Physics 12b Midterm Exam 2/5/15

- This exam should be completed in **five hours**, working by yourself. Breaks are permitted.
- In your studies, do not use problem/solution books, or look at problems or exams from previous years of Phys 2 or 12.
- Please use a blue book, and put your name on it.
- The exam counts for \(\sim 15\%\) of the quarter grade. To get partial credit, show as much work as you can!
- You may use Griffiths, your class notes (or copies of those on reserve), your homework assignments (and/or the solutions), a calculator, and math tables. No other textbooks or problems/solutions should be used.
- Your work is due in the Ph12 IN box in east Bridge, by **10:30 AM, Thursday, 2/12/15**. **No class on 2/12/15: see you at the Student Faculty Conference.**
- **Reading** for next couple of weeks: Griffiths sections 2.5, 2.6, chapter 3, sections 4.1, 4.2.
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Problems: The exam consists of four problems. The point values for the individual parts of each problem are indicated in brackets [ ].

Problem 1. This problem consists of three short, unrelated questions. You should be able to answer them using your physical intuition and a minimum of algebra.

[3] a) A negative muon ($\mu^-$) has a mass $m_{\mu}c^2 = 106$ MeV. It can be bound to a proton by the Coulomb force to form a system analogous to the hydrogen atom. Use the Bohr model of the atom to estimate the wavelength (in Å) of the photon emitted when the $n = 2$ state of the muonic atom decays to the $n = 1$ ground state.

[3] b) The hydrogen molecule (H$_2$) has a vibrational frequency of $\bar{\hbar}\omega = 0.52$ eV and a dissociation energy of 4.48 eV. (This latter is the binding energy of the molecular ground state relative to two separated hydrogen atoms). In a molecule containing tritium (HT), one of the nuclear masses is three times larger, but the potential governing the internuclear separation is the same as that of H$_2$. What is the vibrational frequency and dissociation energy of the HT molecule?

[3] c) An harmonic oscillator is initially in the state $\psi(x,t=0) = \frac{1}{\sqrt{2}}[\phi_0(x) - i\phi_1(x)]$, where $\phi_n(x)$ is the n’th normalized stationary state. What is $\langle \hat{p} \rangle$ for all $t > 0$?

Problem 2. A particle of mass $m$ moves in a one-dimensional “half-square well” potential:

$$V(x) = \begin{cases} +\infty & x < 0 \\ -V_0 & 0 < x < a \\ 0 & x > a \end{cases}$$

with $V_0 > 0$.

[4] a) Give a transcendental equation whose roots determine the energies of the bound states.

[3] b) What are the conditions on $V_0$ and $a$ such that there is only one bound state?

[3] c) Assume that a neutron and a proton interact by a potential of this type and that $a = 2$ fm. It is known that there is only one bound state (the deuteron) and that this state is very weakly bound; ie, its binding energy is nearly negligible in comparison with the depth of the well. Use these facts to estimate the depth, $V_0$ (in MeV), of the neutron-proton interaction.
Problem 3. Angular momentum in 2D. For a particle in two dimensions \((x, y)\), the angular momentum operator is \(\hat{L} = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x\), where \(\hat{p}_x\) and \(\hat{p}_y\) are the \(x\) and \(y\) components of the momentum.

2] a) Show that \(\hat{L}\) is hermitian.

3] b) Evaluate the commutators \([\hat{x}, \hat{L}]\); \([\hat{y}, \hat{L}]\); \([\hat{x}^2 + \hat{y}^2, \hat{L}^2]\).

3] c) Suppose that the Hamiltonian for a particle of mass \(m\) moving in two dimensions \((x, y)\) is \(\hat{H} = \omega \hat{L}\), where \(\omega\) is a positive constant (this would be the case for a charged particle in a magnetic field). Use Ehrenfest’s theorem to derive the equations by which \(\langle \hat{x} \rangle\) and \(\langle \hat{y} \rangle\) evolve. What trajectory does the centroid of the wave packet follow?

3] d) If the Hamiltonian is given, instead, by \(\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + V(x, y)\) where \(V(x, y)\) is a time-independent potential, evaluate \(\frac{d}{dt} \langle \hat{L} \rangle\) and show how it is related to the classical torque.

Problem 4. Minimum energy of the harmonic oscillator. The energy of a linear harmonic oscillator is \(E = \frac{p^2}{2m} + Cx^2/2\).

2] a) Show, using the minimum uncertainty relation (equality rather than inequality), that the energy can be written

\[ E = \frac{\hbar^2}{8m} < x^2 > + \frac{C < x^2 >}{2} \]

3] b) Then show that the minimum energy of the oscillator is \(\hbar\omega/2\) where \(\omega\) is the oscillator angular frequency \(\omega = \sqrt{C/m}\). This minimum energy, a direct consequence of the uncertainty principle, is known as the zero-point energy.

3] c) With this minimum energy, what are the limits in \(x\) of the classical motion of the oscillator? What are the values of the expectation values for the kinetic and potential energies, \(\langle p^2/2m \rangle\) and \(\langle Cx^2/2 \rangle\)? How do they compare with the virial theorem?