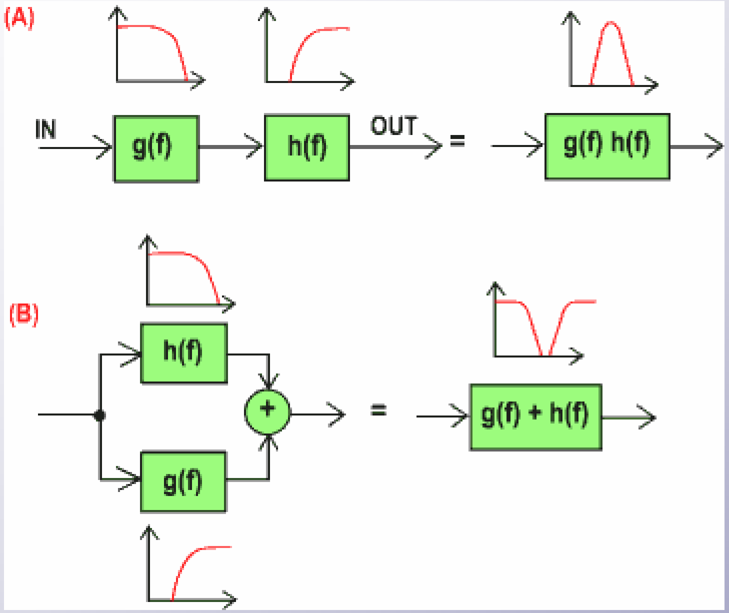
$$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}M\dot{x}_0{}^2 + mgl\left(1 - \cos\frac{x - x_0}{l}\right) \simeq \frac{1}{2}m\dot{x}^2 + \frac{1}{2}M\dot{x}_0{}^2 - \frac{1}{2}\frac{\left(x - x_0\right)^2}{l}mg$$

$$L_{back} = \frac{1}{2}\mu\dot{y}^2 + \left(M + \mu + m\right)gl\left(1 - \cos\frac{y - x_0}{l}\right) \simeq \frac{1}{2}\mu\dot{y}^2 + \frac{1}{2}\frac{\left(y - x_0\right)^2}{l}\left(M + \mu + m\right)g$$

Backreaction Lagrangian depends on the total mass of the system

Problem

Electronics filters analogy

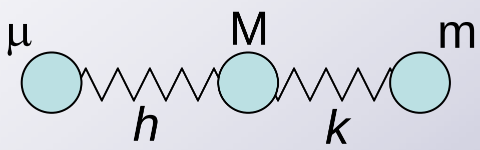


Since we consider the input impedance of analog filters to be infinite we can combine them in a linear way.

Doing the same thing when we connect mechanical systems means considering the mass of the system infinite

Problem

Backreaction Lagrangian Example #2



$$L_{tot} = L + L_{back}$$

$$L = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} M\dot{x}_0^{\ 2} + \frac{1}{2} k\left(x - x_0\right)^2$$

$$L_{back} = \frac{1}{2}\mu\dot{y}^2 + \frac{1}{2} h\left(y - x_0\right)^2$$