Detecting an association between Gamma Ray and Gravitational Wave Bursts

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(December 10, 2001)

If γ-ray bursts (GRBs) are accompanied by gravitational wave bursts (GWBs) the correlated output of two gravitational wave detectors evaluated in the moments just prior to a GRB will differ from that evaluated at times not associated with a GRB. We can test for this difference independently of any model of the GWB signal waveform. If we invoke a model for the GRB source population and GWB radiation spectral density we can find a confidence interval or upper limit on the root-mean-square GWB signal amplitude in the detector waveband. To illustrate we adopt a simple, physically motivated model and estimate that initial LIGO detector observations coincident with 1000 GRBs could lead us to exclude, with 95% confidence, associated GWBs with $h_{\mathrm{RMS}} \gtrsim 1.7 \times 10^{-22}$. This result does not require the detector noise be Gaussian or that any inter-detector correlated noise be measured or measurable; it does not require advanced or *a priori* knowledge of the source waveform; and the limits obtained on the wave-strength improve with the number of observed GRBs.

Gamma Ray Bursts (GRBs), which are known to lie at cosmological distances, likely arise from shocks in a relativistic fireball that is triggered by rapid accretion on to a newly formed black hole [1]. The violent formation of a black hole is likely to produce a substantial gravitational wave burst (GWB); thus, we expect GRBs to be preceded by GWBs that are candidate sources for the new gravitational wave (GW) detectors now under construction [2,3].

Proposed GRB progenitors include coalescing or merging binary systems (*e.g.,* neutron star or black hole binaries, white dwarf - black hole mergers, Helium core - black hole mergers) and hypernovae or collapsars (*e.g.,* failed SNe Ib and single or binary Wolf-Rayet star collapse) [1]. In fact, statistical evidence points to at least three different subclasses of GRBs [4]; so, the actual progenitors may include these as well as other systems. Matched filtering (MF) — the method at the focus of most of the gravitational wave detection literature — requires detailed knowledge of the actual GWB waveform; consequently, it cannot be used to detect the distinct GWB associated with a GRB. Additionally, since GRBs occur at cosmological distances (*i.e.,* $z \gtrsim 1$), the signal-to-noise ratio (SNR) of any individual GWB burst will almost certainly be insufficient for a high confidence detection with these new detectors. Detection techniques other than MF that aim to detect distinct GWBs will perform even worse.

Here we suggest an alternative method for detecting a GWB/GRB *association.* If GWBs are associated with GRBs, the correlated output of two GW detectors will be different in the moments immediately preceding a GRB (*on-source*) than at other times, not associated with a GRB (*off-source*). A statistically significant difference between on- and off-source cross-correlations would support a GWB/GRB *association* and, by implication, represent a detection of gravitational waves by the detector pair. (While we focus on GRBs in this paper any plausible class of astronomical events can serve as a trigger.)

We can test for a difference between the on- and off-source cross-correlation populations using Student's t-test, *without specifying an* a priori *model for the signal waveform, source, or source population.* The observed t statistic can also be used to set a confidence interval (CI) or upper limit (UL) on the root-mean-square (RMS) amplitude of GWB signals associated with GRBs, where the average is over the source population. If we specify even a rudimentary GWB source model, the CI/UL constrains the model.

We restrict attention here to the two full-length LIGO detectors (denoted $\mathcal{D}_i$, $i = 1, 2$). These detectors are nearly identically oriented and lie ~ 3000 Km apart. We do not require that the detector noise statistics be Gaussian, although we do require here that they be approximately stationary. Without loss of generality we assume the noise has zero mean and denote its *one-sided* power spectral density (PSD) by $S_i(f)$. Noise cross-correlated between $\mathcal{D}_1$ and $\mathcal{D}_2$ is assumed only to be stationary (but not necessarily Gaussian) and weak compared to the intrinsic noise of both the detectors.

a. The on-source and off-source distributions. Suppose that a GWB, associated with a GRB, is incident from direction $\vec{n}$ on the GW detector $\mathcal{D}_i$ at time $t_a^{(i)}$. The *lag* δt, equal to $t_a^{(2)} - t_a^{(1)}$, depends only on $\vec{n}$, which we know from the GRB observation. The lag is also the same as the difference $t_\gamma^{(2)} - t_\gamma^{(1)}$, where $t_\gamma^{(i)}$ is the arrival time at detector $\mathcal{D}_i$ of the GRB.

Assuming that any associated GWB precedes the

GRB, focus attention on the output $x_i(t)$, for $0 \leq t_\gamma^{(i)} - t \leq T$, of detector $\mathcal{D}_i$. Choose T as long, but no longer, than necessary to insure that x_i includes the possible GWB signal. From the $x_i(t)$ we compute the weighted cross-correlation

$$\begin{aligned} X &:= \langle x_1, x_2 \rangle \\ &:= \iint_0^T dt\, dt'\, x_1(t_\gamma^{(1)} - t) Q(|t-t'|) x_2(t_\gamma^{(2)} - t'). \end{aligned} \quad (1)$$

The filter kernel Q is at our disposal: we discuss its choice in section c below.

The collection of X computed for each of N_{on} GRBs form a set $\mathcal{X}_{\text{on}}$ of *on-source* events. To complement that set, we also construct a set $\mathcal{X}_{\text{off}}$ of N_{off} *off-source events,* using data segments x_i corresponding to random sky directions and arrival times not associated with any GRB.

The sample sets $\mathcal{X}_{\text{off}}$ and $\mathcal{X}_{\text{on}}$ are drawn from populations whose distributions we denote p_{off} and p_{on}. For T much greater than the detector noise auto- and cross-correlation times, the central limit theorem [5] implies that p_{off} is normal,

$$p_{\text{off}}(X) \simeq N(X;\, \mu_{\text{off}}, \sigma), \quad (2a)$$

with mean and variance

$$\mu_{\text{off}} := \mathrm{E}[\langle n_1, n_2 \rangle], \quad (2b)$$

$$\sigma^2 := \mathrm{E}\left[(\langle n_1, n_2 \rangle - \mu_{\text{off}})^2 \right]. \quad (2c)$$

Here $n_i(t)$ denotes noise from detector i and $\mathrm{E}[\cdot]$ represents an ensemble average across the detector output. Note that μ_{off} is just the detector noise cross-correlation evaluated at the lag δt. (A weak assumption behind equation 2a is that the noise cross correlation does not vary significantly over the lag δt, which should be the case for terrestrial noise sources.)

Now suppose that GRBs are preceded by GWBs. Elements of $\mathcal{X}_{\text{on}}$ then take the form

$$X = \langle n_1, n_2 \rangle + \langle h_1, n_2 \rangle + \langle n_1, h_2 \rangle + \langle h_1, h_2 \rangle, \quad (3)$$

where $h_i(t)$ is detector i's response to the incident GWB. Define P_i by

$$P_i := 4 \int_0^\infty df\, |\widetilde{h}_i(f)|^2 / S_i(f). \quad (4)$$

If $\overline{P_i}$, the average of P_i over the source population, is much less than unity, then p_{on} is also a normal distribution with variance σ^2 and mean

$$\mu_{\text{on}} = \mu_{\text{off}} + \overline{s}, \qquad \text{where} \quad (5a)$$

$$s := \langle h_1, h_2 \rangle \quad (5b)$$

and $\overline{s}$ is, again, an average of s over the source population.

b. Student's t-test. Pose the null hypothesis

$$H_0:\ p_{\text{off}}(X) = p_{\text{on}}(X)\,. \quad (6)$$

Rejecting H_0 supports a GWB/GRB association. Since p_{on} and p_{off} are normal and differ, if at all, only in their means, we can test H_0 using Student's t-test [6].

The t statistic is defined from $\mathcal{X}_{\text{on}}$ and $\mathcal{X}_{\text{off}}$ by

$$t := \frac{\hat\mu_{\text{on}} - \hat\mu_{\text{off}}}{\Sigma} \sqrt{\frac{N_{\text{on}} N_{\text{off}}}{N_{\text{on}} + N_{\text{off}}}}, \quad (7a)$$

$$\Sigma^2 = \frac{\hat\sigma_{\text{on}}^2 (N_{\text{on}} - 1) + \hat\sigma_{\text{off}}^2 (N_{\text{off}} - 1)}{N_{\text{on}} + N_{\text{off}} - 2}, \quad (7b)$$

where $\hat\mu_{\text{on}}$ and $\hat\mu_{\text{off}}$ ($\hat\sigma_{\text{on}}^2$ and $\hat\sigma_{\text{off}}^2$) are the *sample* means (variances) of $\mathcal{X}_{\text{on}}$ and $\mathcal{X}_{\text{off}}$, respectively.

The expectation value of t, averaged over the source population and across the detector noise processes, is

$$\mu_t := E[t] = \frac{\overline{s}}{\sigma} \sqrt{\frac{N_{\text{on}} N_{\text{off}}}{N_{\text{on}} + N_{\text{off}}}}. \quad (8)$$

The relative orientation of the two LIGO detectors guarantees that $h_1(t)$ and $h_2(t)$ are very nearly identical. It follows that, for LIGO, $\overline{s}$ is non-negative; correspondingly, the expectation value $\mathrm{E}[t]$ of t is positive in presence of a GWB/GRB association and zero otherwise.

The actual value of t, given any observed sets $\mathcal{X}_{\text{on}}$ and $\mathcal{X}_{\text{off}}$, will vary from μ_t. The distribution of t is normal for large $N_{\text{on}} + N_{\text{off}}$ and is, more generally, tabulated in any standard statistics text [5]. Consequently, we can find a t_0 such that, when H_0 is true ($\mu_t = 0$) t is greater than t_0 in less than a fraction α (*e.g.,* 5%) of all observations. This is our test: if we observe t greater than t_0 we reject H_0 and conclude that we have found evidence of a GWB/GRB association with significance $1-\alpha$ (*e.g.,* 95%).

c. The filter kernel Q. If we knew the signal $h_i(t)$ corresponding to each GRB trigger we could construct a Q that maximizes s:

$$Q(\tau) = \int_{-\infty}^{\infty} df\, e^{2\pi i f \tau} \frac{\widetilde{h}_1(f) \widetilde{h}_2^*(f)}{S_1(|f|) S_2(|f|)}, \quad (9)$$

where $\widetilde{h}_i$ is the Fourier transform of h_i. For the LIGO detectors, the h_i are identical. Denoting their common functional form $h(t)$, the optimal Q depends only on the GWB signal spectral density $|\widetilde{h}(f)|^2$.

More generally, we can put any knowledge we have of the signal's spectral density shape into Q. For LIGO we can choose Q to match the signal model irrespective of the details of the possible GWB waveforms if $|\widetilde{h}(f)|^2$ is independent of signal parameters. This happens, for instance, in the case of the quadrupole formula waveform of an inspiraling binary. For GWBs associated with GRBs there is no reason to believe that $|\widetilde{h}(f)|^2$ will be known *a* *priori,* let alone that it have this special property. Lacking detailed knowledge, we recommend adopting Q given by equation 9 with $|\widetilde{h}(f)|^2$ assumed to be unity in the detector bandwidth.

d. Setting upper limits. Having specified Q we can test H_0 as described above to rule on the presence of a GWB/GRB association, *independently of any model for the GWB signal or its source.* Alternatively, we can use the observed t to establish a confidence interval (CI) or upper limit (UL) on μ_t, and hence $\overline{s}$, which is related to the GWB wave-strength (cf. eq. 8, 5b). If we invoke a model for the spectral density $|\widetilde{h}(f)|^2$ and spatial distribution of GWB/GRB sources, this becomes a physical constraint on the model.

To obtain a CI/UL on $\overline{s}$ given an observed t, we construct a *confidence belt* [5], of desired confidence, in the t–μ_t plane. We follow the construction of [7], which unifies the treatment of CIs and ULs. ([7, table X] tabulate UL/CIs appropriate to our case: the mean μ of a Gaussian variable x when $\mu \geq 0$ and x itself is the observation.) From observed sets $\mathcal{X}_{\text{on}}$ and $\mathcal{X}_{\text{off}}$ the corresponding t is computed. From t and the confidence belts the corresponding CI/UL on $\overline{s}$ can be read-off.

To interpret the CI/UL, imagine an ensemble of observations and corresponding pairs of sample sets $\mathcal{X}_{\text{on}}$ and $\mathcal{X}_{\text{off}}$. Corresponding to this ensemble of observations is an ensemble of values t and CI/ULs. If the confidence level is ϵ (*e.g.,* 95%), then a fraction ϵ of these CI/ULs will include the actual value of $\overline{s}$ (and a fraction $1-\epsilon$ will not). It is in this sense that the CI/UL corresponding to the observed t is said to bound $\overline{s}$ with confidence ϵ.

To measure the effectiveness of the proposed test we evaluate the UL *most likely* to be placed on $\overline{s}$ if H_0 is, in fact, true. When H_0 is true the most likely *observed* t is zero. Denoting the corresponding UL on μ_t as $\mu_{t,\max}$ the UL on $\overline{s}$ is

$$\frac{\overline{s}}{\sigma} \leq \mu_{t,\max}\sqrt{\frac{N_{\text{on}}+N_{\text{off}}}{N_{\text{on}}N_{\text{off}}}} \tag{10a}$$

$$= \begin{cases} \mu_{t,\max}\sqrt{2/N_\gamma} & (N_{\text{on}} = N_{\text{off}} = N_\gamma) \\ \mu_{t,\max}/\sqrt{N_{\text{on}}} & (N_{\text{off}} \gg N_{\text{on}}) \end{cases} \tag{10b}$$

Since the duty cycle of GRBs is low, the size of the off-source sample can be made much larger than the size of the on-source sample. Even if both sample sets are the same size, however, the limit obtained will be weaker by only a factor of $2^{1/2}$.

The upper limit $\mu_{t,\max}$ corresponding to an observed t of zero and different degrees of confidence is given in [7, table X]. For reference we note that $\mu_{t,\max}$ is 1.00 for 68.27%, 1.64 for 90%, 1.96 for 95%, and 2.58 for 99% confidence.

A derived CI/UL on $\overline{s}$ implies, within the context of a GWB/GRB source model, a CI/UL on the RMS GWB signal amplitude in the detector band, with the average over the source population. As an example, suppose that each GRB is accompanied by the formation of a several solar mass black hole and a corresponding millisecond timescale GWB in the source rest frame. Assume further that $|\widetilde{h}(f)|^2$ is approximately constant in the corresponding KHz bandwidth B_s. (This is consistent with numerical models of supernova core collapse [8,9] and with the formation or ring-down of all but the most rapidly rotating solar mass black holes [10].) At the detector, the signal power from a source at redshift z lies in the bandwidth B_s/z', where z' is equal to $1+z$.

For simplicity, assume that the detector noise PSDs $S_i(f)$ are identical and equal to a constant S_0 in the detector bandwidth B_d, which we take to be approximately 100 Hz about a central frequency of 150 Hz. Outside the detector band we set S_i equal to infinity. (This is a rough approximation to the actual shape of the noise PSD of LIGO [11].) Finally, note that B_s is much larger than B_d, so that B_s/z' completely overlaps B_d for some large range of z'.

With these assumptions,

$$s = \int_{-\infty}^{\infty} df\, |\widetilde{h}(f)|^2 \widetilde{Q}(f) = \frac{2A^2 B_d}{S_0^2} \qquad \text{and} \tag{11}$$

$$\sigma^2 = \frac{T}{4}\int_{-\infty}^{\infty} df\, S_1(|f|)S_2(|f|)|\widetilde{Q}(f)|^2 = \frac{TB_d}{2S_0^2}, \tag{12}$$

where A is defined by

$$\int_{-\infty}^{\infty} df\ |\widetilde{h}(f)|^2 = \frac{2A^2 B_s}{z'}. \tag{13}$$

From equations 11, 12 and 13 it follows that

$$\frac{\overline{s}}{\sigma} = \mathrm{E}\left[\frac{2\sqrt{2}A^2 B_d}{\sqrt{TB_d}S_0}\right] \simeq \frac{2\sqrt{2}\ \overline{A^2}\ B_d}{\sqrt{TB_d}S_0}, \tag{14}$$

where we have replaced A^2 by its mean over the source population (a good approximation when A is sharply peaked about its mean.) From equations 10 and 14 and assuming that H_0 is true we find

$$\overline{A^2} \leq A^2_{\max} = \frac{\mu_{t,\max}}{2\sqrt{2}}\left[\frac{TB_d}{N_{\text{on}}}\right]^{1/2}\frac{S_0}{B_d}, \tag{15}$$

with $N_{\text{off}} \gg N_{\text{on}}$ and $\mu_{t,\max}$ obtained from the confidence belt construction [7] with $t=0$.

We expect that different GWBs will have different waveforms and durations. Define the RMS signal power in the detector band by

$$h^2_{\text{RMS}} := \overline{\left[\frac{2}{\tau}\int_{f\in B_d} df\, |\tilde{h}(f)|^2\right]}, \tag{16}$$

where $h(t)$ is the GWB waveform, τ its duration in the detector band, and the average is over the source population. In our example — broadband bursts whose bandwidth includes the detector band — we can approximate

$1/\tau$ by the detector bandwidth B_d. Combining equations 16, 15 and 13 we find the UL on h_{RMS}:

$$h^2_{\mathrm{RMS}} \leq \left[1.7\times 10^{-22}\right]^2 \frac{\mu_{t,\max}}{1.96}\left(\frac{T}{0.5\,\mathrm{s}}\frac{1000}{N_{\mathrm{on}}}\right)^{1/2} \times \frac{S_0}{\left(3\times 10^{-23}\,\mathrm{Hz}^{-1/2}\right)^2}\left(\frac{B_d}{100\,\mathrm{Hz}}\right)^{3/2}. \quad (17)$$

The reference values of B_d and S_0 are characteristic of the initial LIGO detectors [11]. For T (cf. eq. 1) we assume GRBs are generated by internal shocks in the fireball; then, the GRB/GWB delay is approximately 0.1 sec in the source rest frame [12]. To accommodate GRBs at redshifts $z \leq 4$ we take $T \sim 0.5$ sec. Finally, $\mu_{t,\max}$ equal to 1.96 corresponds to a 95% confidence UL [7].

If, on the other hand, GRBs are generated when the fireball is incident on an external medium, then [13, eq. 3.6] with $n_1 = 1$, $\alpha = 1$, $E_{51} = 10$, and $\Gamma \gtrsim 100$ gives a source rest-frame delay $\lesssim 100$ sec, in which case T should be 500 s and the corresponding UL on h_{RMS} is 9.4×10^{-22}.

Two final notes are in order. To calculate the X (cf. eq. 1), which are at the heart of our analysis, we must know accurately the GRB source direction. Bright bursts in the BATSE3B catalog have positional accuracies of $\delta\theta \lesssim 1.5°$ [14]. The corresponding uncertainty in s is $\lesssim 5\%$, which does not affect significantly the UL on $\overline{s}$.

Finally, the proposed BATSE follow-on — SWIFT — is not an all-sky GRB detector. It will have greater sensitivity than BATSE, but observe only a fraction of the sky at any one time. If SWIFT pointing favors the sky normal to the LIGO detector plane, LIGO's sensitivity to GWBs from observed GRBs will be maximized, increasing the sensitivity of the test described here.

e. Conclusions. Gamma-ray bursts (GRBs), which are believed to be associated with the violent formation of a stellar mass black hole, may well be immediately preceded by a gravitational wave burst (GWB). If we compare the correlated output of two gravitational wave detectors immediately preceding a GRB to the correlation at other times, not associated with a GRB, then a statistically significant difference is evidence for a GRB/GWB association.

We can test for this difference — independent of any model of the GRB/GWB source or GWB waveform — using Student's t-test. Alternatively, we can set an upper limit (UL) or confidence interval (CI) on the RMS GWB amplitude in the detector waveband, averaged over the source population. This CI/UL constrains any GRB/GWB model we do invoke.

This analysis has several important advantages over matched filtering, which is the method at the focus of most of the gravitational wave detection literature. In particular, it becomes more sensitive as the number of observed GRBs increases, does not require any knowledge of the GWB waveforms, is insensitive to the presence of non-Gaussian detector noise, and does not require statistical independence of the detectors or knowledge of their correlated noise. It is thus a powerful addition to the growing arsenal of analysis techniques aimed at making gravitational wave detection an astronomical tool.

ACKNOWLEDGMENTS

We gratefully acknowledge the hospitality of the LIGO Laboratory at Caltech, where the work described here was begun. LSF is glad to acknowledge discussions with B. Barish, who drew attention to the subtleties of upper limit analyses. SDM acknowledges a fruitful discussion with E. S. Phinney regarding GRBs. This work was supported by National Science Foundation grants PHY93-08728, PHY95-03084 and PHY98-00111.

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