

ABOUT THE DETECTION OF GRAVITATIONAL WAVE BURSTS

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Several filtering methods for the detection of gravitational wave bursts in interferometric detectors are presented. These are simple and fast methods which can act as online triggers. All methods are compared to matched filtering with the help of a *figure of merit* based on the detection of supernovae signals simulated by Zwerger and Müller.

1. Introduction

Supernovae have been historically the first envisaged sources of gravitational waves (GW). Although binary inspirals or even periodic GW emitters like pulsars seem to be nowadays more promising sources, impulsive sources of GW such as supernovae should also be considered in the data analysis design of interferometric detectors currently under construction (LIGO, VIRGO). Impulsive GW sources are typically collapses of massive stars, leading to the birth of a neutron star (type II supernova)^{1,2,3} or of a black hole⁴; mergers of compact binaries can also be considered as impulsive sources⁵.

The problem with such sources is that the emitted waveforms are very poorly predicted, unlike the binary inspirals. As a consequence, this forbids the use of matched filtering for the detection of GW bursts. The filtering of such bursts should therefore be as general and robust as possible and with minimal *a priori* assumptions on the waveforms. A drawback is of course that such filters will be sensitive to non-stationary noise as well as to GW bursts; spurious events, e.g. generated by transient noise, should be eliminated afterwards when working in coincidence with other detectors. But, on the other hand, burst filters could help to identify and understand these noise sources, which would be useful especially during the commissioning phase of the detector.

All the filters presented here are dedicated to GW bursts detection and are compared by studying their performance to detect a reference sample of GW burst signals, numerically computed by Zwerger and Müller (ZM).²

Throughout the following, we assume that the detector noise is white, stationary and Gaussian with zero mean. For numerical estimates, we chose the flat (ampli-

tude) spectral density to be $h_n \simeq 4 \times 10^{-23}/\sqrt{\text{Hz}}$ and the sampling frequency $f_s \simeq 20 \text{ kHz}$, so the standard deviation of the noise is $\sigma_n = h_n \sqrt{f_s/2} \sim 4 \times 10^{-21}$. The value chosen for h_n corresponds approximately to the minimum of the sensitivity curve of the VIRGO detector ⁹; around this minimum, the sensitivity is rather flat, in the range [200 Hz,1kHz], which is precisely the range of interest for the gravitational wave bursts we are interested in. This validates then our assumption of a white noise ; otherwise, we can always assume that the detector output has been first whitened by a suitable filter ¹⁰.

2. General filters

2.1. *Filters based on the autocorrelation*

The noise being whitened, its autocorrelation is ideally a Dirac function and in practice vanishes outside of zero. The autocorrelation of the data $x(t)$

$$A_x(\tau) = \int x(t)x(t+\tau)dt \quad (1)$$

should then reveal the presence of some signal (which is surely correlated). The information contained in the autocorrelation function can be extracted in different ways. We have studied two of them and built so two non-linear filters. The first one computes the maximum of $A_x(\tau)$ and has already been described in ⁶. In the following, we will refer to this filter as the Norm Filter (NF). A similar approach has been developed independently by Flanagan and Hughes in the context of the detection of binary black hole mergers ¹¹.

Another possibility is to look at the norm of the autocorrelation function (NA Filter) :

$$\|A\| = \sqrt{\frac{1}{N} \sum_{k=2}^N A(k)^2}, \quad (2)$$

where $A(k)$ denotes the discrete autocorrelation of N data x_i . The sum is here initiated at the second bin according to the fact that the noise (uncorrelated) contributes essentially to the first bin. Note that the only parameter for these two filters is the window size N . The behavior of the NA filter with noise only is not known analytically and its characteristics (mean and standard deviation) have to be found numerically (adding some complexity to this filtering method).

2.2. *The Bin Counting method*

This filter (BC) computes the number of bins in a window of size N whose value exceeds some threshold $s \times \sigma_n$. The threshold s is chosen by maximizing the signal to noise ratio (SNR) when detecting the signals of the ZM catalogue (for more details, see ⁶).

2.3. *Linear Fit Filters*

This filter fits the data to a straight line in a window of size N . If the data are pure white noise with zero mean the slope and the offset of the fitted line are zero on average, so this can well discriminate between the two cases : only noise or noise+signal. The slope and the offset of the fit can easily be computed as a function of time and of the x_i (see ⁷).

In fact, the slope and the offset are two *correlated* random variables. By computing their Covariance Matrix, one reduces them to two *uncorrelated* normal random variables, X_+ and X_- , with

$$X_{\pm} = \left(\frac{X \pm Y}{\sqrt{2(1 \pm \alpha)}} \right), \text{ with } \alpha = -\sqrt{\frac{3}{2} \left(\frac{N+1}{2N+1} \right)} = COV(X, Y) \quad (3)$$

where $X = \text{SNR}(\text{slope}) = |\text{slope}|/\sigma_{\text{slope}}$, $Y = \text{SNR}(\text{offset})$. Finally, the sum $X_+^2 + X_-^2$ is a χ^2 like random variable and gives us a new filter, ALF (Advanced Linearfit Filter).

2.4. The Peak Correlator

Filtering by correlating the data with peak (or pulse) templates is justified by the fact that simulated supernovae GW signals exhibit one (or more) peaks. The pulse templates have been built from truncated Gaussian functions. The method and results are explained in ⁶.

3. Performance and efficiency of the filters

3.1. Definition of a false alarm rate

We *arbitrarily* set the false alarm rate for each of the filters to be 10^{-6} (72 false alarms per hour for a sampling frequency $f_s = 20$ kHz). This high rate is required because the signals we look for are very weak. False alarms will be discarded later when working in coincidence.

3.2. The Zwerger and Müller Catalogue

The catalogue of Zwerger and Müller ⁸ contains 78 gravitational-wave signals. Each of them corresponds to a particular set of parameters (e.g initial distribution of angular momentum). All the signals are computed for a source located at 10 Mpc. We can then re-scale the waveforms in order to locate the source at any distance d .

Since the signal waveforms are here known, we can explicitly derive the optimal SNR provided by the Wiener filter matched to each of them, and then compute the maximal distance of detection. We will then be able to build a benchmark for the different filters by comparing their results (detection distances) to the results of the Wiener filter (we consider here optimally polarized GW's, along the interferometer arms).

The mean distance obtained for the Wiener Filter, averaged over all the signals, is about $\bar{d}_{\text{opt}} \simeq 25.4 \text{ kpc}$, which is of the order of the diameter of the Milky Way.

3.3. Estimating a filter power

The optimal (Wiener) filtering allows to detect the i^{th} signal in the Catalogue emitted by a source located up to a distance $d_i^{(W)}$. Similarly, a filter \mathcal{F} is able to detect the same signal up to a distance $d_i^{(\mathcal{F})}$; of course $d_i^{(\mathcal{F})}$ is averaged over many noise realizations (about 1000) in a Monte Carlo simulation. The detection performance of the filter \mathcal{F} for this signal is simply defined as the distance of detection relative to the optimal distance of detection : $d_i^{(\mathcal{F})}/d_i^{(W)}$. The global performance of \mathcal{F} is then estimated as the detection performance averaged over all the waveforms of the catalogue:

$$\rho = \frac{1}{78} \sum_{i=1}^{78} \frac{d_i^{(\mathcal{F})}}{d_i^{(W)}}. \quad (4)$$

For a given filter, and a given source located at a distance d , one can also evaluate a *detection efficiency* ϵ , which is the number of detections n over the total number of noise realisations \mathcal{N} . This efficiency (averaged over all the signals of the catalogue) will characterize the *practical* behaviour of the filter.

3.4. Comparison of the filtering methods : Performance

The results for the different filters are reported in Table 1. We also give the average distance of detection $\bar{d} = \frac{1}{78} \sum_{i=1}^{78} d_i^{(\mathcal{F})}$ for all the filters.

Table 1: Performance of the different filters. L means linear filter and NL means non-linear filter.

Filter	Optimal	NF	NA	BC	PC	X_+	X_-	ALF
d (kpc)	25.4	11.5	11.4	10.9	18.5	21.6	22.2	22.4
ρ	1	0.46	0.47	0.43	0.73	0.79	0.80	0.81
Linearity	L	NL	NL	NL	L	L	L	NL

The three first filters NF, NA and BC (all non-linear) have a performance slightly below one half, while the PC have a performance greater than 0.7. Both X_+ (or X_-) and ALF can reach a performance around 0.8. Note that ALF has been in fact implemented with a sampling of 20 different window sizes, sufficient to cover the variety of signals (with a non-significant loss of generality). If implemented with a single window size, as the other filters NF, NA and BC, its performance decreases down to 0.72.

3.5. Comparison of the filtering methods : Detection efficiency

For the Wiener filter, one can show that the mean efficiency is roughly 50% for signals located at a distance $d_i^{(W)}$. If $\bar{\rho}$ is the mean performance of a given filter,

one would expect a 50% detection efficiency at $d_i^{(\mathcal{F})} \simeq \bar{\rho} \times d_i^{(W)}$. In fact, such efficiency is reached for a smaller distance.

If one defines the *effective performance* ρ_{eff} as the ratio $d_i^{(\mathcal{F})}/d_i^{(W)}$ for which the detection efficiency is about 50%, then $\rho_{eff} \simeq 0.74$ for ALF and X_- and $\rho_{eff} \simeq 0.71$ for X_+ . This definition gives an idea of the efficiency one can reach in practice, and has to be taken into account when choosing between different online triggers.

4. Conclusion

We have discussed several filters to be used as triggers for detecting GW bursts in interferometric detectors. They are all sub-optimal but their performance is close to the one obtained with the Wiener Filter.

Concerning the detection of Zwerger-Müller-like signals, we note that none of the BC, NF and NA filters is efficient enough to cover the whole Galaxy in average (but their window sizes have not yet been optimized), contrary to ALF and PC (and optimal) filters. A few signals can be detected at distances beyond 50 kpc, the distance of the Large Magellanic Cloud (LMC). It is clear that this class of signals will be detected by the first generation interferometric detectors such as VIRGO only if the supernovae occur inside our Galaxy or in the very close neighbourhood.

Finally, all the filters studied here can be implemented on line without problem, due to use of FFT's (for the NA and the PC) or to simple recursive relations between filter outputs in successive windows (NF,BC or ALF).

Correlations and coincidences between those filters are under study in order to either reduce background (hence a quantifiable loss of signal) or lower detection thresholds (hence a gain of a few % in performance and efficiency).

References

1. S. Bonazzola and J.-A. Marck, *Astron. Astrophys.* **267**, 623 (1993).
2. T. Zwerger and E. Müller, *Astron. Astrophys.* **320**, 209 (1997).
3. M. Rampp, E. Müller and M. Ruffert, *Astron. Astrophys.* **332**, 969 (1998).
4. R.F. Stark and T. Piran, *Phys. Rev. Lett.* **55**, 891 (1985).
5. M. Ruffert and H.-Th. Janka, *Astron. Astrophys.* **338**, 535 (1998).
6. N. Arnaud, F. Cavalier, M. Davier and P. Hello, *Phys. Rev. D* **59**, 082002 (1999).
<http://www.lal.in2p3.fr/virgo/DAD/dadweb.html>
7. F. Cavalier, M. Davier, P. Hello and T. Pradier, "Triggers for the detection of impulsive sources of gravitational waves", internal VIRGO report VIR-NOT-LAL-1390-128 (1999)
8. <http://www.mpa-garching.mpg.de/~ewald/GRAV/grav.html>
9. The "official" VIRGO sensitivity curve is available at :
<http://www.pg.infn.it/virgo/presentation.htm>
10. M. Beccaria, E. Cuoco and G. Curci, "Adaptive Identification of VIRGO-like noise spectrum", internal VIRGO report VIR-NOT-PIS-1390-096 (1997).
11. E.E. Flanagan and S.A. Hughes, *Phys. Rev. D* **57**, 4535 (1998).
12. B.S. Sathyaprakash and S.V. Dhurandhar, *Phys. Rev. D* **44**, 3819 (1991).