

Measurement of Optical Response of a Detuned Resonant Sideband Extraction Interferometer LIGO-P060007-00-R

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We report on the optical response of a suspended-mass detuned resonant sideband extraction (RSE) interferometer with power recycling. The purpose of the detuned RSE configuration is to manipulate and optimize the optical response of the interferometer to differential displacements (induced by gravitational waves) as a function of frequency, independently of other parameters of the interferometer. The design of our interferometer results in an optical gain with two peaks: an RSE optical resonance at around 4 kHz and a radiation pressure induced optical spring at around 41 Hz. We have developed a reliable procedure for acquiring lock and establishing the desired optical configuration. In this configuration, we have measured the optical response to differential displacement and found good agreement with predictions at both resonances and all other relevant frequencies.

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Currently the first generation of ground-based laser interferometric gravitational wave (GW) observatories, including LIGO[1], VIRGO[2], GEO[3] and TAMA[4], are in operation. Together, they form a global network for the detection and study of GWs and their astrophysical sources. However, the expected event rate for detectable GWs is still very low with the present sensitivity, and more sensitive detectors are required in order to detect significant numbers of sources. Advanced LIGO [5, 6] is one of the next-generation of planned gravitational wave detectors which will employ a detuned signal mirror in order to manipulate the frequency response of the detector. Such configurations hold the possibility of circumventing the free-mass Standard Quantum Limit (SQL) on the measurement of small displacements [7–9] at frequencies between those where shot noise dominates and those where radiation pressure noise dominates.

Long-baseline gravitational wave detectors are based on Michelson interferometers, which are designed to be sensitive to small differential displacements of the arms. The Initial LIGO, VIRGO, and TAMA300 detectors add Fabry-Perot resonant optical cavities in the arms, and a power recycling cavity formed from a mirror between the laser and the Michelson, to enhance the optical gain of the displacement measurement. The next generation of detectors aims to improve on the sensitivity by making use of higher-powered lasers and more complex optical configurations.

Signal recycling (SR) was proposed by Meers[10] as a means to reduce the shot noise contribution to the displacement signal over a narrow band near DC. By using a signal mirror at the asymmetric port of the Michelson to form a signal recycling cavity, the storage time of the signal sidebands can be increased, leading to enhanced sensitivity near DC but reduced sensitivity at higher frequencies; the detector bandwidth is reduced. Thus, it is most effective to use this technique with interferometers whose arms are optical delay-lines (such as GEO) or low-finesse cavities. High-gain power recycling (PR) is used to enhance the overall optical gain, resulting in a configuration referred to as dual recycling.

Mizuno [11] proposed a complementary configuration, called resonant sideband extraction (RSE). In this configuration, the interferometer arms are high finesse cavities, the power recycling gain is correspondingly lower, and a signal extraction cavity is used to resonantly extract the signal sidebands from the arms, thereby enhancing the signal bandwidth. The finesse of the relatively low-loss arm cavities can be made large, with RSE compensating for the resulting loss of bandwidth. The required optical gain of the higher-loss power recycling cavity is correspondingly reduced, resulting in both higher overall optical gain, and much reduced requirements for compensation of the thermal load on the transmissive optics in the power recycling cavity (beam splitter and arm input test masses). As interferometric detectors move to higher

laser power, this thermal loading can be a severe problem. Thus, Advanced LIGO has chosen as its baseline optical configuration a power-recycled Michelson interferometer with high-finesse Fabry-Perot arms and an RSE signal extraction cavity [12].

When all other noise sources are reduced, the sensitivity of such an interferometer is limited by quantum sensing noise, which enters the interferometer through the laser light incident at the symmetric port and the vacuum incident at the asymmetric port of the Michelson (where the gravitational wave signal exits) [13, 14]. At high frequencies, photon shot noise dominates; this contribution to the sensitivity can be reduced by increasing the laser power. At low frequencies, quantum fluctuations in the radiation pressure on the test masses dominate; this contribution can be reduced by decreasing the laser power. In the presence of additional noise sources such as seismic noise and thermal noise in the test masses or their suspensions, the laser power can be chosen to optimize sensitivity. By making use of a signal cavity, the frequency dependence of the quantum limit on the sensitivity can be manipulated between the extremes of signal recycling, resonant sideband extraction, or in-between (“detuned”).

A detuned signal cavity will exhibit optical resonances at frequencies that can be chosen to optimize sensitivity in the presence of other noise sources. A peak at higher frequency arises from the unbalanced response of a GW sideband resonating in the detuned SR cavity (the RSE optical resonance). A peak at lower frequency arises because the GW sidebands induced by the differential displacement of arm cavities sees a signal cavity detuned from resonance, forming a radiation pressure induced opto-mechanical spring.

The optical response and noise spectra of the detuned RSE optical configuration was analyzed by Buonanno and Chen [8] using the KLMTV formalism [15]. Figure 1 shows the relationship between the input vacuum field a_i (carrying the quantum noise) to the signal port output field b_i , the input laser power I_0 , and the gravitational wave strain signal h . These quantities are related by Eq.(2.26) in Ref.[8]. The relation between a and h determines the quantum-limited strain sensitivity for the interferometer. The sensitivity as a function of frequency exhibits two dips in between the frequency regions where radiation pressure noise and the shot noise dominate, corresponding to the optical spring and detuned RSE optical resonances. Buonanno and Chen [7–9] showed that in this configuration, “ponderomotive squeezing” will induce correlations in the quantum noise. The possibility exists of achieving quantum-limited sensitivity beyond the “standard” quantum limit in the frequency region around these two resonances.

To measure the optical gain of the interferometer response to a strain h , the end test masses are displaced differentially by application of an external sinusoidal force. Typically, the applied displacement is large, so that $h \gg a_i$ and the input vacuum a_i is ignored. In that approximation, the ratio between h and b_i in Eq.(2.26)

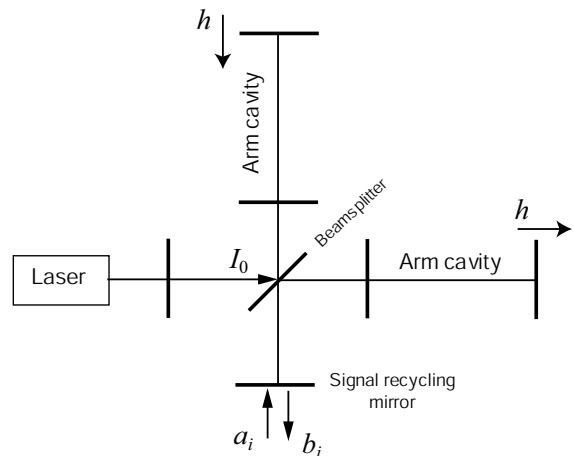


FIG. 1: Schematic view of an RSE interferometer. a_i and b_i are the vacuum input and signal output. h is the signal field due to the differential motion of the arm lengths induced by a gravitational wave strain. I_0 is the input laser power at the beamsplitter, enhanced by the power recycling cavity.

of Ref. [8] is the optical gain,

$$\left| \frac{b_\zeta}{h} \right| = \sqrt{2\kappa\tau} \cdot \left| \frac{D_1 \sin \zeta + D_2 \cos \zeta}{M \cdot h_{\text{SQL}}} \right|, \quad (1)$$

where b_ζ is the output field with readout phase ζ . M and D_i are defined as:

$$M = 1 + \rho^2 e^{4i\beta} - 2\rho e^{2i\beta} \left(\cos 2\phi + \frac{\kappa}{2} \sin 2\phi \right), \quad (2)$$

$$D_1 = -(1 + \rho e^{2i\beta}) \sin \phi, \quad D_2 = -(-1 - \rho e^{2i\beta}) \cos \phi. \quad (3)$$

In these equations, $2\beta = 2\text{atan}\Omega/\gamma$ is the net phase gained by the laser light due to a sinusoidal GW with angular frequency Ω in the arm cavity, $\gamma = Tc/4L$ is the half bandwidth of the arm cavity, T is the power transmissivity of the arm cavity input mirrors, L is the length of the arm cavity, τ is the amplitude transmissivity and ρ is the amplitude reflectivity of the SR mirror, and ϕ is the detuning of the signal cavity from carrier resonance. κ is an effective coupling constant which relates the mirror motion to the output signal,

$$\kappa = \frac{2(I_0/I_{\text{SQL}})\gamma^4}{\Omega^2(\gamma^2 + \Omega^2)}, \quad (4)$$

where I_0 is the input light power at the beamsplitter enhanced by the PR gain. I_{SQL} is the light power needed by a conventional interferometer (with no signal cavity) to reach the SQL at sideband frequency $\Omega = \gamma$:

$$I_{\text{SQL}} = \frac{mL^2\gamma^4}{4\omega_0}, \quad (5)$$

where m is the mass of each arm cavity mirror, and ω_0 is the carrier angular frequency. h_{SQL} in Eq.(1) is the SQL

for gravitational wave strain measurement, given by

$$h_{\text{SQL}} = \sqrt{\frac{8\hbar}{m\Omega^2 L^2}}. \quad (6)$$

As mentioned above, the optical gain described by Eq. (1) exhibits a detuned RSE optical resonance peak and an optical spring peak. The shape of the optical gain as a function of frequency is somewhat different than the (inverse of the) shape of the quantum-limited sensitivity curve; owing to the assumption that $h \gg a_i$, quantum noise correlations will not be evident when the optical gain is measured.

Several small-scale (“table-top”) experiments have been used to study optical configurations similar to that planned for Advanced LIGO, and developed prototypes for the control topology required to operate them[16–19]. The results of these experiments formed the basis for the Advanced LIGO design [12]. Subsequently, Somiya *et al.* [20] reported a measurement of the optical spring using low input power (no power recycling) and light masses. Although they measured the slope of the optical spring effect, the resonant peaks associated with the optical spring and with the detuned RSE were not seen clearly. Recently Corbitt *et al.* [21] reported the observation of an optical spring in a detuned single Fabry-Perot cavity. They also discussed the parametric instability in high power stored cavities. **LIGO folks: please point us to other relevant references involving experiments in the AdLIGO-like configuration, or experiment or theory with optical springs.**

The Caltech 40 meter (40m) prototype interferometer was originally developed as a test bed for the initial LIGO optical configuration and control system, and currently it is used as a test bed for Advanced LIGO [22, 23]. The optical configuration of the 40m (Fig. 2) is chosen to be similar to the optical configuration envisioned for Advanced LIGO. The arm cavities were chosen to have the same finesse as Advanced LIGO (around 1200); the input test mass mirrors (ITMs) have power transmission of 0.5%. The power recycling gain is designed to be 15, and a signal recycling cavity detuning is chosen to increase the detector bandwidth.

The light source is a 10 W continuous Nd:YAG laser which has a frequency stabilization servo, a pre-mode cleaner, and an intensity stabilization servo. Phase modulated RF sidebands are placed on the input beam at 33.2 MHz and 166 MHz using electro-optic modulators in a Mach-Zehnder interferometer. The light is attenuated, and 1 W is injected into a 13 meter mode cleaner (MC) which consists of 3 suspended mirrors forming a triangular cavity with 13 m half-length. This mode cleaner serves to further stabilize the laser frequency, while transmitting the carrier light and both pairs of RF sidebands. The beam passes through a Faraday isolator, reflective mode-matching telescope, and PZT-actuated mirrors which steer the beam into the main interferometer.

All ten core optics (three for the mode cleaner and seven for the main interferometer) are suspended as single

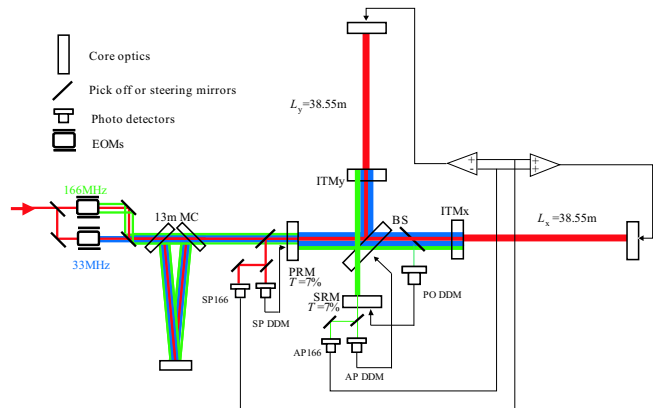


FIG. 2: Schematic diagram of the experimental setup of the 40m interferometer. The optical configuration, a Michelson interferometer with Fabry-Perot arms, power recycling, and detuned RSE, is similar to Advanced LIGO configuration.

pendula; they behave like free masses above the pendulum resonant frequency (around 0.8 Hz) and thus respond easily to the optical spring (the optomechanical rigidity is much larger than the mechanical rigidity of the mirror mount). The suspended optics are placed on passive seismic isolation stacks, within a single vacuum volume. (In Advanced LIGO, multiple pendulum suspensions and active seismic isolation systems will be used).

The main interferometer has five degrees of freedom that require length control: the common and differential modes of the two Fabry-Perot arm cavities, the Michelson fringe, the PR cavity and the detuned SR cavity. Output beams are monitored with RF photodiodes, and length sensing signals are derived from demodulations at 33.2, 133, 166 and 199 MHz. Length control servos are implemented in a digital system to allow dynamical re-configuration of the control topology and the signal filtering during and after lock acquisition. The typical control bandwidth of these servos is 300 Hz.

Because of the complexity of the optical configuration and the coupling of all the RF sidebands in the detuned signal cavity, lock acquisition and control of the interferometer is far more challenging than in Initial LIGO. Full lock acquisition and control in the desired configuration was first achieved in November 2005, through a process that will be described in a later publication. The buildup of carrier and RF sideband fields in the interferometer were then observed to be qualitatively as expected. Arm cavity losses were somewhat higher than expected, however. The achieved power recycling gain is about 5 and the arm cavity finesse is about 1200. The total power inside each arm is about 1.9 kW.

In order to measure the optical response of the interferometer to differential arm length changes (such as would arise in the presence of a gravitational wave), external sinusoidal forces are applied to the suspended optics at the ends of the arms via magnetic actuation, through the servo loop controlling that length degree of freedom. The

error signal for that servo loop is extracted at the asymmetric port of the detector, after transmission through the signal cavity. The light is detected at the signal port, demodulated at 166 MHz, whitened, digitized, filtered, and then fed back to the differential displacement of the two suspended optics at the ends of the arm cavities.

The optical gain of the differential mode of the arms is measured as the spectral transfer function from the in-loop feedback signal to the error signal. This transfer function includes, in addition to the desired optical response, the actuator pendulum transfer function which has a f^{-2} behavior above the pendulum resonant frequency of 0.8 Hz. There are also known whitening and anti-aliasing filters, and time and phase delays associated with the conversion from analog to digital and from digital to analog. The delays are measured with another simpler optical configuration consisting of a single Fabry-Perot arm cavity, and compensated.

Fig 3 shows the measured optical gain (solid line) and the prediction from Eq. (1) (dashed line). In Fig 3, the peak at 41 Hz is due to the optical spring resonance and the peak at approximately 3600 Hz is due to the optical resonance of the signal sideband in the RSE signal cavity. Table I shows the parameters which are used to calculate the model of the optical gain for the differential mode of the arms.

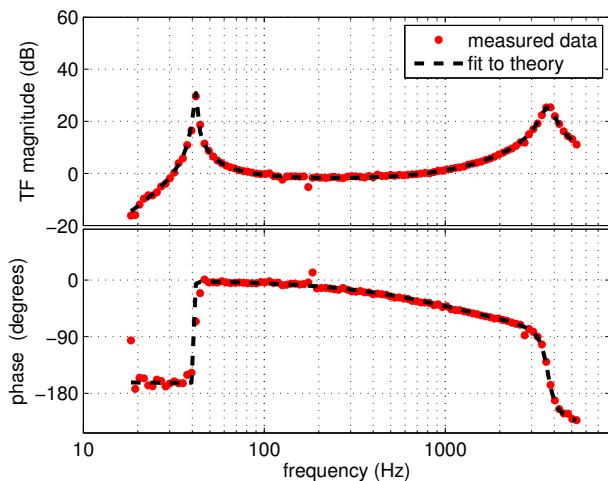


FIG. 3: The magnitude (top) and phase (bottom) response of the 40m interferometer to a swept-sine excitation of the arm length differential mode. The solid line shows the measured response, while the dashed line is the prediction. The peak at 41 Hz is due to the optical spring resonance and the peak at around 3600 Hz is due to the optical resonance of the signal sideband in the RSE signal cavity.

Degeneracies of parameters which determine the optical response of the interferometer were investigated in [25]. Of particular note are equations (13) of [25], which shows the relation between the ITM transmittance, the signal mirror reflectivity, and the signal cavity detuning phase in determining the free RSE optical resonant fre-

TABLE I: 40m parameters.

Quantity	Symbol & value
Power at beam splitter	$I_0 = 4.2 \text{ W}$
Laser angular frequency	$\omega_0 = 1.8 \times 10^{15} \text{ sec}^{-1}$
End-mirror mass	$m = 1.276 \text{ kg}$
SR mirror transmissivity	$\tau = \sqrt{0.07}$ (amplitude)
Laser power to reach SQL	$I_{\text{SQL}} = 2.3 \text{ kW}$
Arm cavity half bandwidth	$\gamma = Tc/4L = 2\pi \times 1550 \text{ sec}^{-1}$
Arm cavity length	$L = 38.55 \text{ m}$
Input test mass transmissivity	$T = 0.005$ (power)
SR cavity detuning	$\phi = \pi/2 - 0.39 \text{ rad}$
Homodyne phase	$\zeta = 0.22 \text{ rad}$

quency and equation (49) which relates these quantities, along with the circulating power, to the ponderomotive rigidity (the optical spring peak). In producing Fig 3, we assume the ITM and signal mirror transmittance to be at their design values, and vary the detune phase, signal cavity losses, circulating laser power, and measurement quadrature to fit the theoretical curve to the measured data. The values thus obtained are consistent with our expectations based on the interferometer design, and on other measurements made with the interferometer.

The quadrature ζ in Eq. (1) can be chosen by changing the RF demodulation phase of the 166 MHz local oscillator because the upper +166 MHz sideband is designed to be resonant in the combined signal and power cavities, while the lower -166 MHz sideband is not resonant there. This unbalanced RF sideband at the detection port makes it possible to choose the quadrature ζ [24]. In this measurement, ζ is determined to be 0.22 rad by fitting the measured response in Fig 3.

The Michelson asymmetry combined with the detuned signal cavity causes the control RF sidebands to be imbalanced in the interferometer. This leads to demodulation phase dependent offsets in the error signals derived from these sidebands. These offsets are largely indistinguishable from actual length deviations, and are strongly mixed among the short degrees of freedom. A lack of secondary diagnostics to precisely determine the offset in the signal cavity length sensing means that the signal cavity detuning was not known, operationally, to a precision greater than a few percent. Thus, we treat the detuning here as a partially free parameter, and the exact detuning used to produce the theoretical curve in Fig 3 was determined through a fit to the data.

The form of the measured optical gain is in good agreement with the prediction, thus building confidence in both the theory and practical implementation of the RSE interferometer's response to differential displacement. In particular, the theoretical prediction for the quantum-limited sensitivity achievable by Advanced LIGO is supported. However, we do not report on the measured noise spectrum or its comparison with prediction in this paper. Other noise sources make it impossible at present to achieve quantum-limited sensitivity in the frequency

range of interest for GW detection with the 40m interferometer.

Note that the 180° phase advance in the optical spring resonance results in an instability [9], but in practice it was not a problem to acquire operational lock for this experiment because the control bandwidth was about 300 Hz, well above the optical spring resonance at 41 Hz. The servo unity gain frequency in this measurement lies between the optical spring resonance and the RSE optical resonance, in a region where the phase of the optical gain is fairly flat. This will probably not be the case in Ad-

vanced LIGO and other next-generation GW detectors, and the stability of the control servos must be carefully considered in the design.

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