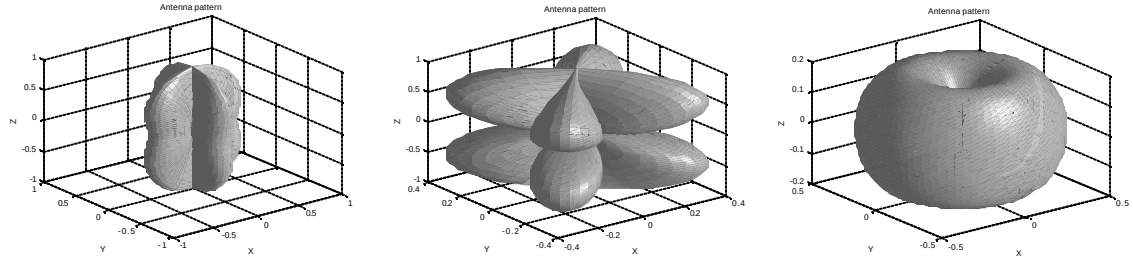
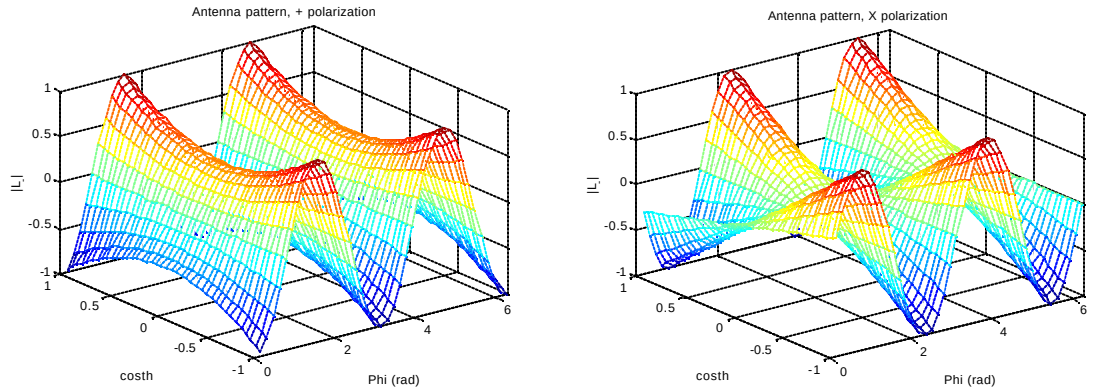
LIGO antenna pattern (without knowing any GR).

(a simple pedagogical note by AJW, 4/01/01)

ETM response to quadrupole wave must be of the form: $dLx = x^i T^{ij} x^j$
where x^i is displacement from ITM to ETM (take to be unit x, y vectors with length L_{arm})
and T^{ij} in the "transverse traceless" gauge must be built out of the vectors characterizing the GW: the direction of motion w^j, and the direction of the GW strain perpindicular to the direction (transverse) and each other, u^i and v^j.
It must be traceless and anti-symmetric under u <-> v, so it must be of the form:
$T^{ij} = u^i u^j - v^i v^j + a (u^i v^j - v^i u^j)$
where I don't know what a is, but it doesn't matter, since that term is zero when $x^i T^{ij} x^j$ is formed. This gives, for $x^i T^{ij} x^j / L^2$, in the (Lx-Ly)/2 combination:



left: psi=0 (h_+); middle: psi = $\pi/4$ (h_x); right: L_+ for psi=0.



left: psi=0 (h_+); right: psi = $\pi/4$ (h_x).
Generated with the following code:

```
% construct GW direction and x/y polarization vectors
hz = [-sinth*cos(phi) -sinth*sin(phi) -costh];
wx = [-hz(2) hz(1) 0]./sqrt(hz(1)^2+hz(2)^2);
wy = cross(hz,wx);
hx = cos(psi)*wx+sin(psi)*wy;
hy = cross(hz,hx);

% convert to Lx and Ly with the antenna pattern
Lx = (hx(1)^2-hy(1)^2);
Ly = (hx(2)^2-hy(2)^2);
Lm = 0.5.*(Lx-Ly);
Lp = 0.5.*(Lx+Ly);
```

```
cLc(I,J) = Lm;
```

These patterns agree perfectly with the usual formulae for the antenna pattern:

```
% analytic form of antenna pattern
FLm = 0.5*(1+costh^2)*cos(2*phi)*cos(2*psi)
      -costh*sin(2*phi)*sin(2*psi);
FLp = sinth^2*cos(2*psi);
```