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Are Reaction Chains Needed for AdLIGO HAM Optics?

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1 OVERVIEW

It is possible that the HAM cavity optics for Advanced LIGO can be controlled entirely with magnetic actuators. If so, then there is a potentially substantial simplification that can be made on the GEO triple pendulum suspension as applied to AdLIGO- the removal of the reaction chain.

The reaction chain is an essential part of the GEO suspension because they require a very quiet platform from which to apply forces to their sensitive optics.

This quiet platform is also essential for AdLIGO as well. However, the displacement noise spectrum of the seismic platform is quite low, and the noise requirements for the HAM cavity optics are not so strict. Because the force exerted by a voice coil actuator is relatively forgiving of the coil/magnet spacing around the 'sweet spot', the noise force from a moving coil can be quite low if the force itself is low.

This document analyzes the issues involved.

2 CALCULATION

The AdLIGO seismic isolation platform noise requirement is $x_{sei}(f)=10^{-11}$m/root Hz at 1Hz, falling about as $f^{-1.7}$. Thus at 10Hz it is $2\text{x}10^{-13}$m/root Hz, and at 100Hz it is projected to be about $4\text{x}10^{-15}$m/root Hz. We assume that the voice coil is rigidly attached to the platform and thus has the same noise spectrum.

The mode cleaner is required to have longitudinal noise less than $3\text{x}10^{-17}$m/root Hz at 10Hz and $3\text{x}10^{-19}$m/root Hz at 100Hz. The mass of the mode cleaner mirror is 3.5kg, and the noise force F(f) on the mirror resulting in this motion is given by

$$F(f) = M\omega^2 x(f))$$

The values obtained are F= $4\text{x}10^{-13}$N/root Hz over 10-100Hz.

The voice coil is expected to exert no more than F_{max}=1 microNewton on the mode cleaner mirror. To estimate the noise force due to motion of the coil, we need to know how the force varies with distance. A Mathematica model of the LIGO OSEM supplied by Mark Barton shows that, so long as the magnet is positioned within 1mm of the 'sweet spot' of the coil, then the gradient of the force with displacement in any direction is about (dF/dx)/F=.1/mm, or 100/m.

The noise force is then simply estimated as

$$F(f) = F_{max} \times \frac{(dF/dx)}{F} \times x_{sei}(f)$$

This evaluates to $2x10^{-17}$N/root Hz at 10Hz and $4x10^{-19}$N/root Hz at 100Hz. This is at least four decades smaller than requirements over the whole bandwidth.

It is necessary to analyze the noise force on the penultimate mass as well, since it will be acted upon by an even larger force. However, its noise motion will be attenuated by the final pendulum stage. If we assume that the penultimate mass has the same mass as the mirror, then the noise force is given by the formula above, with F_{max} adjusted appropriately for the needs of the servo. The mirror noise motion will approximate that of a free mass for frequencies 10Hz and above. If we assume further that the pendulum frequency of the last stage is 1Hz, then this noise motion will be attenuated at the mode cleaner mirror by $(1Hz/f)^2$. Thus the noise motion of the mode cleaner mirror due to force fluctuations on the penultimate mass is

$$x(f) = F_{max} \times \frac{(dF/dx)}{F} \times x_{sei}(f) \times \frac{1}{m\omega^2} \times \left(\frac{1Hz}{f}\right)^2$$

This noise motion is within specs if the maximum force on the penultimate mass is 2N or less. This seems quite likely as 2N is sufficient to push the suspension roughly a centimeter at DC. This estimate is rather crude and can be refined by use of the detailed suspension model, but is unlikely to be far off.

There is a complementary approach to this calculation. One can define an ‘effective isolation’ of platform motion to the mirror through the coil by calculating how much smaller sinusoidal motion of the mirror is from that of the platform. This is

$$\frac{x_{mirror}}{x_{sei}} = F_{max} \times \frac{(dF/dx)}{F} \times \frac{1}{m\omega^2}$$

For the parameters given above, this effective isolation is about 10^{-10} at 100Hz. This is much larger than the isolation of the mirror suspension itself.

3 OTHER FACTORS

The calculation above restricted itself to the mode cleaner and to stationary noise motion within the bandwidth of 10-100Hz. It did not consider higher frequencies, but in all cases the requirements were most strongly challenged at 10Hz. Also, it did not consider that servo loops would be closed around the mirror/actuator unit. These factors make the estimates above conservative.

Narrow noise peaks are a separate issue. The platform and suspension cage are likely to have relatively large amplitude motion at a few widely separated resonance frequencies. A reaction chain would filter these out, but at the price of introducing peaks of its own. The reaction chain peaks will tend to be below the GW band like those of the suspension itself and so not an issue. The cage and platform peaks will be in the GW band but at fairly large frequencies where the noise force requirements are very easy to meet.

The calculation for the recycling mirrors must consider only the penultimate mass, since there will be no direct actuation on those mirrors. In addition, the noise requirements are at least 10x less stringent over the 10-100Hz band. This implies that a reaction chain for the recycling mirrors is also unnecessary, though a more detailed calculation should be done.

4 RECYCLING MIRROR CALCULATION

The longitudinal noise requirements for the recycling mirrors are that it be less than 4×10^{-16}m/root Hz at 10Hz and 1.5×10^{-17}m/root Hz at 100Hz. The actuation on the recycling mirrors will be only through the penultimate mass, so any force noise there will be filtered as $1/f^2$ at and above 10Hz. In the same manner as for the mode cleaner mirror we can specify the force noise requirment at the recycling mirror penultimate mass as:

$$x(f) = F_{max} \times \frac{(dF/dx)}{F} \times x_{sei}(f) \times \frac{1}{m\omega^2} \times \left(\frac{1Hz}{f}\right)^2$$

where now the mass is 10kg. Inserting values as before shows that the noise requirements are met if the maximum force on the penultimate mass is less than about 80N. This is nearly enough to lift the penultimate mass. The force required to move the penultimate mass one fringe is of the order of milliNewtons. So long as the overall force required for control does not exceed this by more than a few orders of magnitude, it appears that there are orders of noise margin, sufficient to operate the recycling mirror without a reaction chain as well.