Physics 12b Assignment VII 2/26/15

Reading: Griffiths sections 2.5, 2.6.

Problems: Due in Ph12 IN box in 1st floor East Bridge, 10:30 AM, Thursday, 3/5/15. Note that some of these problems require solving long coupled linear equations. They should not be hard, but if you wish to use Mathematica, you are welcome to do so. Please make sure the results are simplified as much as possible, so that they look like similar cases in the book (so that you can gain insight into things like resonance phenomena), and attach your work. Also, if the problem asks you to sketch the potential or solutions, you are encouraged to use computer graphics (Mathematica, MATLAB, python, ...) to do so.

VII.1 Tunnelling through rectangular barrier.

(a) Griffiths problem 2.33.

(b) Show that the transmission coefficient for low energy scattering off of a thick (length L) barrier of height $V_0$ (so that $E/V_0 \ll 1$ and $\kappa L \gg 1$) can be approximated by

$$T \approx \frac{16}{\kappa^2} \frac{E}{V_0} \exp \left( -\frac{2L}{\hbar} \sqrt{\frac{2mV_0}{\bar{E}}} \right).$$

(c) Evaluate the transmission coefficient for an electron of total energy 2 eV incident upon a rectangular barrier of height 4 eV and thickness 1Å, using the “exact” expression, and using the approximate result from part (b). Repeat, for a barrier thickness of 100Å.

VII.2 Transmission resonances for potential well. In the Ramsauer-Townsend effect, the transmission of low energy electrons through a dilute gas of atoms is maximized at resonance; the gas of atoms becomes transparent at certain resonance energies. This was an early indication that electrons behave like waves.

(a) Consider transmission of a beam of particles across a finite square potential well of width 2a and depth $-V_0$ (with $V_0 > 0$). Write the transmission coefficient $T(\bar{E})$ in terms of the scaled energy $\bar{E} = E/V_0$ and the parameter $g^2 = 2m(2a)^2V_0/\hbar^2$, and nothing else.

(b) Write down the values of the scaled energy at the transmission peaks, $\bar{E}_n$ (with integral $n$), in terms of g. Do the same for the transmission minima.

(c) In the limit $g^2 \gg 1$, show that the minima of $T$ will fall on a curve which is well approximated by $T_{min} = 4\bar{E}$.

(d) Plot $T(\bar{E})$ for $g^2 = 10^5$, with the above curve superimposed, showing the first 4 or 5 resonances.

(e) Zoom in on one of the peaks, $\bar{E}_n$, with a Taylor expansion. Show that the transmission in the close vicinity to one of those peaks can be written as a Lorentzian:

$$T(E) = \frac{\Gamma^2/4}{(E - E_n)^2 + \Gamma^2/4}$$

(here, $E$ and $E_n$ are the unscaled energy and energy of peak transmission, respectively). Write the expression for $\Gamma$ in terms of $E_n$ and $g$, and also in terms of the original parameters of the problem.
(f) Show that $\Gamma$ is related to the time it would take for a classical particle to cross the well.

(g) Plot $T(E)$ near resonance.

**VII.3 The S-matrix in 1 dimension.** Griffiths problem 2.52.

**VII.4 Simple derivation of WKB.** A particle of total energy $E$ impinges upon a potential barrier of arbitrary shape, entering at $x = x_1$ and exiting at $x = x_2$.

(a) Model the barrier as a succession of square barriers, by dividing the interval $[x_1, x_2]$ into many smaller sub-intervals of length $\Delta x_i$. In this way, calculate an approximate expression for the transmission coefficient $T$ for the whole barrier, knowing that $T_i \approx \exp\left[-\frac{2}{\hbar} \sqrt{2m(V(x_i) - E)\Delta x_i}\right]$ for the $i$th rectangular barrier.

(b) In cold emission, electrons are drawn from a metal (at room temperature) by an externally supported electric field. The potential well that the metal presents to the free electrons before the electric field is turned on is depicted in the figure below. After application of the constant electric field $E$, the potential at the surface slopes down as shown in the figure, thereby allowing electrons in the "Fermi sea" to tunnel through the potential barrier. If the surface of the metal is taken as the $x = 0$ plane, the new potential outside the surface is

$$V(x) = \Phi + E_F - eEx,$$

where $E_F$ is the Fermi level of the electrons in the metal (discussed in Ph 12c), and $\Phi$ is the work function of the metal. Using the results of part (a), calculate the approximate transmission coefficient for cold emission (the *Fowler-Nordheim* equation).