A Simulation for LIGO I Seismic Noise

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Abstract

An important component of the residual motions of the LIGO mirror surfaces is due to propogation of ground seismic noise through the stacks and suspensions to the masses. I discuss the first version of a simulation intended to produce realistic Monte–Carlo data of the machine output due to this noise source.

1 Simulating LIGO I Seismic Noise

1.1 Aim of the Seismic Simulation

The aim of this work is to generate Monte Carlo simulated data for the displacement of the LIGO I optics in response to seismic noise in the ground. Initially, we concentrate on the frequency range 10 to 500Hz.

1.2 The LIGO I Vibration Isolation Stacks

Two different stack designs are employed, one in the so-called 'beam splitter chambers' (BSCs) and the other in the 'horizontal access modules' (HAMs). We concentrate on the BSC assembly as both the end test masses (ETMs) and intermediate test masses (ITMs) of LIGO I are located in BSC chambers.

The BSC vibration isolation stack has been described in detail elsewhere. A picture of an assembled BSC vibration isolation stack is shown in figure 1. We wish to simulate motion of the surfaces of the ITMs and ETMs in response to seismic noise in the ground.

The coupling of ground motion to motion of the reflecting surface of a suspended mirror is considered as two couplings in series. The first coupling is of ground motion to motion of the optical table on which the pendulum suspension is mounted. For now we use a state space model developed by Hytec as a physical model for this coupling. The second coupling is from the optical table through the steel wire suspension to the mass itself. For this coupling we employ a simple analysis of the transfer function of a pendulum with a resonant frequency of 0.74Hz and structural damping.

Figure 1: A BSC vibration isolation stack.

1.3 The BSC Stack Transfer Function

We concentrate on mirror motion parallel to the beam direction. There are 6 ground coordinates that couple noise into this mirror coordinate. They are translation parallel to the beam axis (the u axis), horizontal translation perpendicular to the beam axis (along the v axis), vertical translation (along the w axis), and the three rotation or 'tilt' coordinates about u, v, and w referred to as α, β and γ, respectively.

1.4 The Hytec Model of the BSC Stacks

Figure 2 shows the model parameterization of the mechanical elements of the stack. Element 1 is the floor. Element 2 is the model representation of the support tubes. Element 3 represents the support table on which the masses and springs shown in figure 1 are seated. The three mass elements in each of the four columns are elements 4 through 15 as shown in the figure. The optical table and the down tube are represented by element 16.

The Hytec state space model employs the 3 displacements and 3 rotations of each of the 16 mass elements, and the time rates of change of each of these coordinates, as the 192 basis degrees of freedom. The differential equations of the state space model relating these degrees of freedom are

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}u \\ y &= \bar{C}x + \bar{D}u, \end{aligned} \tag{1}$$

where x is a 180 element 'vector' of the degrees of freedom of all the elements excluding the floor, $\dot{x}$ are the rates of change of the elements of x, u are the 12 floor (element 1) degrees of freedom, the 'inputs' to the system, and y are the 12 optical table (element 16) degrees of freedom, the 'outputs'. The Hytec state space model provides the matrix elements of $\bar{A}$, a 180×180 matrix, $\bar{B}$, a 180×12 matrix, $\bar{C}$, a 12×180 matrix, and $\bar{D}$, a 12×12 matrix.

The Hytec state space model of the stack is used to generate the magnitude of the frequency response between various stack ground and table degrees of freedom Figure 3 shows the results.

1.5 Designing Filters to Replicate Measured Frequency Response

We concentrate on one significant coupling between the ground and the table, the $u_1 \rightarrow u_{16}$ coupling. Between 10 and 500Hz we notice that the frequency falloff is very well approximated by frequency response magnitude $|A(f)|$ given by

$$|A(f)| = \frac{1}{1 + \left[\frac{f}{10\mathrm{Hz}}\right]^8}. \tag{2}$$

We therefore divide the frequency response by this function to obtain a 'whitened' response. Figure 4 shows the whitened frequency response between 10 and 1kHz. The smooth

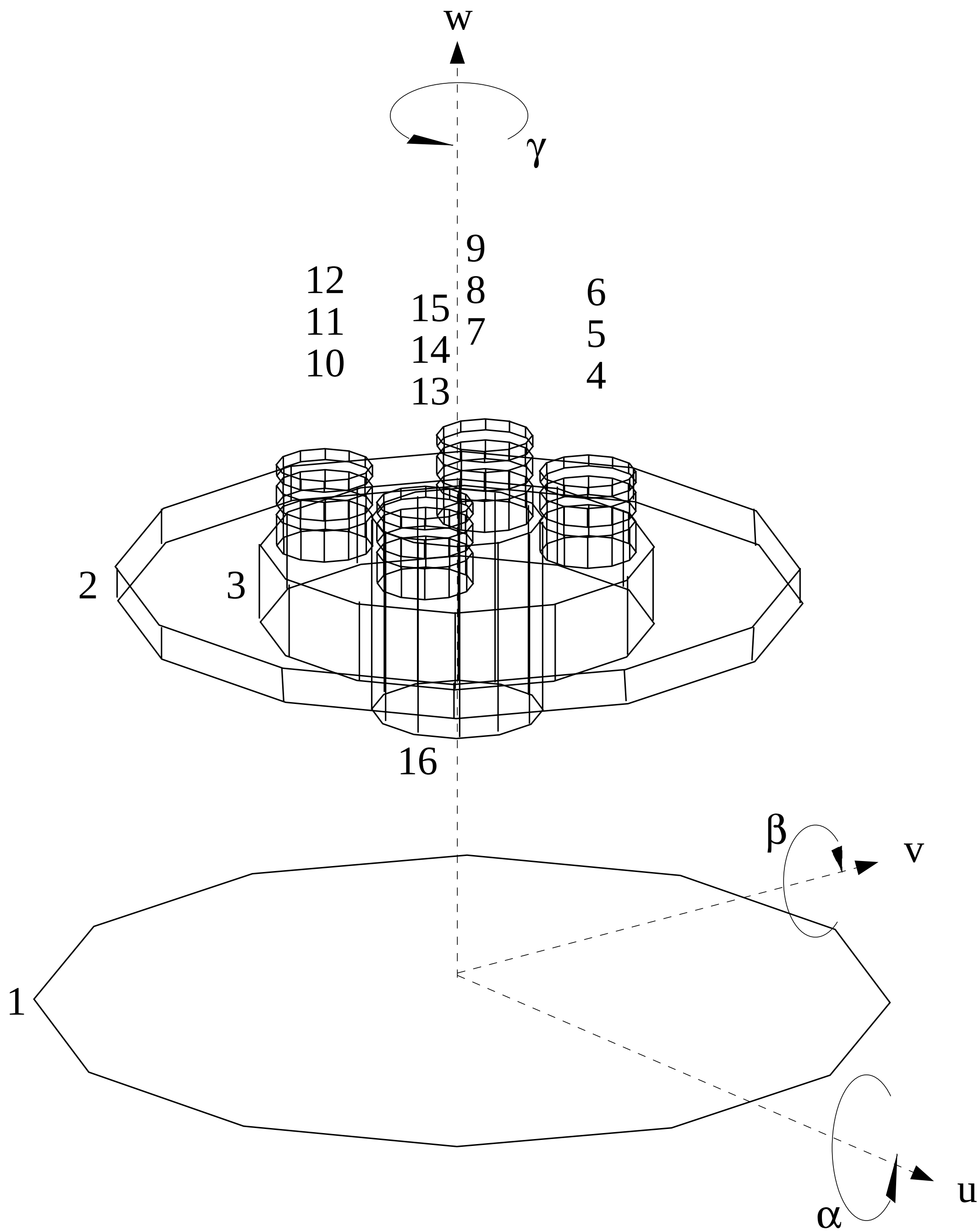

Figure 2: Hytec model representation of the elements of the stack.

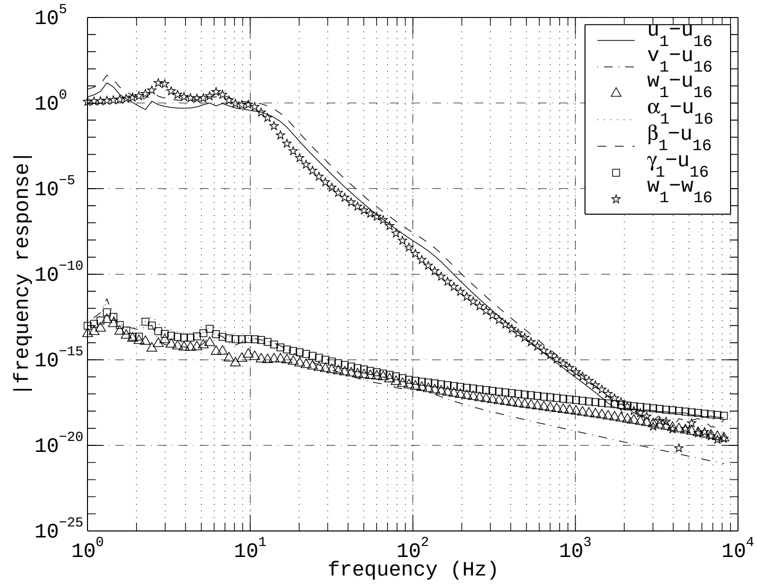

Figure 3: Frequency response of the u component of table motion to excitations in the 6 ground degrees of freedom. Also shown is the frequency response between vertical motion of the ground and vertical motion of the optical table.

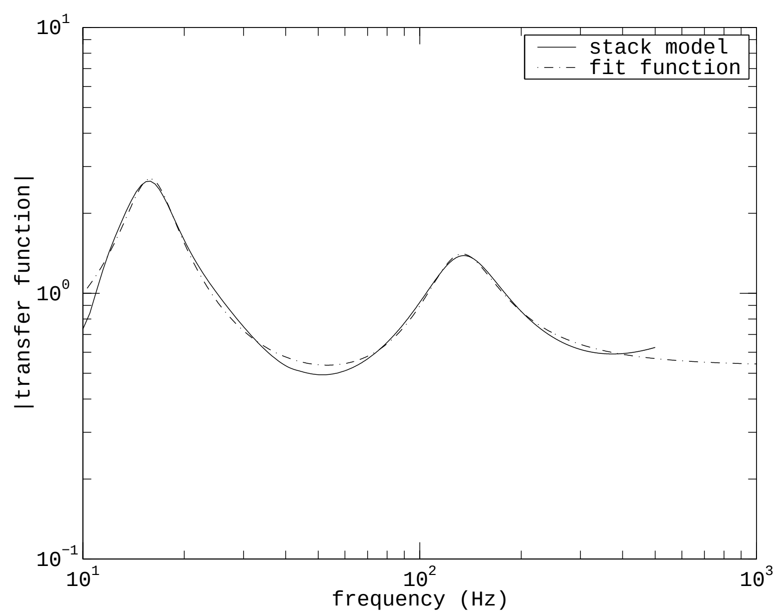

Figure 4: Whitened frequency response between 10 and 500Hz, and results of fit to four zeros and four poles

line is the response given by the model. The dotted line is a fit to the whitened response to be discussed below.

The task is to find a set of poles and zeros having a frequency response close to that given by the model. One approach is to take the magnitude and phase of the frequency response from the model, inverse fourier transform it, and use the resulting time series as a set of FIR filter taps. While this method yields the correct frequency response, the number of filter taps needed is given roughly by the ratio of the Nyquist frequency (8192Hz) to the minimum frequency important in the simulation (10Hz) . An 800 point convolution for every data point makes the Monte Carlo very inefficient.

The frequency response is very smooth, so we try instead to find an IIR filter capable of producing the frequency response shown in figure 4. We notice that the frequency response contains peakes, that the peaks are skewed to one side and that the peaks seem to sit on a pedestal that is roughly frequency independent. The last point causes us to look for a filter with an equal number of poles and zeros. The first point causes us to look for a filter that resembles two ordinary Breit-Wigner resonances, one for each peak. In terms of frequency response, our fit function $|T(f)|$ takes the form

$$|T(f)| = g\sqrt{\frac{\left((f^2 - {f_0^{z1}}^2)^2 + {\Gamma^{z1}}^2 f^2\right)\left((f^2 - {f_0^{z2}}^2)^2 + {\Gamma^{z2}}^2 f^2\right)}{\left((f^2 - {f_0^{p1}}^2)^2 + {\Gamma^{p1}}^2 f^2\right)\left((f^2 - {f_0^{p2}}^2)^2 + {\Gamma^{p2}}^2 f^2\right)}}, \tag{3}$$

where g is a gain, f_0^{pi} and Γ^{pi} are the approximate frequency and full width of the ith peak (2 of them in this case), and f_0^{zi} and Γ_0^{zi} are of similar magnitude, being properties of inverse Breit Wigner peaks introduced to make the number of poles and zeros equal, and to provide the skewness.

How do these quantities relate to the positions of poles and zeros in the s plane? To ensure that the differential equation representing the filter we construct has real solutions, we require that the poles and zeros are in complex conjugate pairs. To ensure the stability of the filter, we require that the real parts of the pole frequencies are negative, and the real parts of the zero frequencies are positive. Equation 3 has two roots in the numerator and two in the denominator per peak, so the s plane representation of our fit function should have a pair of poles and a pair of zeros for each peak, each pair being complex conjugates of each other. For each peak, the poles and zeros are therefore of the form

$$\begin{aligned} s_1^p &= -\alpha^2 + i\beta^2 \\ s_2^p &= -\alpha^2 - i\beta^2 \\ s_1^z &= +\gamma^2 + i\delta^2 \\ s_1^z &= +\gamma^2 - i\delta^2, \end{aligned} \tag{4}$$

where s_1^p and s_2^p are the two pole frequencies, and s_1^z and s_2^z are the two zero frequencies. In terms of these quantities, the transfer function in the s plane $T(s)$ factors into a product of terms of the form

$$T(s) = g\frac{(s - s_1^z)(s - s_2^z)}{(s - s_1^p)(s - s_2^p)}. \tag{5}$$

Substituting with $s = i2\pi f$ and finding the modulus of the result we obtain the modulus of the transfer function in frequency space

$$|T(f)| = g\sqrt{\frac{\left(f^2 - \frac{1}{(2\pi)^2}(\gamma^4 + \delta^4)\right)^2 + \frac{\gamma^4 f^2}{\pi^2}}{\left(f^2 - \frac{1}{(2\pi)^2}(\alpha^4 + \beta^4)\right)^2 + \frac{\alpha^4 f^2}{\pi^2}}} \tag{6}$$

Comparing equations 3 and 6 we see that

$$\begin{aligned} f^p &= \frac{1}{2\pi}\sqrt{\alpha^4 + \beta^4} \\ \Gamma^p &= \frac{\alpha^2}{\pi} \\ f^z &= \frac{1}{2\pi}\sqrt{\gamma^4 + \delta^4} \\ \Gamma^z &= \frac{\gamma^2}{\pi}. \end{aligned} \tag{7}$$

Our procedure is to guess the values of the parameters f^p, f^z, Γ^p, Γ^z for each peak, compute initial guesses for α, β, γ, δ using equations 7, and perform a nonlinear fit to the whitened transfer function from the Hytec model using equation 6 as a fit function. The zero and pole positions are calculated using equations 4. Because the fit parameters are squared to get the zero and pole positions, the stability of the resulting filter is garunteed as long as the fit converges.

The solid line in figure 4 is the result of the nonlinear fit with initial guesses $f^{p1} = f^{z1} =$ 18Hz, $\Gamma^{p1} = \Gamma^{z1} =$ 10Hz, $f^{p2} = f^{z2} =$ 140Hz, $\Gamma^{p2} = \Gamma^{z2} =$ 100Hz, and $g = 0.4$. After the fit the final values of these parameters were $f^{p1} =$ 15.8945Hz, $f^{z1} =$ 18.0299Hz, $\Gamma^{p1} =$ 5.1310Hz, $\Gamma^{z1} =$ 36.0553Hz $f^{p2} =$ 131.3042Hz, $f^{z2} =$ 110.1697Hz, $\Gamma^{p2} =$ 60.1579Hz, $\Gamma^{z2} =$ 147.8138Hz, and $g = 0.5340$.

1.6 Model for the ground noise

Measurements at the sites indicate that the power spectral density of seismic noise in the ground is roughly flat at a level of few $\times\, 10^{-9}\mathrm{m}/\sqrt{\mathrm{Hz}}$ between 1 and 5Hz, falling as f^{-2} above 5Hz. For simplicity we assume that the ground noise is Gaussian and uncorrelated and has a power spectral densith of $3 \times 10^{-9}\mathrm{m}/\sqrt{\mathrm{Hz}}$ below 5 Hz. If the Nyquist frequency is f_N, then the procedure for obtaining noise with power spectral density P_N is to take the output of a Gaussian random number generator generating numbers with unit standard deviation and multiply by a factor of $P_N\sqrt{f_N}$.

The falloff above 5Hz is simulated by filtering the initially flat noise with a 2 pole Butterworth filter at 5Hz. The poles of this Butterworth filter are at $\pm 10\pi i$ in the s plane. This simplistic set of assumptions will be relaxed in better developed versions of the Monte Carlo.

1.7 Stack Transfer Function Poles and Zeros in the s plane

The procedure for obtaining table-like noise is to take noise representing the motion of the floor, apply a time-domain filter to yield noise having the same power spectral shape as the whitened frequency response of figure 4, and finally apply a Butterworth filter equivalent to the dewhitening function of equation 2 to yield the transfer function initially predicted by the Hytec model for motion of the table.

Equation 2 is equivalent to an 8 pole Butterworth filter, with the poles at the s plane, and no zeros. In practice it was discovered that 8 pole digital Butterworth filters in the z-plane (discrete time) representation tended to be unstable. We chose instead to dewhiten with two cascaded 4-pole Butterworth filters, which was always stable. Table 1.7 shows the zeros and poles in the s plane.

stack model	poles	$-16.119521327446 \pm 98.5585444441585i$
		$-188.991668072757 \pm 803.070248556014i$
	zeros	$113.271084624815 \pm 1.774071354679i$
		$464.370753369455 \pm 513.345428504745i$
butterworth	poles	$20\pi exp[in\pi/4], n = -3, -1, +1, +3$

Figure 5 shows the power spectral density of uncorrelated Gaussian noise treated as if it was produced at 16384 samples per second, filtered using the 3 stages (1 for whitened model response and 2 4–pole Butterworths) discussed above. Overlaid on the plot is the stack response directly from the model, multiplied by the power spectral density of the noise. The agreement is very good in the important region between 10 and 50Hz, and continues to be very good up to 500Hz.

1.8 Transfer Function of the Pendulum

We treat the table transfer function as that of a simple pendulum. The resonant frequency of the pendulum is 0.74Hz, the Q of the resonance being 1.85×10^5 if the pendulum swings freely. In practice, however, the differential and common length servos will considerably reduce the Q of the pendulum mode, so it is unrealisic to use the value of Q to derive the pendulum motion. Furthermore, the resonant frequency is a factor of ~ 10 below the lower bound of our target frequency range of 10-50Hz. For now, therefore, we treat the pendulum transfer function as a Butterworth filter having 2 poles at 0.74Hz. In the s plane, the pole positions are $s_M^p = \pm(2\pi \times 0.74)i$. Again, the digital filter poles and zeros are related to these by a bilinear transform.

1.9 Summary of the Simulation Method

The simulation is implemented in `matlab`. An initialization function imports the Hytec model of a BSC stack, calculates the transfer function from the ground to the table and performs the nonlinear fit to deduce IIR filter poles and zeros. The Butterworth filters for dewhitening the stack frequency response, for representing the pendulum transfer function above 10Hz, and for representing the f^{-2} falloff in ground noise amplitude above 5Hz, are

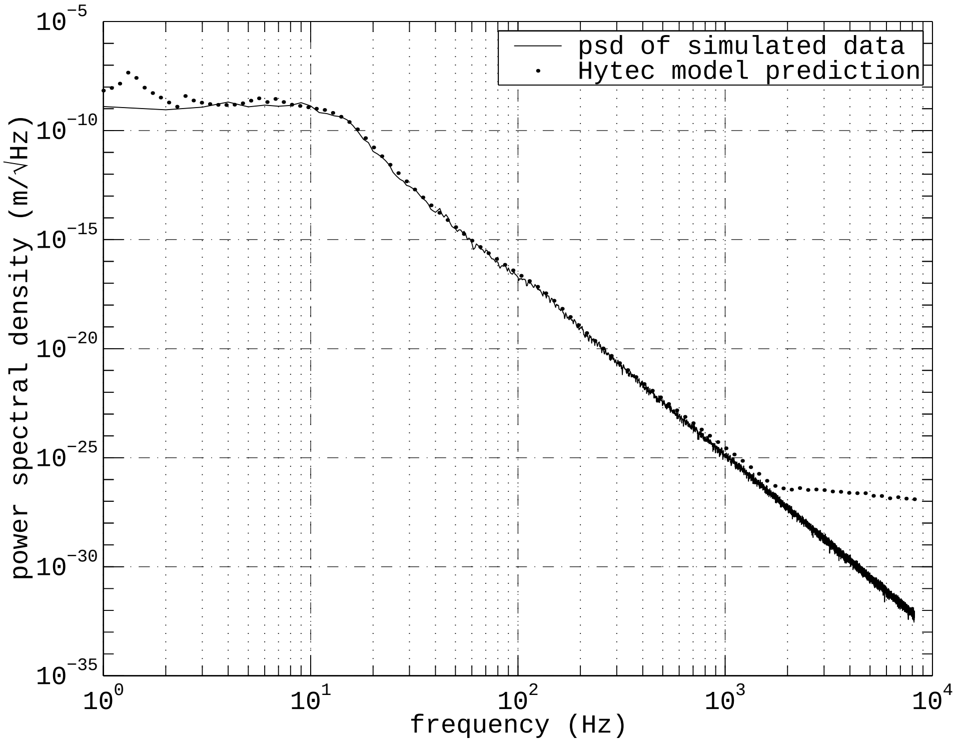

Figure 5: Simulation filter on uncorrelated Gaussian noise compared to the transfer function predicted by the Hytec stack model.

calculated. One second of Gaussian distributed random numbers, normalized to represent a power spectral density of $3 \times 10^{-9}\mathrm{m}/\sqrt{\mathrm{Hz}}$ is generated and run through the filter banks. The states of the filters (taps kept to allow smooth continuation between 1 second data segments) are stored in a state structure. Subsequent 1 second segments of data are generated by calling a second function with the structure containing the filter states and random number seeds as arguments.

Practical details on running the simulation can be found at:

`http://gravity.phys.psu.edu/~lsf/SimData`.

The current release version of the code is preliminary and will be modified to account for couplings of other floor degrees of freedom to the motion of the mirror. A more sophisticated model for the filtering effect of the pendulum suspension will also be included. Questions or comments on this work or the seismic part of the simulated data code can be sent to me at:

`edaw@ligo.mit.edu`