LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Calibration of the 40m Fabry-Perot Michelson Interferometer

Seiji Kawamura, Osamu Miyakawa, and Shihori Sakata

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of the LIGO Project.

California Institute of Technology
LIGO Project – MS 51-33
Pasadena CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu

ABSTRACT

The concise and reliable calibration method for the 40m prototype in a Fabry-Perot Michelson configuration is described.

1. Introduction

The 40m prototype at Caltech is now working with the Fabry-Perot Michelson Interferometer configuration. We produced the first displacement sensitivity spectrum of the prototype using a concise and reliable calibration method. The calibration consists of two steps:

1. Calibration of the mirror motion
2. Calibration of the displacement spectrum

In the following sections the procedure at each step will be described.

2. Calibration of the mirror motion

The mirror motion can be calibrated best with the mid-fringe locking in a simple Michelson interferometer. Figure 2 shows the schematic diagram of the method.

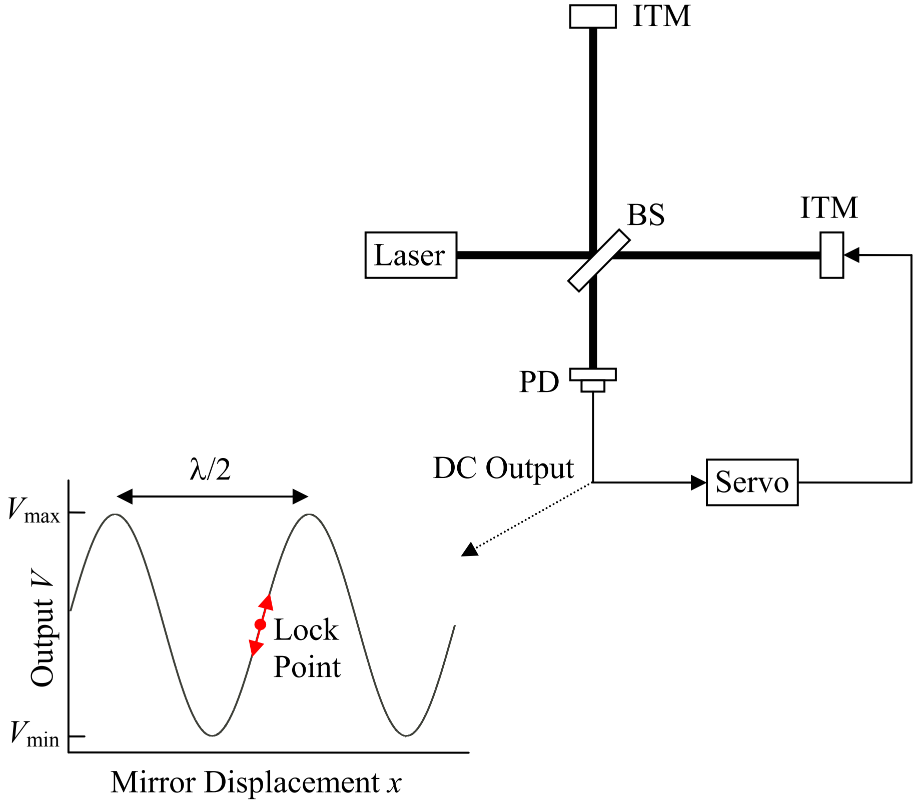

Fig. 2 Schematic diagram of the mid-fringe locking in a Michelson interferometer.

First the Fabry-Perot Michelson should be turned into a simple Michelson interferometer by blocking the beams in front of the end mirrors. Then we lock the Michelson interferometer to the mid-fringe using the DC output of the photodetector at the anti-symmetric port (or symmetric port). The output of the photodetector, V, is expressed by the mirror displacement, x, as follows,

$$V = \frac{V_{\mathrm{max}} - V_{\mathrm{min}}}{2} \sin(2\pi \frac{x}{(\lambda/2)}) + \frac{V_{\mathrm{max}} + V_{\mathrm{min}}}{2} ,$$

where λ is the wavelength of the light. Therefore the derivative, dV/dx, at the mid-fringe, where x=0, is

$$\left.\frac{\mathrm{d}V}{\mathrm{d}x}\right|_{x=0} = \frac{V_{\max} - V_{\min}}{2} \cdot \frac{4\pi}{\lambda}.$$

We excite one of the input test masses at 100Hz with appropriate amplitude. The resultant variation in the DC output of the photodetector gives the actual motion of the mass according to the above equation.

Note that we locked the Michelson interferometer very loosely so that the mass can be regarded completely free at the calibration frequency.[1] The loop gain of the servo system at 100 Hz was -25 dB which gives an error of less than 5% in calibration at 100Hz.

The results are summarized in Table 1.

Table 1 Excitation of the mass and the corresponding motion.

Excitation	Mirror motion
5,000 counts in amplitude at 100Hz injected at ITMX_LSC_EXC with dewhitening filter off	1.1×10^{-9} $\mathrm{m_{pp}}$

This calibration has an accuracy of ~10%. The calibration is valid unless the magnet-coil coupling changes significantly, which is unlikely. Thus we can use the same calibration value until any significant changes in the magnet-coil coupling occur.

3. Calibration of the displacement spectrum

Figure 1 shows the block diagram of the differential-mode servo system in the 40m Fabry-Perot Michelson interferometer.

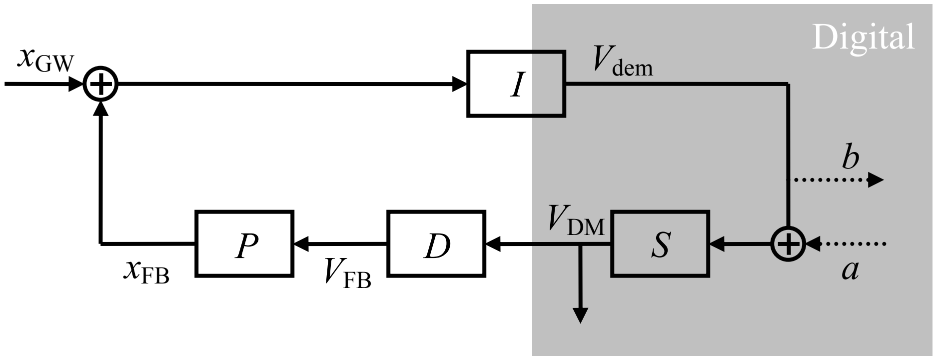

Fig. 1 Block diagram of the differential-mode servo system. The system is digital inside the gray area.

Here each symbol represents the following:

I: Interferometer sensitivity
S: Servo filter for the differential-mode arm cavity length control
D: Dewhitening filter or bypass (no filter)
P: Pendulum response

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[1] This is not a necessary condition for the calibration. If the gain at the calibration frequency is not negligible, we should just take the effect of the servo into consideration.

x_{GW}: Displacement signal by gravitational waves
V_{dem}: Demodulated signal
V_{DM}: Differential-mode feedback signal
V_{FB}: Feedback voltage to the coils
x_{FB}: Feedback displacement

The output of the interferometer to be measured is the feedback output of the differential-mode servo, V_{DM}, as shown in Fig. 1. If V_{DM} contains only gravitational wave signals,

$$V_{\mathrm{DM}} = \frac{IS}{1 - ISDP} x_{\mathrm{GW}} .$$

In reality V_{DM} consists of only noise. Thus the noise corresponding to the gravitational wave signals in terms of displacement, x_{noise}, is obtained by

$$x_{\mathrm{noise}} = \frac{1 - ISDP}{IS} V_{\mathrm{DM}} .$$

A main portion of this coefficient can be obtained by measuring the closed-loop transfer function from *a* to *b*, $T_{\mathrm{a \to b}}$, in Fig. 1.

$$T_{\mathrm{a \to b}} = \frac{ISDP}{1 - ISDP} .$$

Thus

$$x_{\mathrm{noise}} = \frac{DP}{T_{\mathrm{a \to b}}} V_{\mathrm{DM}}$$

The rest of the coefficient is obvious. *D* was measured to have two zeros at 15Hz, two poles at 100Hz, and a DC gain of 8dB with the dewhitening filter on, and just a flat response with a DC gain of 8dB with the dewhitening filter off. *P* is well known to have a frequency response of f^{-2} above the resonance frequency. Note that the low pass filter due to the coil inductance and the series resistance has a corner frequency much higher than our interesting frequency range (up to 7 kHz). Thus the effect is negligible.

Even without knowing the DC gain of *P*, we can now construct x_{noise} with an arbitrary unit and make use of the excitation of ITMX to calibrate the arbitrary unit. We take the spectrum with the excitation on. The resultant peak in the displacement spectrum should match the calibrated motion of the mirror. Note that we should be careful to take into consideration that the peak value of the excitation in the spectrum has a unit of $\mathrm{m}_{\mathrm{rms}} / \sqrt{\mathrm{Hz}}$, which depends on the bandwidth of the spectrum taken. Table 2 shows the excitation used and the peak value in the spectrum.

Table 2 Excitation of the mass used for calibration.

Excitation	Peak value
200 counts in amplitude at 100Hz injected at ITMX_LSC_EXC with dewhitening filter off	$1.3 \times 10^{-11} \mathrm{m}_{\mathrm{rms}} / \sqrt{\mathrm{Hz}}$ (BW=1.5Hz)

The resultant calibrated displacement spectrum as of Nov. 25, 2003 is shown in Fig. 3.

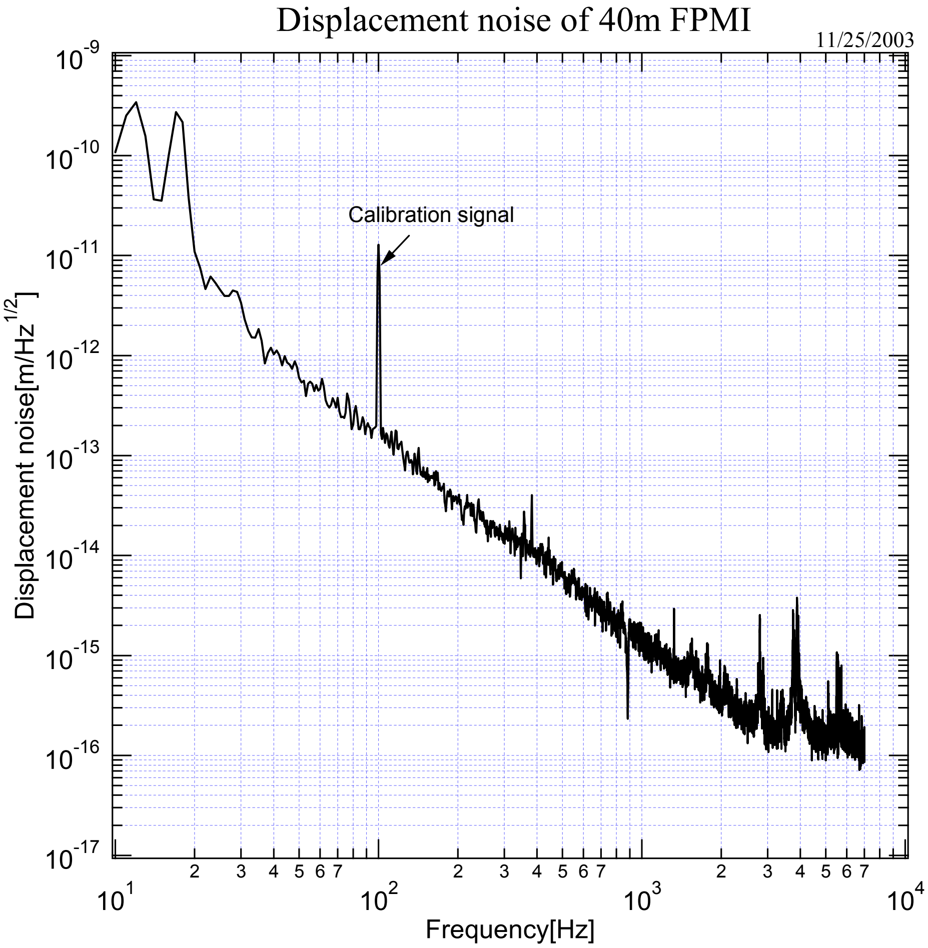

Fig. 3 Calibrated sensitivity spectrum of the 40m Fabry-Perot Michelson interferometer.