LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Document Type **LIGO-T950011-14 - D** May. 23, 96

Suspension Design Requirements

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DRAFT

This is an internal working note
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1 INTRODUCTION

1.1. Purpose

This Design Requirements Document (DRD) for Suspension System (SUS) identifies the information necessary to define the SUS subsystem and quantify its relationship to other subsystems.

1.2. Scope

SUS will develop and provide the suspension system for all the suspended components of Core Optics Component (COC) and Input/Output Optics (IOO).

1.3. Definitions

SUS is the system which suspends, protects, damps, and actuates the optics.

1.4. Acronyms

- LOS1: Large Optics Suspension 1
- LOS2: Large Optics Suspension 2
- SOS: Small Optics Suspension

Acronyms for names of subsystems should be referred to [1] LIGO-1401051 Rev. B: LIGO DETECTOR Construction Phase Implementation Plan (p. 13).

1.5. Applicable Documents

1.5.1. LIGO Documents

[1] LIGO-1401051 Rev. B: LIGO DETECTOR Construction Phase Implementation Plan
[2] LIGO-E950018-02-E: LIGO Science Requirements Document
[3] LIGO-E960022-00-D: LIGO Vacuum Compatibility, Cleaning Methods and Procedures
[4] LIGO-E950099-01-D: Core Optics Components Requirements (1064 nm)
[5] LIGO-T960019-00-D: Frequency, Intensity and Oscillator Noise in the LIGO
[6] LIGO-T952007-01-I: Alignment Sensing/Control Design Requirements Document
[7] LIGO-T960070-00-D: Framework of Range Requirement of Suspension Actuator
[8] LIGO-T960065-02-D: Seismic Isolation Design Requirements Document
[9] LIGO-960090-00-D: Thermal Noise in HAM OS
[10] LIGO-T960040-00-D: Response of Pendulum to Motion of Suspension Point
[11] LIGO-T950060-00-D: Naming Convention and Interface Definition for SUS
[12] LIGO-T960058-00-D: Length Sensing and Control Design Requirements Document
[13] LIGO-M950046-F: LIGO Project System Safety Management Plan
[14] LIGO-T950061-01-D: Interferometer Requirements Flowdown to SUS

[15] LIGO-T960074-00-D: Suspension Preliminary Design

[16] LIGO-T960086-00-D: Suspension Test Plan

[17] LIGO-P940003-00-R: Thermally Excited Vibrations of the Mirrors of Laser Interferometer Gravitational-Wave Detectors

[18] LIGO-P940011-00-R: Suspension Losses in the Pendula of Laser Interferometer Gravitational-Wave Detectors

[19] LIGO-P940012-00-R: Mirror-Orientation Noise in a Fabry-Perot Interferometer Gravitational Wave Detector

1.5.2. Non-LIGO Documents

2 GENERAL DESCRIPTION

2.1. Specification Tree

This document is part of an overall LIGO detector requirement specification tree. This particular document is circled in Fig. 1.

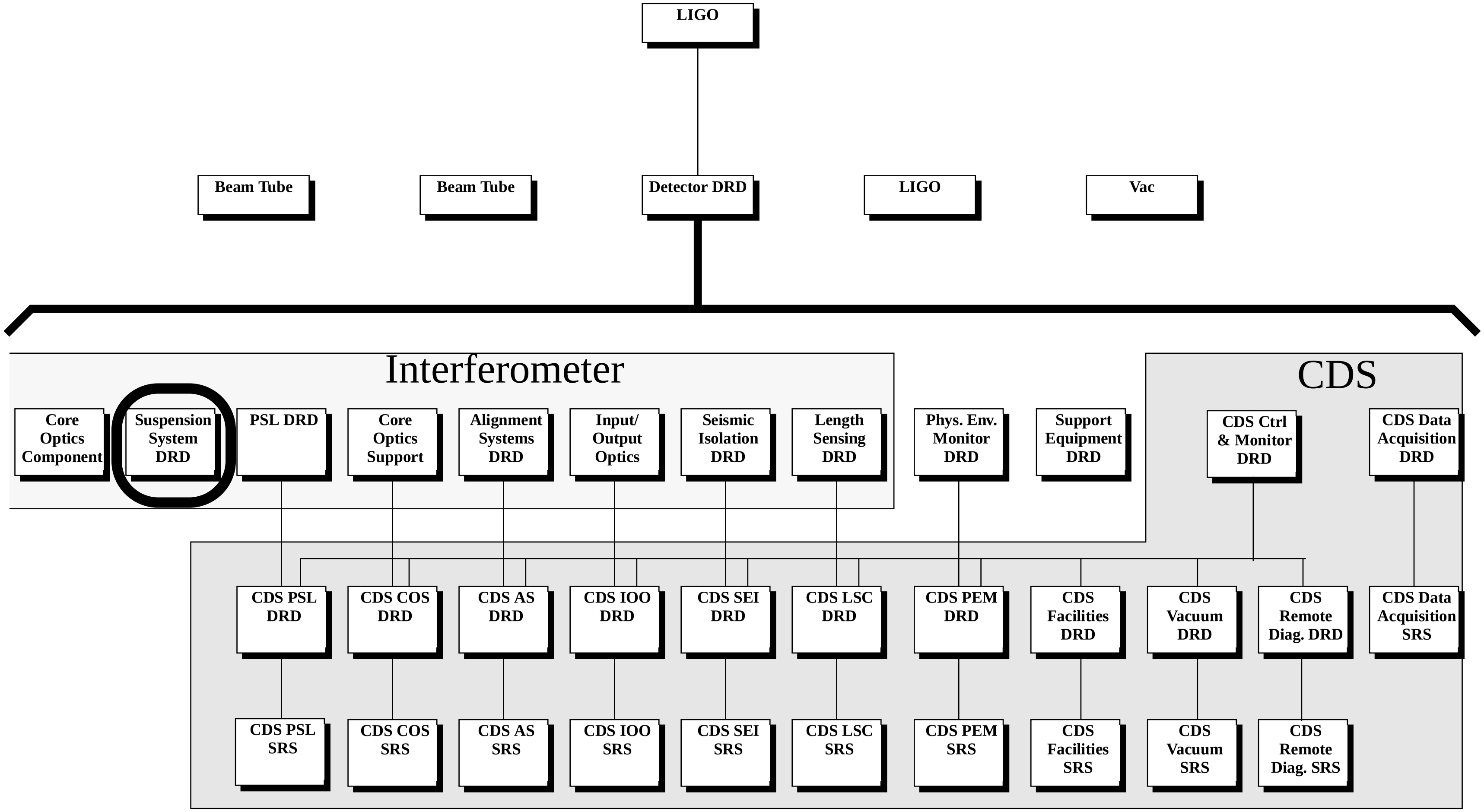


Figure 1: Overall LIGO detector requirement specification tree. SUS DRD is circled.

2.2. Product Perspective

SUS relates to the rest of the system as shown in Fig. 2.

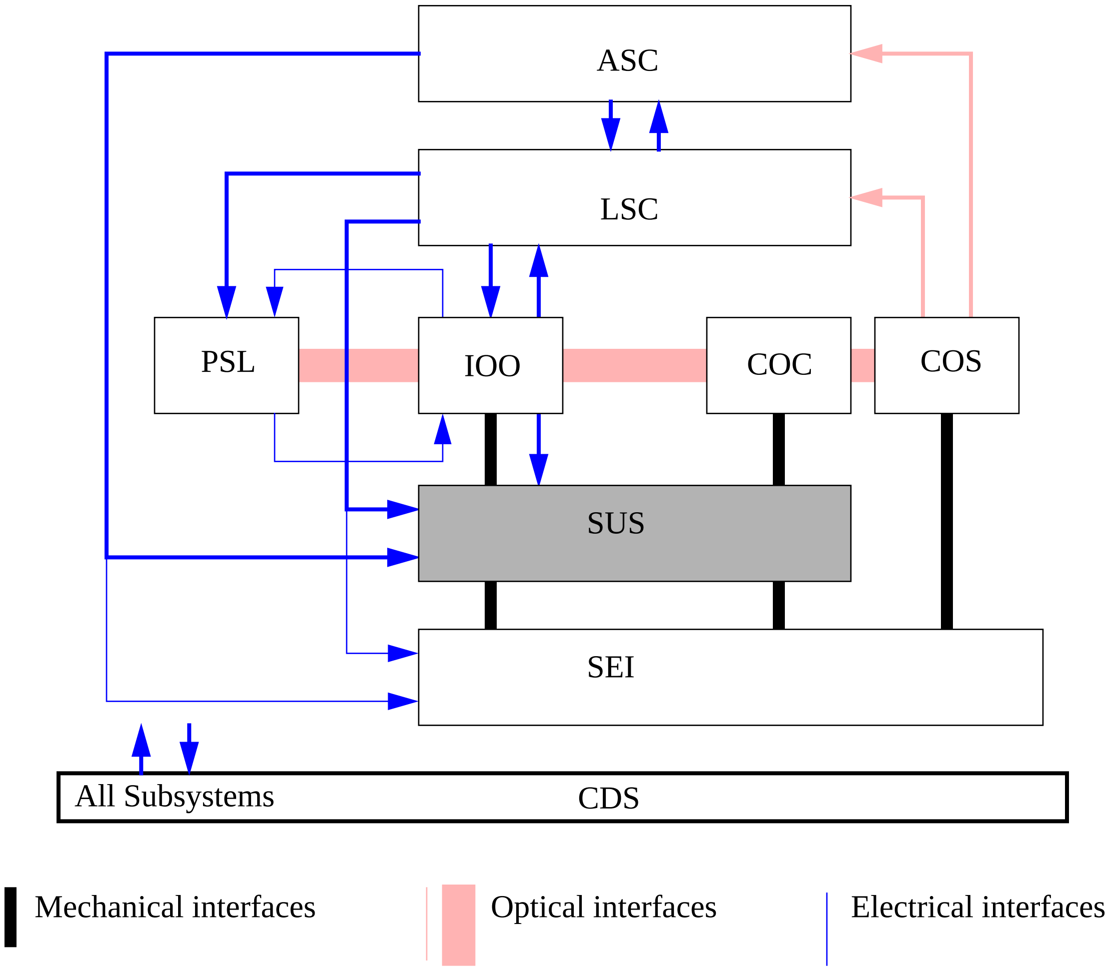


Figure 2: Relationship of SUS to the rest of the detector subsystem. SUS is shaded.

2.3. Product Functions

The main functions of SUS are:

- Suspend a test mass to allow it to move freely horizontally for detection of gravitational waves.
- Isolate an optical component from ground motion by suspending the component.
- Damp the optical component’s motion in position and orientation using the local suspension’s sensors and actuators.
- Provide control inputs for applying forces and torques to the suspended component in response to signals from the LSC and ASC systems.

- Protect the optical components by limiting motion from external disturbance.
- Hold the optical components firmly during installation.
- Reduce the effect of stray/scattered light from the optical component.

2.4. General Constraints

- The initial LIGO must have a sensitivity specified in [2] LIGO-E950018-02-E: LIGO Science Requirements Document, which SUS must not preclude.
- LIGO must operate continuously, therefore SUS must be designed with high reliability and low mean time to repair.
- LIGO interferometers have strict vacuum-compatibility requirements which constrain the material choices for the SUS components to those materials compatible with [3] LIGO-E960022-00-D: LIGO Vacuum Compatibility, Cleaning Methods and Procedures.

2.5. Assumptions and Dependencies

2.5.1. Assumption in SUS

2.5.1.1 Single Pendulum

The suspension system employs a single pendulum as opposed to a multi-stage pendulum.

2.5.1.2 Suspension Type

There will be three types of the suspension system depending on the size of the suspended optical component: Large Optics Suspension 1 (LOS 1), Large Optics Suspension 2 (LOS 2), and small optics suspension (SOS). A list of suspended optical components for each suspension system is shown in Table 1.

Table 1: List of suspended optical components

SUS type	*Suspended Optical Components (Subsystem)*
LOS 1	Test Masses (CO), Recycling Mirror (CO), Large Folding Mirror (CO), Mode Matching Mirrors (IOO), Faraday Isolator (IOO)[a] **TBD**
LOS 2	Beamsplitter (CO)
SOS	Mode Cleaner Mirrors (IOO), Small Folding Mirrors (IOO), Small Pick-offs (IOO)

[a] An adapter ring will be fitted to the Faraday Isolator to fit it into the standard assembly.

2.5.1.3 Damping by Suspension’s Sensor

A suspended component is or is not damped using the sensor signals from its respective suspension assembly depending on the type of the optical component and the state of the interferometer.

Table 2 summarizes the state of the interferometer in which optical components are damped by the suspension's sensor.

Table 2: States of interferometer operation in which optical components are damped by the suspension's sensor

Motion	*Test Mass*	*Beamsplitter, Recycling Mirror, MC Mirror, Folding Mirror, Mode Matching Mirror*	*Pick-off, Faraday Isolator*
Longitudinal	Before and during acquisition	Always	Always
Transverse	Always	Always	Always
Orientation	Before acquisition	Before acquisition	Always

2.5.1.4 Suspension's Actuator

The suspension's actuator is used to correct fluctuations on the time scale that is shorter than well below the microseismic peak (f > 0.03 Hz). The stack support actuator is used to correct fluctuations of much longer time scales than the microseismic peak (principally the tidal disturbances).

2.5.2. Assumption in Other Subsystems

2.5.2.1 Size of Optics

The size, wedge, and optical clear aperture of the suspended optics (COC) are listed in [4] LIGO-E950099-01-D: Core Optics Components Requirements (1064 nm).

2.5.2.2 Mode Cleaner Output Frequency Noise

Allowed frequency noise of the light coming out of the mode cleaner (IOO) is $1 \times 10^{-4}\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 100 Hz with $f^{-0.5}$ dependence above 100 Hz and f^{-2} **TBD** dependence below 100 Hz (See [12] LIGO-T960058-00-D: Length Sensing and Control Design Requirements Document).

2.5.2.3 Beam Spot Offset

The requirement of the beam spot offset from the center of the optic is 1 mm for test masses (ASC) and 3 mm for mode cleaner mirrors (ASC).

2.5.2.4 Level of Mode Cleaner

The requirement of the level of the mode cleaner (IOO) is $3 \times 10^{-4}\mathrm{rad}$ **TBD**.

2.5.2.5 Vibrational Loss of Bare Substrate

The required vibrational loss of the bare substrate of the test mass (COC) is 3×10^{-7} and that of the mode cleaner mirror (IOO) is **TBD**.

3 REQUIREMENTS

3.1. Requirements Flowdown

Performance requirements of SUS are derived from system requirements of the interferometer noise and detector availability. Fig. 3 shows the noise and availability requirements flowdown to the SUS subsystem.

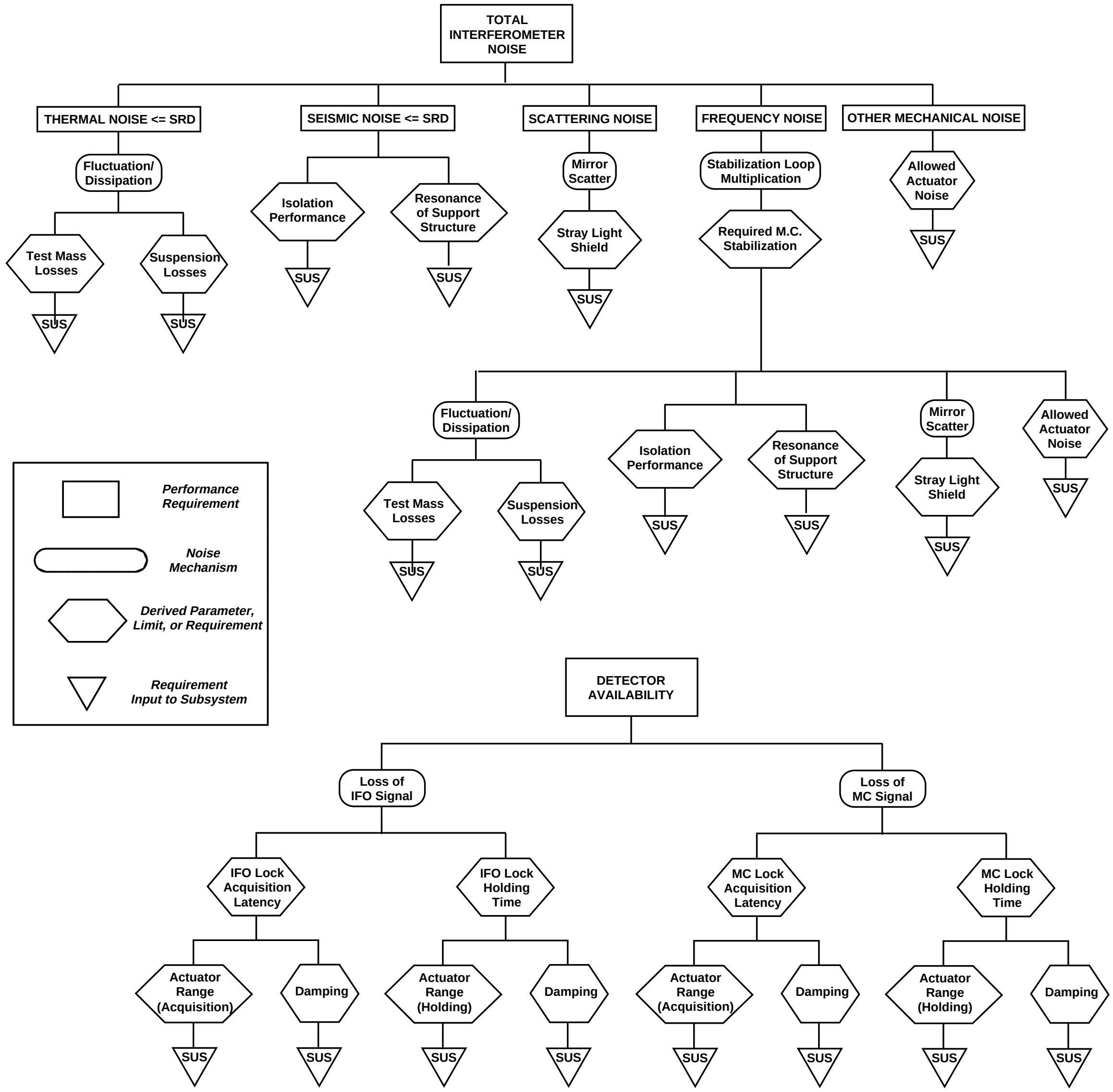


Figure 3: Interferometer noise and detector availability requirements flowdown to SUS.

3.2. Characteristics

3.2.1. Performance Characteristics

3.2.1.1 Detector Availability

3.2.1.1.1 Range

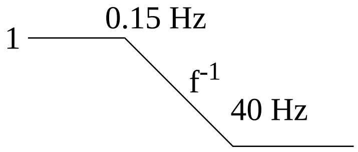
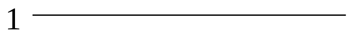
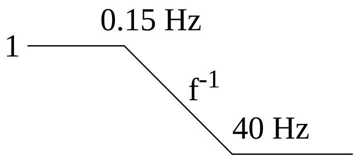
The actuator range is required to provide:

- continuous operation of the LSC system,
- smooth acquisition of the LSC system,
- proper initial alignment, and
- continuous operation of the ASC system.

The requirements of the actuator range are expressed by a DC (DC: defined as f << 0.15 Hz) peak-to-peak motion DC (DC: defined as f << 0.15 Hz) in displacement and orientation and a weighting function (transfer function from the output driver voltage to force normalized in such a way that it is unity at DC.). The weighting function represents the frequency dependence of the range (See [7] LIGO-T960070-00-D: Framework of Range Requirement of Suspension Actuator). Table 3 shows the requirement of the suspension range for displacement (operation and acquisition mode) and orientation of the mass.

These requirements are common to all the three suspensions. Coordination of the requirements between the actuator range and the stack isolation is explained in [8] LIGO-T960065-02-D: Seismic Isolation Design Requirements Document.

Table 3: Requirement of the suspension actuator range.

Mode		*DC Peak-to-Peak Motion*	*Weighting Function*
Displacement	Operation	40 μm_{pp}	
	Acquisition	40 μm_{pp}	
Orientation		1 $mrad_{pp}$	

3.2.1.1.2 Damping

The magnitude and quality of damping the suspended component is required to provide:

- stable operation of the LSC system,
- smooth acquisition of the LSC system, and
- negligible up-conversion noise of the spurious interferometer.

The requirements of damping the suspended component are expressed by the height of the residual bump around the pendulum and pitch/yaw frequencies in the transfer function from (horizontal, vertical, and pitch/yaw) motion of the suspension point to (horizontal and pitch/yaw) motion of the suspended mass. This height of the residual bump is required to be less than 3.

3.2.1.2 Interferometer Noise

3.2.1.2.1 Transfer Function of Suspension

The transfer function of the suspension is required to provide sufficient isolation above 35 Hz to meet the LIGO sensitivity requirement. Table 4 shows the requirements of the transfer function of the suspension system from (horizontal, vertical, and pitch/yaw) motion of the suspension point to (horizontal, vertical, and pitch/yaw) motion of the suspended mass except wire violin modes. Coordination of the requirements between the suspension transfer function and the stack isolation is explained in [8] LIGO-T960065-02-D: Seismic Isolation Design Requirements Document. See

Table 4: Requirements matrix of the transfer function of the suspension system from motion of the suspension point to motion of the suspended mass (f > 40Hz).

Transfer Function	*To*	*Horizontal (m)*	*Vertical (m)*	*Pitch (rad)*
From				
Horizontal (m)		$< \left(\frac{f_p}{f}\right)^2$ m/m f_p = 0.74 Hz (LOS1/2), 0.84 Hz (SOS)	Trivial	$< \alpha \times \left(\frac{f_p}{f}\right)^2$ m/m α = 100 (LOS1/2), 30 (SOS)
Vertical (m)		$< 3\times10^{-5}\times\left(\frac{f_v}{f}\right)^2$ m/m	$< \left(\frac{f_v}{f}\right)^2$ m/m f_v = 13 Hz (LOS1/2), 16 Hz (SOS)	$< \beta\times\left(\frac{f_v}{f}\right)^2$ rad/m $\beta = 3\times10^{-2}$ (LOS1/2), 1×10^{-2} (SOS)

3.2.1.2.2 Resonance of Suspension Support Structure

The resonance of the suspension support structure is required not to preclude the LIGO sensitivity requirement. Table 5 shows the requirements for the frequency and Q of the resonance of the suspension support structure. See [9] LIGO-960090-00-D: Thermal Noise in HAM OS.

Table 5: Requirements of the (average effective) thermal loss of the suspension system.

Mode	*LOS1/LOS2*	*SOS*
Resonance Frequency	> 160 Hz	> 150 Hz
Q	< 300	< 300

3.2.1.2.3 Thermal Loss

The thermal loss of the suspension system is required to be small enough to meet the LIGO sensitivity requirement. There are three kinds of thermal losses with regard to the suspension system (See Appendix B for detail):

- structural loss of the suspension pendulum, pitch/yaw, and vertical mode,
- structural loss of the mass internal mode due to the suspension attachments,
- viscous loss of the suspension pendulum mode due to interaction between the suspension attachments and the external components.

Requirements for LOS2 will not explicitly established; we simply use the same design for LOS2 as LOS1. Table 6 shows the requirements of the (average effective) thermal loss of the suspension system.

Table 6: Requirements of the (average effective) thermal loss of the suspension system.

Loss Type		*Loss*	
Damping Mechanism	*Mode*	*LOS1*	*SOS*
Structural (Loss: frequency independent)	Vibrational	$< 4\times10^{-7}$	$< 1\times10^{-5}$
	Pendulum	$< 7\times10^{-6}$	$< 5\times10^{-6}$
	Pitch/Yaw	$< 5\times10^{-4}$ / $< 8\times10^{-4}$	$< 3\times10^{-4}$ / $< 9\times10^{-5}$
	Vertical	$< 3\times10^{-3}$	$< 7\times10^{-2}$
Viscous (Loss: linear to frequency)	Pendulum	$< 8\times10^{-7}$ at 100 Hz $< 6\times10^{-9}$ at 0.74 Hz	$< 6 \times 10^{-5}$ at 100 Hz $< 6 \times 10^{-7}$ at 1 Hz

3.2.1.2.4 Control Noise

The control noise of the suspension system in total is required to cause no more than 10% of the LIGO displacement noise per degree of freedom for LOS1/LOS2, that is $1.0\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 40 Hz with f^{-2} dependence and no more than 10% of the required mode cleaner stability for SOS, that is $3.5\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz with $f^{-0.5}$ dependence above 100 Hz and f^{-2} **TBD** dependence below 100 Hz. Table 7 shows the requirements of the control noise per mass expressed in displacement or orientation motion. The beam spot offset of 1 mm (LOS1/LOS2) and 3 mm (SOS) are assumed for the pitch/yaw requirement.

Table 7: Requirements of the control noise per mass (f > 40 Hz).

Mode	*Control Noise*	
	LOS1/LOS2	*SOS*
Displacement	$< 5\times10^{-20}\times\left(\frac{40\mathrm{Hz}}{f}\right)^2\mathrm{m}/\sqrt{\mathrm{Hz}}$	$< 2\times10^{-19}\times\left(\frac{100\mathrm{Hz}}{f}\right)^2\mathrm{m}/\sqrt{\mathrm{Hz}}$ (f < 100 Hz) $< 2\times10^{-19}\times\left(\frac{100\mathrm{Hz}}{f}\right)^{0.5}\mathrm{m}/\sqrt{\mathrm{Hz}}$ (f > 100 Hz)
Pitch/Yaw	$< 2\times10^{-17}\times\left(\frac{40\mathrm{Hz}}{f}\right)^2\mathrm{rad}/\sqrt{\mathrm{Hz}}$	$< 6\times10^{-17}\times\left(\frac{100\mathrm{Hz}}{f}\right)^2\mathrm{rad}/\sqrt{\mathrm{Hz}}$ (f < 100 Hz) $< 6\times10^{-17}\times\left(\frac{100\mathrm{Hz}}{f}\right)^{0.5}\mathrm{rad}/\sqrt{\mathrm{Hz}}$ (f > 100 Hz)

3.2.1.2.5 Stray Light Shield

TBD

3.2.2. Physical Characteristics

3.2.2.1 Size Constraints

The size constraints for the SUS subsystem is:

- Three kinds of suspension system must accommodate the corresponding optical component, and must satisfy the corresponding condition of optical clear aperture.
- The suspension system must provide a proper vertical position for the suspended components.

The Size and the optical clear aperture of representative components and the height of the beam for three kinds of chambers are summarized in Table 8 and Table 9.

Table 8: Size and optical clear aperture of suspended components.

Physical Quantity	*LOS1*	*LOS2*	*SOS*
Diameter of Suspended Component	25 cm	25 cm **TBD**	7.62 cm
Thickness of Suspended Component	10 cm	4 cm **TBD**	2.54 cm
Weight of Suspended Component	10.7kg	6.2 kg **TBD**	0.25 kg
Required Optical Clear Aperture	24 cm (Fore) 19 cm (Back)	11 cm **TBD**[a]	2 cm
Wedge Angle of Suspended Component	**TBD**	**TBD**	**TBD**

[a] The angle of the incident beam is 45 degrees from perpendicular.

Table 9: Beam height for chambers.

Physical Quantity	*BSC Chamber*		*HAM Chamber*
	Test Mass	*Beamsplitter*	
Beam Height	**TBD**	**TBD**	**TBD**

3.2.3. Interface Definitions

See [11] LIGO-T950060-00-D: Naming Convention and Interface Definition for SUS.

3.2.4. Reliability

Mean Time Between Failures (MTBF) should be **TBD**.

3.2.5. Maintainability

Mean Time To Repair (MTTR) should be less than **TBD**.

3.2.6. Environmental Conditions

3.2.6.1 Natural Environment

3.2.6.1.1 Temperature and Humidity

TBD

3.2.6.1.2 Atmospheric Pressure

TBD

3.2.6.1.3 Seismic Disturbance

TBD

3.2.6.2 Induced Environment

3.2.6.2.1 Electromagnetic Radiation

TBD

3.2.6.2.2 Acoustic

TBD

3.2.6.2.3 Mechanical Vibration

TBD

3.2.7. Transportability

All items shall be transportable by commercial carrier without degradation in performance. As necessary, provisions shall be made for measuring and controlling environmental conditions (temperature and accelerations) during transport and handling. Special shipping containers, shipping and handling mechanical restraints, and shock isolation shall be utilized to prevent damage. All containers shall be movable for forklift. All items over 100 lbs. which must be moved into place within LIGO buildings shall have appropriate lifting eyes and mechanical strength to be lifted by cranes.

3.3. Design and Construction

3.3.1. Materials and Processes

3.3.1.1 Finishes

- Metal components must have quality finishes on all surfaces, suitable for vacuum finishes.
- All materials must have non-shedding surfaces.

3.3.1.2 Materials

All materials used inside the vacuum must comply with [3] LIGO-E960022-00-D: LIGO Vacuum Compatibility, Cleaning Methods and Procedures.

3.3.1.3 Processes

3.3.1.3.1 Welding

TBD

3.3.1.3.2 Annealing

TBD

3.3.1.3.3 Cleaning

All materials used inside the vacuum chambers must be cleaned in accordance with [3] LIGO-E960022-00-D: LIGO Vacuum Compatibility, Cleaning Methods and Procedures.

3.3.2. Component Naming

All components shall be identified using the LIGO Detector Naming Convention (document **TBD**).

3.3.3. Workmanship

TBD

3.3.4. Interchangeability

All LOS1 components must be interchangeable between LOS1 systems except for components that depend on wedges. All SOS components must be interchangeable between SOS systems except for components that depend on wedges.

3.3.5. Safety

This item shall meet all applicable NSF and other Federal safety regulations, plus those applicable State, Local and LIGO safety requirements. A hazard/risk analysis shall be conducted in accordance with guidelines set forth in section 3.3.2. of [13] LIGO-M950046-F: LIGO Project System Safety Management Plan.

No special considerations for human safety are presented by the suspension design, except for the normal precautions taken with electronics. The suspension actuator electronics will be capable of delivering voltages up to 150 Volts.

The following special precautions must be taken to assure safety of the equipment:

- the suspension hardware and the suspended component are ultrahigh-vacuum components and must be handled according to approved LIGO procedures to prevent contamination
- the magnet/standoff assemblies are extremely delicate and must be protected during all operations prior to mounting the suspended component into the suspension structure
- tools fabricated from metal or other hard substances should be kept away from the suspended component's polished faces to prevent scratching or marring of these surfaces
- suspended components must always be properly locked into the safety cage before any movement of the suspension structure is attempted

3.3.6. Human Engineering

The suspension design must allow for coarse alignment of the suspended component relative to the suspension support structure to be accomplished on a clean bench outside the vacuum chamber. This is required because the alignment procedure is quite delicate and requires that personnel aligning an optic have comfortable access to the components. Alignment procedures within the vacuum chamber will be restricted to adjustments of the orientation of the suspension support structure. Fine alignment will be accomplished through the suspension actuators.

3.4. Documentation

3.4.1. Specifications

TBD

3.4.2. Design Documents

- Suspension Final Design

3.4.3. Engineering Drawings and Associated Lists

TBD

3.4.4. Technical Manuals and Procedures

3.4.4.1 Procedures

Procedures shall be provided for, at minimum,

- Initial installation and setup of equipment
- Normal operation of equipment
- Normal and/or preventative maintenance
- Troubleshooting guide for any anticipated potential malfunctions

3.4.4.2 Manuals

TBD

3.4.5. Documentation Numbering

All documents shall be numbered and identified in accordance with the LIGO documentation control numbering system LIGO document **TBD**

3.4.6. Test Plans and Procedures

Test plan is documented in [16] LIGO-T960086-00-D: Suspension Test Plan.

All procedures shall be developed in accordance with **TBD**.

3.5. Logistics

The design shall include a list of all recommended spare parts and special test equipment required.

3.6. Precedence

This section should list the relative importance of requirements (or goals) to be achieved by the design.

3.7. Qualification

Test and acceptance criteria.

4 QUALITY ASSURANCE PROVISIONS

This section includes all of the examinations and tests to be performed in order to ascertain the product, material or process to be developed or offered for acceptance conforms to the requirements in section 3.

4.1. General

This should outline the general test and inspection philosophy, including all phases of development.

4.1.1. Responsibility for Tests

Who is responsible for testing.

4.1.2. Special Tests

4.1.2.1 Engineering Tests

List any special engineering tests which are required to be performed. Engineering tests are those which are used primarily for the purpose of acquiring data to support the design and development.

4.1.2.2 Reliability Testing

Reliability evaluation/development tests shall be conducted on items with limited reliability history that will have a significant impact upon the operational availability of the system.

4.1.3. Configuration Management

Configuration control of specifications and designs shall be in accordance with the LIGO Detector Implementation Plan.

4.2. Quality conformance inspections

Design and performance requirements identified in this specification and referenced specifications shall be verified by inspection, analysis, demonstration, similarity, test or a combination thereof per the Verification Matrix, Appendix 1 (See example in Appendix). Verification method selection shall be specified by individual specifications, and documented by appropriate test and evaluation plans and procedures. Verification of compliance to the requirements of this and subsequent specifications may be accomplished by the following methods or combination of methods:

4.2.1. Inspections

Inspection shall be used to determine conformity with requirements that are neither functional nor qualitative; for example, identification marks.

4.2.2. Analysis

Analysis may be used for determination of qualitative and quantitative properties and performance of an item by study, calculation and modeling.

4.2.3. Demonstration

Demonstration may be used for determination of qualitative properties and performance of an item and is accomplished by observation. Verification of an item by this method would be accomplished by using the item for the designated design purpose and would require no special test for final proof of performance.

4.2.4. Similarity

Similarity analysis may be used in lieu of tests when a determination can be made that an item is similar or identical in design to another item that has been previously certified to equivalent or more stringent criteria. Qualification by similarity is subject to Detector management approval.

4.2.5. Test

Test may be used for the determination of quantitative properties and performance of an item by technical means, such as, the use of external resources, such as voltmeters, recorders, and any test equipment necessary for measuring performance. Test equipment used shall be calibrated to the manufacture’s specifications and shall have a calibration sticker showing the current calibration status.

5 PREPARATION FOR DELIVERY

Packaging and marking of equipment for delivery shall be in accordance with the Packaging and Marking procedures specified herein.

5.1. Preparation

Equipment shall be appropriately prepared. For example, vacuum components shall be prepared to prevent contamination.

5.2. Packaging

Procedures for packaging shall ensure cleaning, drying, and preservation methods adequate to prevent deterioration, appropriate protective wrapping, adequate package cushioning, and proper containers. Proper protection shall be provided for shipping loads and environmental stress during transportation, hauling and storage.

5.3. Marking

Appropriate identification of the product, both on packages and shipping containers; all markings necessary for delivery and for storage, if applicable; all markings required by regulations, statutes, and common carriers; and all markings necessary for safety and safe delivery shall be provided.

APPENDIX A TRANSFER FUNCTION OF SUSPENSION

A.1. Transfer Function from Horizontal to Horizontal, T_{hh}

The requirement is self-evident.

A.2. Transfer Function from Horizontal to Pitch, T_{hp}

The requirement is obtained by demanding the resultant cavity length variation due to pitch motion with a beam spot offset d of 1 mm (LOS1) and 3 mm (SOS) to be 10% of the transfer function from horizontal to horizontal:

$$d \times T_{hp} < 0.1 \times T_{hh} . \tag{1}$$

A.3. Transfer Function from Vertical to Vertical, T_{vv}

The requirement is self evident. The resultant vertical motion contributes to the interferometer noise due to a misalignment of the optical axis with the local perpendicular to gravity at the test mass chamber, that is 3×10^{-4} for both LOS1 and SOS.

A.4. Transfer Function from Vertical to Horizontal, T_{vh}

The requirement is obtained by demanding the resultant horizontal motion to be 10% of the resultant cavity length variation due to the vertical to vertical transfer function:

$$T_{vh} < 0.1 \times (3 \times 10^{-4}) \times T_{vv} . \tag{2}$$

A.5. Transfer Function from Vertical to Pitch, T_{vp}

The requirement is obtained by demanding the resultant cavity length variation due to pitch motion with a beam spot offset d of 1 mm (LOS1) and 3 mm (SOS) to be 10% of the resultant cavity length variation due to the vertical to vertical transfer function:

$$d \times T_{vp} < 0.1 \times (3 \times 10^{-4}) \times T_{vv} . \quad (3)$$

APPENDIX B THERMAL NOISE

B.1. LOS1

B.1.1. Vibrational Thermal Noise

The vibrational thermal noise requirement for the quadrature sum of all four test masses is $8 \times 10^{-20} \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz with $f^{-0.5}$ frequency dependence.

Thermal noise due to internal vibrations of the test masses was estimated following the method used in [17] LIGO-P940003-00-R: Thermally Excited Vibrations of the Mirrors of Laser Interferometer Gravitational-Wave Detectors:

$$\tilde{x}_{IFO}(f) = 4.2 \times 10^{-20} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \cdot \left(\frac{100\ \mathrm{Hz}}{f}\right)^{1/2} \cdot \left(\frac{\varphi(f)}{10^{-7}}\right)^{1/2} \quad (4)$$

for the noise contribution to the interferometer from the four test masses, accounting for the finite spot sizes of the beams on the end and vertex masses, and summing over the appropriate vibrational modes of the test masses. Therefore the average effective loss of vibrational modes due to the sum of loss in the attachments and the intrinsic loss of the substrate must be less than 4×10^{-7}.

B.1.2. Pendulum Thermal Noise

The pendulum thermal noise requirement for the quadrature sum of the four test masses is $1 \times 10^{-19} \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz (thus $5 \times 10^{-20} \mathrm{m}/\sqrt{\mathrm{Hz}}$ per test mass) with $f^{-2.5}$ frequency dependence.

The spectral density of displacement due to the suspension fiber thermal noise is (See [18] LIGO-P940011-00-R: Suspension Losses in the Pendula of Laser Interferometer Gravitational-Wave Detectors):

$$\tilde{x}^2_{pend}(f) = \frac{4k_B T}{M} \cdot \frac{\omega_0^2 \varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4 \varphi^2(f)]} \quad (5)$$

where $k_B = 1.4 \times 10^{-23}$ J/K, T = 295 K, M = 10.7 kg, and $\omega_0 = 0.74 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. $\varphi(f)$ must be less than 7×10^{-6}.

B.1.3. Pitch and Yaw Thermal Noise

Suspension pitch/yaw thermal noise couples with a beam spot offset from the center of mass to produce cavity length variations (See [19] LIGO-P940012-00-R: Mirror-Orientation Noise in a Fabry-Perot Interferometer Gravitational Wave Detector). The displacement noise caused by the pitch/yaw thermal noise must be less than 10% of the LIGO sensitivity, that is, $1\times10^{-20}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per degree of freedom at 100 Hz (thus $5\times10^{-21}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per test mass per degree of freedom) with $f^{-2.5}$ frequency dependence. Since the requirement for the beam spot offset (ASC) is 1 mm, the required angle fluctuation is $5\times10^{-18}\mathrm{rad}/\sqrt{\mathrm{Hz}}$ per test mass per degree of freedom at 100 Hz with $f^{-2.5}$ frequency dependence.

The spectral density of orientation due to the pitch/yaw thermal noise is:

$$\tilde{\theta}^2(f) = \frac{4k_BT}{I}\cdot\frac{\omega_0^2\varphi(f)}{\omega[(\omega_0^2-\omega^2)^2-\omega_0^4\varphi^2(f)]} \tag{6}$$

where $k_B = 1.4\times10^{-23}$ J/K, T = 295 K, $I = 5.1\times10^{-2}$ kgm^2, and $\omega_0 = 0.6\times2\pi$ or $0.5\times2\pi$ are Boltzman’s constant, the temperature, the momentum of inertia, and the resonant angular frequency (for pitch and yaw), respectively. $\varphi(f)$ must be less than 5×10^{-4} for pitch and 8×10^{-4} for yaw, respectively.

B.1.4. Vertical Thermal Noise

Vertical thermal noise contributes to the interferometer noise due to a misalignment of the optical axis with the local perpendicular to gravity at the test mass chamber, that is 3×10^{-4}. This is the average misalignment for each test mass chamber, both sites, both 4km arms, and can be used for all (although some are strictly at 0 and others at 6e-4). The displacement noise caused by the vertical thermal noise must be less than 10% of the LIGO sensitivity, that is, $1\times10^{-20}\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz (thus $5\times10^{-21}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per test mass) with $f^{-2.5}$ frequency dependence. Therefore the vertical motion must be less than $1.7\times10^{-17}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per test mass) at 100 Hz with $f^{-2.5}$ frequency dependence. The spectral density of displacement noise due to the vertical thermal noise is:

$$\tilde{z}^2_{vert}(f) = \frac{4k_BT}{M}\cdot\frac{\omega_0^2\varphi(f)}{\omega[(\omega_0^2-\omega^2)^2-\omega_0^4\varphi^2(f)]} \tag{7}$$

where $k_B = 1.4\times10^{-23}$ J/K, T = 295 K, M = 10.7 kg, and $\omega_0 = 13\times2\pi$ are Boltzman’s constant, the temperature, the mass, and the resonant angular frequency, respectively. $\varphi(f)$ must be less than 3×10^{-3}.

B.1.5. Viscous Damping

Viscous thermal noise is required to be less than 20% of the LIGO sensitivity, that is

$3.2 \times 10^{-18} \mathrm{m} / \sqrt{\mathrm{Hz}}$ (thus $1.6 \times 10^{-18} \mathrm{m} / \sqrt{\mathrm{Hz}}$ per test mass) at 100 Hz with f^{-2} frequency dependence

$$\tilde{x}^2_{\mathrm{visc}}(f) = \frac{4 k_B T}{M} \cdot \frac{\omega_0^2 \varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4 \varphi^2(f)]} \tag{8}$$

where $k_B = 1.4 \times 10^{-23}$ J/K, T = 295 K, M = 10.7 kg, and $\omega_0 = 0.74 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. $\varphi(f)$ must be less than 8×10^{-7} at 100 Hz, which corresponds to 6×10^{-9} at 0.74 Hz.

B.2. SOS

Thermal noise requirement for the quadrature sum of the three mode cleaner mirrors is $3.5 \times 10^{-18} \mathrm{m} / \sqrt{\mathrm{Hz}}$ at 100 Hz with $f^{-0.5}$ frequency dependence above 100 Hz and $f^{-2.5}$ frequency dependence below 100 Hz to ensure the required frequency noise ($1 \times 10^{-4} \mathrm{Hz} / \sqrt{\mathrm{Hz}}$ at 100 Hz with $f^{-0.5}$ frequency dependence above 100 Hz and $f^{-2.5}$ frequency dependence below 100 Hz) of the light coming out of the mode cleaner.

B.2.1. Vibrational Thermal Noise

The vibrational thermal noise requirement for the quadrature sum of the three mode cleaner mirrors is 70% of the total thermal noise, that is, $2.5 \times 10^{-18} \mathrm{m} / \sqrt{\mathrm{Hz}}$ with $f^{-0.5}$ frequency dependence.

Thermal noise due to internal vibrations of the mirrors was estimated following the method used in [17] LIGO-P940003-00-R: Thermally Excited Vibrations of the Mirrors of Laser Interferometer Gravitational-Wave Detectors.

$$\tilde{x}_{\mathrm{MCM}}(f) = 2.3 \times 10^{-18} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \cdot \left(\frac{100\ \mathrm{Hz}}{f}\right)^{1/2} \cdot \left(\frac{\varphi(f)}{10^{-5}}\right)^{1/2} \tag{9}$$

for the noise contribution to the mode cleaner from the three mode-cleaner mirrors, accounting for the finite spot sizes of the beams on the flat and curved mirrors, and summing over the appropriate vibrational modes of the mirrors. Therefore the average effective Loss of vibrational modes due to attachments must be less than 1×10^{-5}.

B.2.2. Pendulum Thermal Noise

The pendulum thermal noise requirement for the quadrature sum of all three mode cleaner mirrors is less than 20% of the total thermal noise, that is $7.0 \times 10^{-19} \mathrm{m} / \sqrt{\mathrm{Hz}}$ at 100 Hz (thus $3.5 \times 10^{-19} \mathrm{m} / \sqrt{\mathrm{Hz}}$ per mirror) with $f^{-2.5}$ frequency dependence.

The spectral density of the displacement due to the pendulum thermal noise is:

$$\tilde{x}^2_{pend}(f) = \frac{4k_BT}{M} \cdot \frac{\omega_0^2\varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4\varphi^2(f)]} \qquad (10)$$

where $k_B = 1.4 \times 10^{-23}$ J/K, T = 295 K, M = 0.25 kg, and $\omega_0 = 1 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. φ(f) must be less than 5×10^{-6}.

B.2.3. Pitch and Yaw Thermal Noise

Suspension pitch/yaw thermal noise couples with the beam spot offset from the center of mass to be converted into the cavity length variations. The pitch/yaw thermal noise requirement is less than 20% (for pitch) and 10% (for yaw) of the total thermal noise, that is $7.0\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ (for pitch) and $3.5\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ (for yaw) at 100 Hz (Thus $3.5\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ (for pitch) and $1.8\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ (for yaw) per mirror) with the $f^{-2.5}$ frequency dependence.

Since the requirement for the beam spot offset (IOO) is 3 mm, the required angle fluctuation is $1.2\times10^{-16}\mathrm{rad}/\sqrt{\mathrm{Hz}}$ (for pitch) and $5.8\times10^{-17}\mathrm{rad}/\sqrt{\mathrm{Hz}}$ (for yaw) per test mass per degree of freedom at 100 Hz with the $f^{-2.5}$ frequency dependence.

The spectral density of orientation due to the pitch/yaw thermal noise is:

$$\tilde{\theta}^2(f) = \frac{4k_BT}{I} \cdot \frac{\omega_0^2\varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4\varphi^2(f)]} \qquad (11)$$

where $k_B = 1.4 \times 10^{-23}$ J/K, T = 295 K, $I = 1.0 \times 10^{-4}$ kgm^2, and $\omega_0 = 0.85 \times 2\pi$ or $0.75 \times 2\pi$ are Boltzman's constant, the temperature, the momentum of inertia, and the resonant angular frequency (for pitch and yaw), respectively. φ(f) must be less than 3×10^{-4} for pitch and 9×10^{-5} for yaw, respectively.

B.2.4. Vertical Thermal Noise

Vertical thermal noise contributes to the interferometer noise due to a misalignment of the mode cleaner mirrors from the local verticals, that is 3×10^{-4}. The vertical thermal noise requirement is less than 10% of the total thermal noise, that is, $3.5\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz (thus $1.8\times10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per mirror) with the $f^{-2.5}$ frequency dependence. Therefore the vertical motion must be less than $5.8\times10^{-16}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per mirror at 100 Hz. The spectral density of displacement due to the vertical thermal noise is:

$$\tilde{z}^2_{vert}(f) = \frac{4k_BT}{M} \cdot \frac{\omega_0^2\varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4\varphi^2(f)]} \qquad (12)$$

where $k_B = 1.4 \times 10^{-23}$ J/K, T = 295 K, M = 0.25 kg, and $\omega_0 = 16 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. φ(f) must be less than 7×10^{-2}.

B.2.5. Viscous Damping

The viscous thermal noise requirement is less than 70% of the total thermal noise, that is, $2.5\times10^{-18}\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz (thus $1.2\times10^{-18}\mathrm{m}/\sqrt{\mathrm{Hz}}$ per mirror) with f^{-2} frequency dependence

The spectral density of displacement due to the viscous thermal noise is:

$$\tilde{x}^2_{\mathrm{visc}}(f) = \frac{4k_BT}{M} \cdot \frac{\omega_0^2\varphi(f)}{\omega[(\omega_0^2 - \omega^2)^2 - \omega_0^4\varphi^2(f)]} \tag{13}$$

where $k_B = 1.4 \times 10^{-23}$ J/K, T = 295 K, M = 0.25 kg, and $\omega_0 = 1 \times 2\pi$ are Boltzman's constant, the temperature, the mass, and the resonant angular frequency, respectively. φ(f) must be less than 6×10^{-5} at 100 Hz, which corresponds to 6×10^{-7} at 1 Hz.