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# Sensor constants and cross coupling in a large optic

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## ABSTRACT

There are several possible mechanisms of cross coupling between the modes of a suspended optic. These mechanisms include (but are probably not limited to) intrinsic mechanical coupling, mismatch in the force constants, and mismatch in the sensor constants. This document describes an exploration of the effects of mismatched sensor constants on cross couplings.

The mechanism is conceptually simple. If the sensor constants are not properly tuned, an excitation in one mode can contribute to the error signal in another. The servo will then try to compensate for this perceived motion in the second mode, introducing actual motion to null the spurious error signal. There has recently been some concern that the sensor constants might change over time and that the cross couplings that result would introduce noise in the LIGO.

Presented here are both a theoretical model for sensor-constant-mediated cross couplings and the results of an experimental test of this model done on a large optic *in situ* at Hanford.

## INTRODUCTION

We would like to be able to adjust our suspended optics in the LIGO detector by acting on individual modes: position (pendulum mode), pitch, yaw, or sideways motion. As they are now constructed, however, the optics exhibit some *cross coupling*. If you try to act on one mode (position, for example), you usually wind up introducing some motion in the other modes (pitch, yaw, and side) as well. At this time it is thought that these cross couplings might introduce a significant source of noise in a running observatory, so there is a need to understand their mechanisms and limitations.

There are likely several mechanisms by which motion in one mode can be coupled into another. David Shoemaker has described a procedure for minimizing the cross couplings that arise from mismatched force constants [1]. In this document I will explore the consequences of mismatched sensor constants. Several people have observed indications that the sensor constants may drift over time. A proper study of the long-term stability of the sensor constants would require some time, and if performed at this time it would probably interfere with installation and commissioning of the detector. Before embarking on such a study it is useful to understand how a mismatch in the sensor constants can affect a suspended optic, and that is the purpose of this work.

## THEORY

Before constructing a model for this type of cross coupling, it is useful to establish some notation. Let's define a vector that represents the position of a mirror by the position of each of its magnets.

$$\vec{x} = (x_{UL}, x_{UR}, x_{LL}, x_{LR}),$$

where $x_{UL}$ represents the position of the upper left magnet, etc., along the x-axis. (The axis of the mirror itself would be parallel to this x-axis when the mirror is perfectly aligned.) The mirror's mechanical modes can be represented by a set of vectors that describe position, pitch, and yaw, respectively as*

$$\hat{z} = \frac{1}{2}(+1, +1, +1, +1),$$

$$\hat{p} = \frac{1}{2}(+1, +1, -1, -1),$$

and

$$\hat{y} = \frac{1}{2}(-1, +1, -1, +1).$$

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* Thanks to Stan Whitcomb for suggesting this vector-based notation.

We only need three vectors to describe the mirror, despite the fact that we have four magnets, because all four magnets lie in the same plane: the back surface of the mirror. The fourth basis vector needed to span a complete four dimensional space would be

$$\hat{s} = \frac{1}{2}(+1,-1,-1,+1),$$

which does not correspond to anything the mirror can do physically.

The state of the mirror can be expressed in terms of our three basis vectors as

$$\vec{x} = (\vec{x}\cdot\hat{z})\hat{z} + (\vec{x}\cdot\hat{p})\hat{p} + (\vec{x}\cdot\hat{y})\hat{y}.$$

We can define the mirror's position, pitch, and yaw by the projections of $\vec{x}$ along each of the basis vectors.

$$x_{POS} = \vec{x}\cdot\hat{z},$$

$$x_{PIT} = \vec{x}\cdot\hat{p},$$

and

$$x_{YAW} = \vec{x}\cdot\hat{y}.$$

These mode signals are independent of each other because the basis vectors that define them are orthogonal.

Now, what we have been describing so far is the *physical* state of the mirror. The basis vectors $\hat{z}$, $\hat{p}$, and $\hat{y}$ represent what the mirror is actually doing. The controller, however, measures signals from each of the shadow sensors and tries to calculate the position, pitch, and yaw of the mirror from these signals. It does this by taking its own approximations to the basis vectors and dotting them (forming an inner product) with $\vec{x}$. These approximate basis vectors will not necessarily correspond to the ideal basis vectors, because of differences in the shadow sensors, etc. Let us define the *sensor constants* as these approximations to the basis vectors and denote them by greek letters instead of roman. The sensor constants for position, pitch, and yaw will then be, respectively,

$$\vec{\zeta} = (+\zeta_{UL},+\zeta_{UR},+\zeta_{LL},+\zeta_{LR}),$$

$$\vec{\theta} = (+\theta_{UL},+\theta_{UR},-\theta_{LL},-\theta_{LR}),$$

and

$$\vec{\phi} = (-\phi_{UL},+\phi_{UR},-\phi_{LL},+\phi_{LR}).$$

The individual elements of these sensor constant vectors will include the shadow-sensor sensitivity and the gains of each channel, so we will not normalize these vectors.

To calculate the pitch error signal, for example, the pitch servo forms the product

$$\begin{aligned}\varepsilon_{PIT} &= \vec{\theta}\cdot\vec{x}\\ &= (\vec{x}\cdot\hat{z})(\vec{\theta}\cdot\hat{z}) + (\vec{x}\cdot\hat{p})(\vec{\theta}\cdot\hat{p}) + (\vec{x}\cdot\hat{y})(\vec{\theta}\cdot\hat{y})\\ &= x_{POS}(\vec{\theta}\cdot\hat{z}) + x_{PIT}(\vec{\theta}\cdot\hat{p}) + x_{YAW}(\vec{\theta}\cdot\hat{y})\end{aligned}$$

The error signal is a sum of three separate signals. We would like for the sensor constants to correspond to the actual physical modes of the mirror, *i.e.*

$$\left(\vec{\theta}\cdot\hat{z}\right) = \left(\vec{\theta}\cdot\hat{y}\right) = 0,$$

but this is not usually the case, due to the imbalances mentioned above. If the force constants are properly tuned, the pitch servo only acts on one of the channels, namely $x_{PIT}$. For the rest of this paper, we will assume that the force constants have already been tuned.*

A block diagram of the pitch servo, including sensor-constant cross coupling, is shown in Figure 1. If the sensor constants are not tuned, *i.e.*

$$\left(\vec{\theta}\cdot\hat{z}\right), \left(\vec{\theta}\cdot\hat{y}\right) \neq 0,$$

then driving position or yaw ($x_{POS}$ or $x_{YAW}$) will induce motion in pitch by adding an additional error signal to the pitch loop.

It is straightforward to calculate the mirror pitch for the example shown in Figure 1.

$$\begin{aligned}x_{PIT} &= \frac{G}{1-G}\left[x_{POS}\left(\vec{\theta}\cdot\hat{z}\right) + x_{YAW}\left(\vec{\theta}\cdot\hat{y}\right)\right]\\ &= \frac{G}{1-G}\left[\frac{x_{POS}}{2}\left(\theta_{UL}+\theta_{UR}-\theta_{LL}-\theta_{LR}\right) + \frac{x_{YAW}}{2}\left(-\theta_{UL}+\theta_{UR}+\theta_{LL}-\theta_{LR}\right)\right]\end{aligned} \tag{1}$$

This cross coupling depends linearly on the sensor constants, and in the case where $G \ll 1$, it varies linearly with the overall gain of the pitch servo as well.

Note that, according to this model, we should be able to completely eliminate this source of cross coupling with a suitable choice of sensor constants.

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* It would be straightforward to include cross couplings in the force constants in this model, but that is not necessary for understanding the sensor-constant-mediated cross couplings treated here.

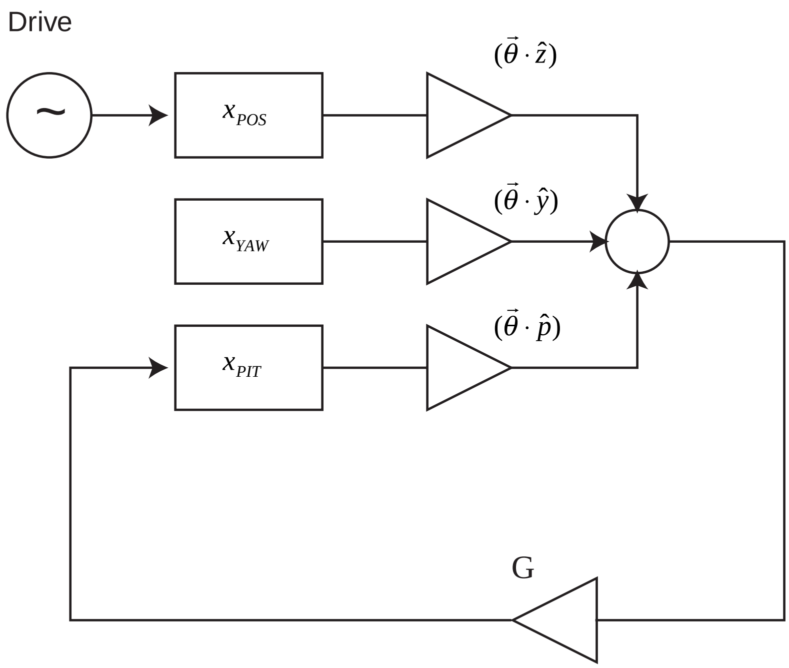


**Figure 1** Block diagram showing how position motion can get coupled into the pitch servo. Here we assume that the force constants have already been tuned, *i.e.* the pitch servo only acts on the mirror's pitch. The gain stage $G$ represents all of the elements in the feedback loop *except* the sensor constants. It includes electronic gains, mechanical transfer functions, etc.

## EXPERIMENTAL TEST OF THE MODEL

This model was tested on a large optic *in-situ* at Hanford by driving position and measuring pitch as a function of both pitch sensor constant gain and overall pitch gain. Pitch was measured using the ISC optical lever, rather than the OSEM shadow sensors. Some control experiments were also performed by varying other parameters, such as one of the yaw sensor constants, and verifying that they had no effect on the position to pitch coupling.

### Experimental setup

Test optic: Hanford 2k interferometer, ITMY, under vacuum

This experiment was done with tuned force constants, obtained by applying David Shoemaker's tuning procedure. The force constants used in this experiment were, as they read on the software control panels,

```
UL->POS = 50.0      UR->POS = 49.0
UL->PIT = 50.0      UR->PIT = 49.0
UL->YAW = 50.0      UR->YAW = 41.0

LL->POS = 47.0      LR->POS = 46.0
LL->PIT = 48.0      LR->PIT = 48.0
LL->YAW = 47.0      LR->YAW = 49.0
```

These force constants minimized the cross couplings, but did not eliminate them. One goal of this experiment was to see if the residual cross couplings were due to mismatched sensor constants, and to see if tuning the sensor constants could further reduce the cross couplings.

The optic's position mode was driven sinusoidally, using a function generator, by injecting a signal into the POS TEST IN port of the suspension controller. A rough measure of the cross coupling with untuned sensor constants was obtained by taking the ratio of the pitch motion, as measured by PIT 2 MON, to the position motion, as measured by POS 2 MON.

| | |
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| Drive Port: | POS TEST IN |
| Drive Voltage: | $20V_{pp}$ |
| Drive Frequency: | $2.0Hz$ |
| Drive Response: | $4V_{rms}/\sqrt{Hz}$ as measured at POS 2 MON, or approximately $4mm_{rms}/\sqrt{Hz}$. |
| Pitch Response: | $4mV_{rms}/\sqrt{Hz}$ as measured at PIT 2 MON, with untuned sensor constants. Note that this corresponds to a cross coupling of approximately $x_{PIT}/x_{POS} \approx 10^{-3}$. |

The pitch motion of the mirror was measured with its ISC optical lever. Individual quadrants of the receiver photodiode were monitored by tapping into the signal cable where it breaks out in equipment rack 2x6; the appropriate arithmetic to convert these signals to pitch and yaw motion was done with three SR560 preamps; and the pitch and yaw signals were then fed into a spectrum analyzer. Each data point plotted below represents the height of the peak found at $2.0Hz$ on the spectrum analyzer, which should be proportional to the angular motion of the mirror. No information about the phase of the signal was recorded, only its amplitude.

**Expected ideal sensor constants**

The expected ideal sensor constants were calculated by driving position, measuring the response of each of the individual shadow sensors, and normalizing each channel to produce the same signal. (The assumption here is that when driving position, each of the magnets moves by the same amount, so each shadow sensor should produce the same signal.) The output of each shadow sensor with the above drive, as measured at the satellite box, was

UL = $420\mu V_{rms} / \sqrt{Hz}$
UR = $372\mu V_{rms} / \sqrt{Hz}$
LL = $415\mu V_{rms} / \sqrt{Hz}$
LR = $230\mu V_{rms} / \sqrt{Hz}$

(Uncertainties were on the order of 10%.) We might then naively expect that the gains in each input matrix channel that would minimize the cross couplings would be

```
UL->XX = 25
UR->XX = 28
LL->XX = 25
LR->XX = 46
```

(These gains are what you must multiply each shadow sensor signal by to get the same output, assuming the magnets are all moving by the same amount.) As we shall see, this expectation does not appear to be justified. The ideal sensor constants appear to be different than what this measurement would suggest.

**Cross coupling vs. sensor constants**

The model described above predicts that the couplings should be linearly proportional to the sensor constants. We may vary these sensor constants by varying the gains of the input matrix elements. The sensor constants $\theta_{XX}$, where XX stands for the appropriate magnet, UL, UR, LL, or LR, should be proportional to the input matrix element gains XX->PIT. The first test of the predictions of this model (Equation 1) was an observation of the pitch motion vs. input matrix gain with a constant drive applied to position, as described above.

The output of the ISC optical lever vs. input matrix gain for three different matrix elements is shown in Figure 2. Two of the pitch input matrix elements and one of the yaw elements were varied. We expect from Equation 1 that the pitch excitation will vary linearly with the pitch matrix elements and not at all with the yaw element. As expected, the pitch motion did not appear to depend on the yaw sensor gain. The pitch motion did appear to be linear in both of the pitch input gains that were varied, but only for grossly mismatched sensor constants. (Note that the bilinear behavior shown in Figure 2 is consistent with the predictions of Equation 1, since the data do not contain any phase information.)

One prediction of Equation 1 was not verified: that the cross coupling should vanish for some choice of sensor constants. Some residual cross coupling remains, even after both force and sensor constants are optimized. Note also that the optimum set of sensor constants appears to coincide with all input gains equal, at 25. Recall that our naive expectation was that the optimum value for LR->PIT would be 46.

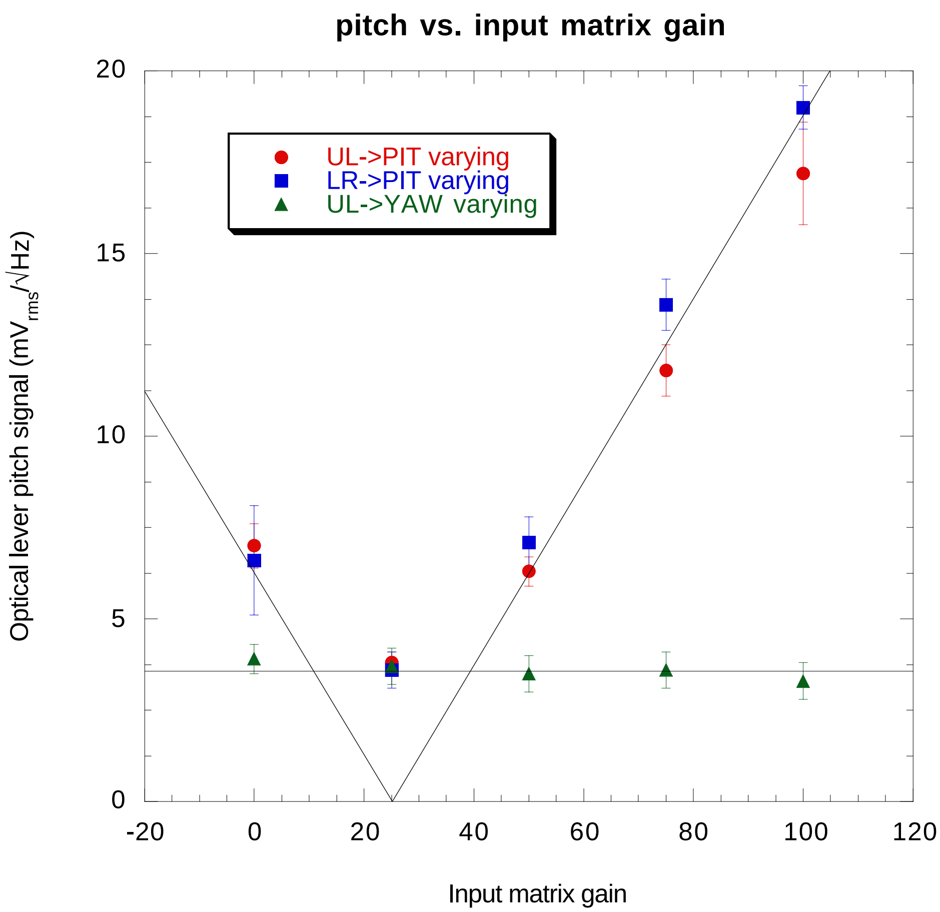


**Figure 2** Observed pitch versus sensor constant gain. Varying the UL->PIT gain varies $\theta_{UL}$, varying LR->PIT varies $\theta_{LR}$, and varying UL->YAW varies $\phi_{UL}$. We don't expect the pitch to depend on the yaw sensor constants, and no dependence is observed. Points are averages of either five to ten individual data points. Error bars are standard deviations of these averages. Lines are qualitative expected behavior, not fits.

## Cross coupling vs. mode gain

The second test made of the predictions of this model was to measure the cross coupling from position to pitch as a function of the pitch mode gain, effectively $G$ in Equation 1. The overall gain $G$ should be proportional to this mode gain, so the pitch excitation should vary with the mode gain the same way Equation 1 varies with $G$. For our case, where $G \ll 1$, this means that the pitch motion should be proportional to $G$. The slope of this linear dependence should be proportional to each $\theta_{XX}$, and hence linearly proportional to each input matrix element gain XX->PIT.

For this test, the position mode was excited at a constant amount, as described above, and the pitch motion was measured as a function of PIT gain with three different values of the input matrix gain UL->PIT: 25, 75, and 100. (All other input matrix element gains were held at 25.) Results are plotted in Figure 3, where good qualitative agreement with the predictions of Equation 1 is revealed.

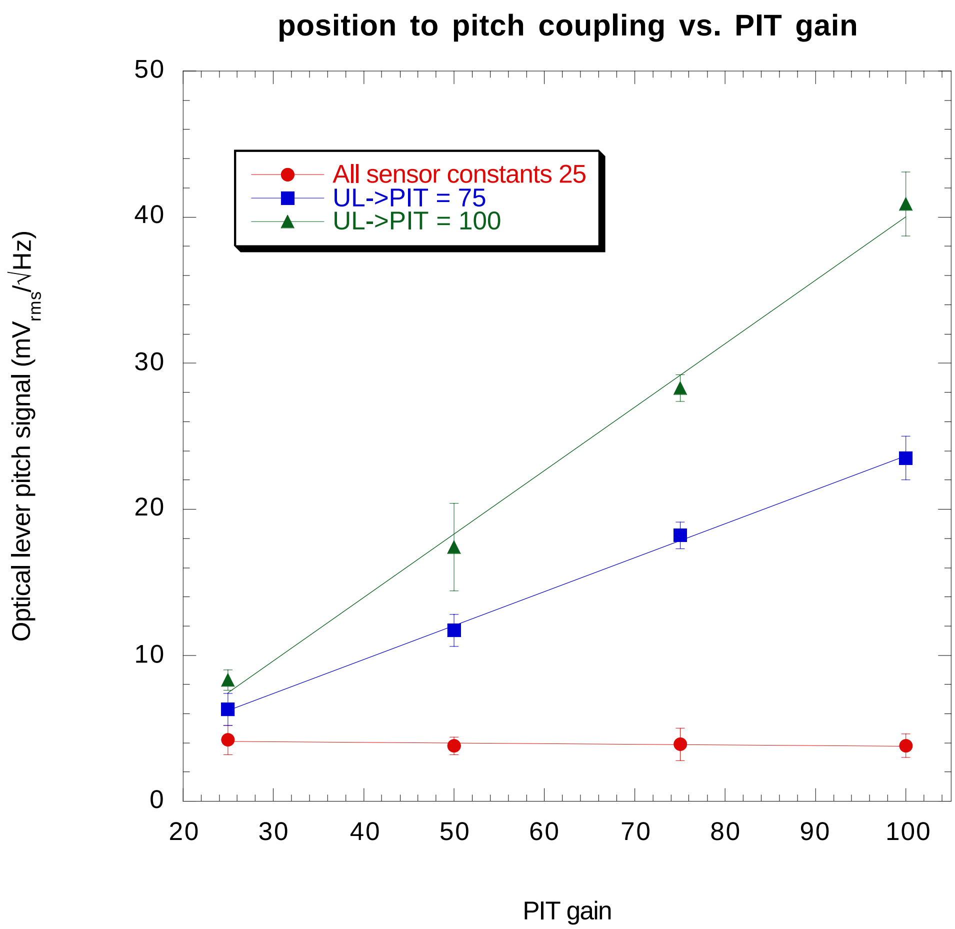


**Figure 3** Position to pitch coupling vs. overall PIT gain, for various sensor constant arrangements. We expect this dependence to be linear (in the regime where $G << 1$), and we expect the slope to vary linearly with each of the sensor constants. Lines are linear fits.

## Control experiments

Four control tests were made in this experiment. The first was a measurement of the dependence of the position to pitch cross coupling vs. one of the yaw sensor constants. The results of this test are shown in Figure 2. No dependence of the position to pitch cross coupling on the yaw input matrix is expected, and as shown in Figure 2, none is observed.

The second test was a measurement of the dependence of position to pitch cross coupling on the overall yaw gain. Again, no dependence is expected between these two quantities, and as shown in Figure 4, none was observed.

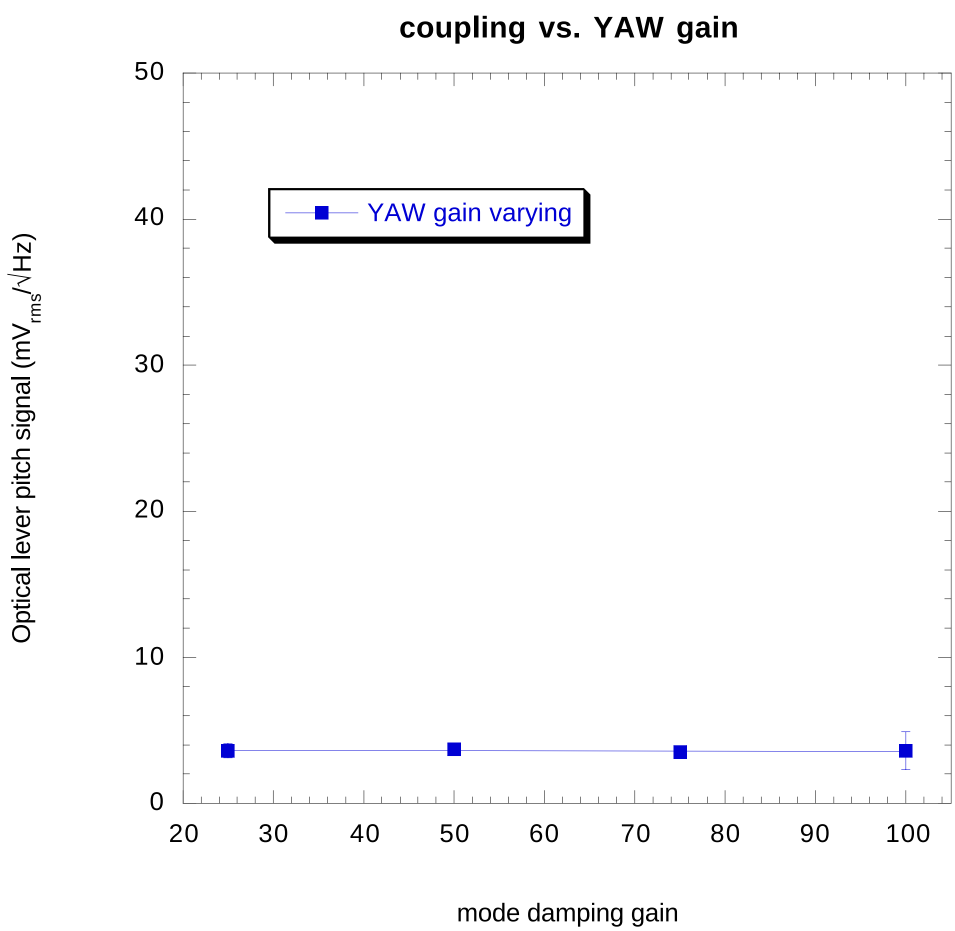


**Figure 4** Position to pitch cross coupling vs. yaw mode gain. No dependence is expected, and none is observed. Line is a linear fit.

The third control test was to vary the overall gain of the side mode and see if that affected the position to pitch coupling. No dependence on side gain is expected, and as shown in Figure 5, none was observed.

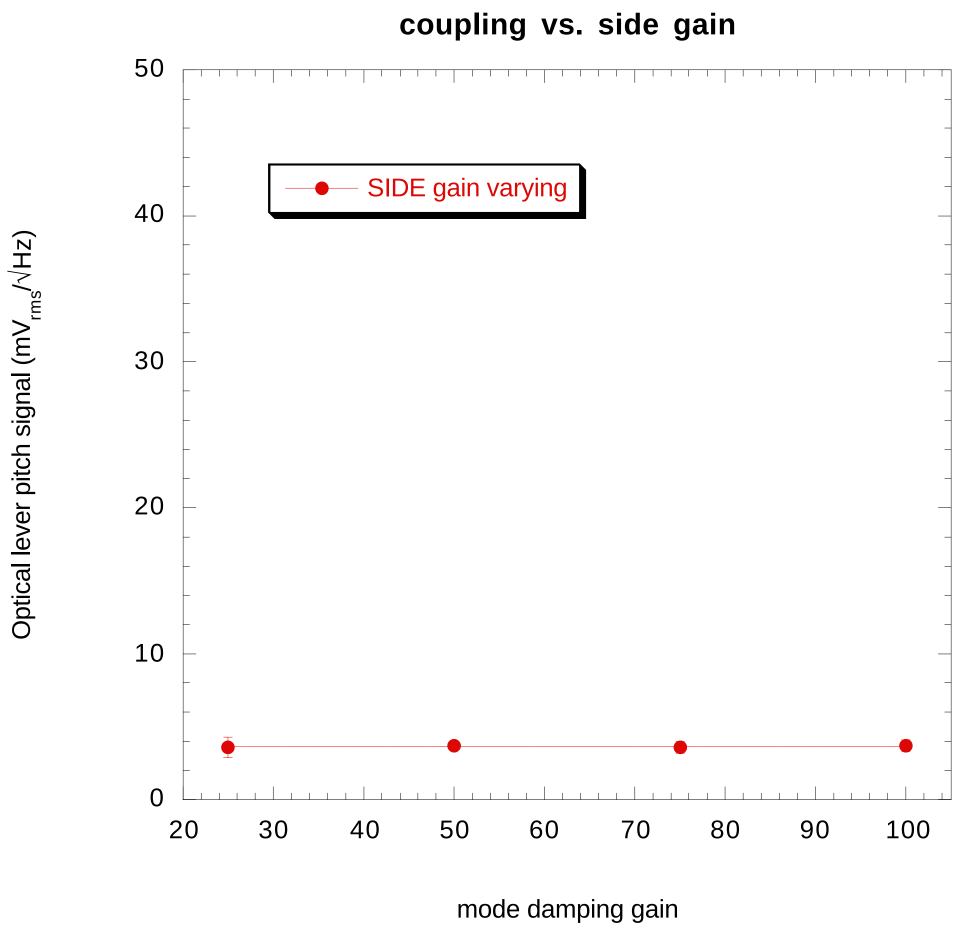


**Figure 5** Position to pitch cross coupling vs. side mode gain. No dependence is expected, and none is observed. The line is a linear fit.

The fourth and final test was to vary the overall position gain and see if that affected the pitch motion. Here some dependence is to be expected, since the position gain should affect the excitation signal $x_{POS}$ in Equation 1, hence affecting the pitch motion $x_{PIT}$. We may expect the excitation signal to vary linearly with the position gain, which in turn means that the pitch motion should vary linearly with the position gain as well. The results of this test are shown in Figure 6, where some dependence is observed, in qualitative agreement with our expectations.

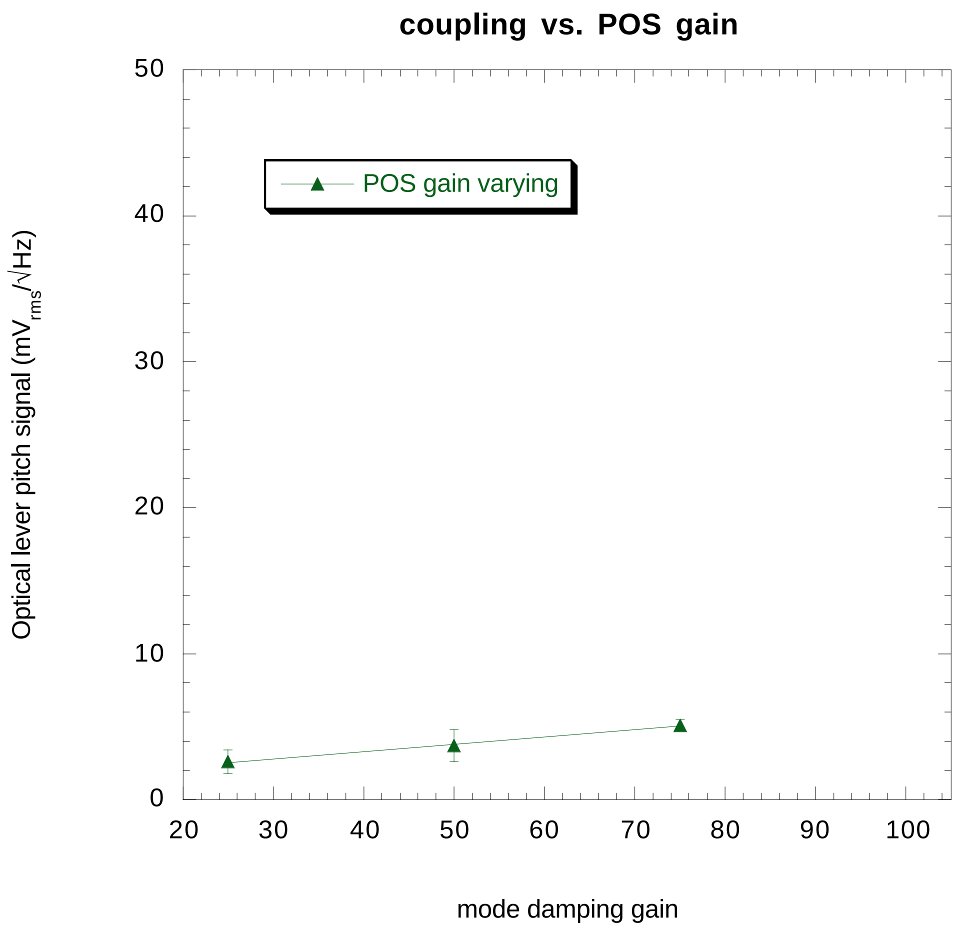


**Figure 6** Variation of the pitch motion vs. position gain. Here some dependence is both expected observed. The line is a linear fit.

## CONCLUSIONS

This model for sensor-constant-mediated cross couplings appears to be correct, but in this optic it only introduces significant cross couplings when the sensor constants are egregiously mismatched. Since this cross coupling depends only weakly on the sensor constants, we probably do not need to worry about moderate drifts in them over time.

A simple way to test whether or not tuning the sensor constants would improve the cross couplings would be to excite one mode and vary the gain of the others, recording the cross coupling as in Figure 3. If you observe any dependence of the cross coupling on $G$, then tuning the sensor constants will probably reduce your cross coupling. If you observe no dependence on $G$, then you won't gain anything by tuning the sensor constants.

The dominant source of cross coupling in this optic, when the force constants are tuned but the sensor constants are not, appears to be independent of both the force constants and the sensor constants. This residual cross coupling appears to dominate over the sensor-constant-mediated effect and is not well understood at this time.

**REFERENCES**

[1] Eric Black, Gabriela Gonzalez, Nergis Mavalvala, and David Shoemaker, *Output matrix tuning for large and small optics controllers*, LIGO-T990094-00-D, (1999).