

# Vibrational Modes of the HAM Seismic Isolation System

## (Initial LIGO SEI)

The modal calculations and visualization in this *Mathematica* notebook are based upon the Hytec mass and stiffness matrices documented in: E. Ponslet, "HAM and BSC Dynamic Models", Hytec Inc., 4/22/98, LIGO-T990128-00 (<http://www.ligo.caltech.edu/docs/T/T990128-00.pdf>)

These mode shapes are for the passive isolation stack and a 12 dof representation of the elastic support structure.

Notes:

(1) The latest version of this document (T990128) was issued later than the model files that are used (and the only ones available at this date) for this analysis. The latest document refers to additional information in the model files which are not present in the available files. The model files used were issued in conjunction with a draft release of T990128 on 4/22/1998. According to Hytec, the additional data in the model files referred to in the later version of T990128-00 do not exist.

(2) The eigenvector components for the optics table (the modal participation factors) are listed for all of the modes. In addition, for all modes that have a significant participation by the optics table, there are modal displacement plots and modal animations. One can read the notebook and display the modes dynamically with either Mathematica or the free MathReader application available from Wolfram research at:

<http://www.wolfram.com/products/mathreader/>

To animate the modes, select the bracket at the right edge of the window, with an arrow at the bottom, and then select the menu item "Cell" --> "Animate Selected Graphics" (or just type Ctrl+Y). The cell bracket with an arrow at the bottom represents a closed set of cells; Double-clicking will open and display the sequence of graphics used to create the animation of the mode.

(3) If you have Mathematica and attempt to recalculate the mode shapes, you will need the data file HAM\_SEI\_MK.txt, which is available in the same directory:

### ■ Version History

#### ■ version 1

1/30/2002 Calculation & visualization of the natural modes based on the Hytec mass and stiffness matrices. [Version 1 was done only for the BSC SEI system, but to keep the versions in synch, I will use the same version numbering for the HAM analysis.] Reads the mass and stiffness matrices from the file MK\_HAM\_SEI.txt. This file is created from the original Hytec Matlab model file (described in T980128) with the Matlab script HytecModel\_2\_Mathematica.m

## ■ version 2

3/26/2002

- a) Corrected the Euler angle convention/sequence used for visualizing the mode shapes.
- b) Added mode number and modal mass in the optics table modal participation factor table (and generated the table for all modes).
- c) Attempted to get a better diagonalization of the modal mass matrix by trying other transformations from the generalized to the standard eigenproblem, but was not successful; There modal mass matrix has some significant, complex off-diagonal terms. Perhaps there is a problem with the *Mathematica* routine?
- d) Generated a list of all modes with any optics table modal participation factor greater than a threshold; Used this list to plot mode shapes relevant to motion of the optics table.

## ■ Initialization

## ■ Test rotation & translation directions

## ■ Read Global Stiffness and Mass Matrices

## ■ Eigenproblem:

```
nDOFs = Round[km]
```

```
66
```

Check that M is positive definite (symmetric &  $m[[i,i]] > 0$ ):

```
Min[Abs[Table[M[[i, i]], {i, 1, nDOFs}]]] > 0
```

```
True
```

```
Md = Flatten[M - Transpose[M]];
```

```
Md.Table[1, {Dimensions[Md][[1]]}] == 0
```

```
True
```



Tr [M]

9009.64

modalMass = Table[Re[Mm][[i, i]], {i, 1, nDOFs}]

{229.778, 84.3494, 50.6785, 250.848, 44.7009, 238.249, 72.4951, 300.521, 238.689, 177.15, 267.201, 138.628, 17.224, 14.6379, 17.2926, 16.5642, 16.5642, 16.5642, 16.5642, 16.5642, 3.17902, 0.771208, 0.771208, 3.17902, 3.17902, 3.21829, 3.30255, 3.17736, 4.49884, 4.49884, 4.49884, 4.49759, 76.2665, 134.05, 115.152, 134.506, 3.32879, 3.15738, 2.21667, 1.9806, 1.9806, 1.9806, 1.9806, 1.9806, 6.71565, 186.731, 7.91356, 13.8448, 11.6694, 138.061, 16.8718, 17.4828, 17.4828, 17.4828, 17.4828, 3.81972, 3.81972, 3.81972, 3.82169, 287.361, 36.7695, 21.7059, 27.3945, 28.9332, 69.8496}

vecs = Reverse[vecs];

opticstableVecs = Transpose[Take[Transpose[vecs], -6]];

threshold = 0.01;

opticstableModes = 1;

Do[

  If[

    Max[Abs[opticstableVecs[[i]]] > threshold,

    opticstableModes = Flatten[{opticstableModes, i}],

  ], {i, 2, nDOFs}];

opticstableModes

{1, 2, 3, 4, 5, 6, 16, 18, 19, 20, 21, 22, 28, 29, 30, 32, 33, 34, 52, 53, 54}

nTableLength = nDOFs;

tableHead = {"freq (Hz)", "x", "y", "z", "Rx", "Ry", "Rz"};

tableLabel = Table[i, {i, nTableLength}];

opticsTableModeShapes = Transpose[Partition[

  Join[Take[freqs, nTableLength], Flatten[Transpose[Take[opticstableVecs, nTableLength]]], nTableLength]];

Print[TableForm[Chop[opticsTableModeShapes, 10^-5], TableHeadings -> {tableLabel, tableHead}]]

	freq (Hz)	x	y	z	Rx	Ry	Rz
1	1.52298	-0.241034	0	0	0	-0.358692	0
2	1.65443	0	0	0	0	0	-0.251354
3	1.82706	0	-0.172791	0	0.0962452	0	0
4	2.36539	-0.0729194	0	0	0	0.221065	0
5	3.11095	0	0.00562605	0	0.375973	0	0
6	3.18398	0	0	0.580093	0	0	0
7	6.13234	0	0	0	0	0	0.0010063
8	6.13363	0	0	0	0	0	0
9	6.13363	0	0	0	0	0	0
10	6.13363	0	0	0	0	0	0
11	6.68404	0	0	0	0	0	0

12	6.68404	0	0	0	0	0	0
13	6.68404	0	0	0	0	0	0
14	6.68404	0	0	0	0	0	0
15	6.68404	0	0	0	0	0	0
16	7.12138	-0.0393238	0	0	0	-0.0157906	0
17	7.13519	0	0	0	0	0	0
18	7.22233	0	0	0	0	0	0.0624402
19	7.26137	0	0.0445386	0	-0.0272548	0	0
20	7.48743	0.0176863	0	0	0	-0.050431	0
21	7.73173	0	0	-0.266041	0	0	0
22	7.89605	0	0.00531524	0	0.0731491	0	0
23	8.72577	0	0	0	0	0	0
24	8.72577	0	0	0	0	0	0
25	8.72577	0	0	0	0	0	0
26	8.72577	0	0	0	0	0	0
27	8.72577	0	0	0	0	0	0
28	9.10424	0	0	0	0	0	0.0258237
29	9.73015	0.0330753	0	0	0	-0.0327643	0
30	9.91126	0	0.0322562	0	0.04678	0	0
31	11.9571	0	0	0	0	0	0
32	11.9921	-0.006875	0	0	0	0.079903	0
33	12.0558	0	0	-0.101556	0	0	0
34	12.1277	0	-0.0103922	0	-0.140529	0	0
35	12.5588	0	0	0	0	0	-0.000317724
36	12.5608	0	0	0	0	0	0
37	12.5608	0	0	0	0	0	0
38	12.5608	0	0	0	0	0	0
39	12.6206	-0.00534757	0	0	0	0.00498781	0
40	12.9703	0	0.00570334	0	0.00826614	0	0
41	13.0942	0	0	0	0	0	-0.0059612
42	13.0974	0	0	0	0	0	0
43	13.0974	0	0	0	0	0	0
44	13.0974	0	0	0	0	0	0
45	13.0974	0	0	0	0	0	0
46	13.0974	0	0	0	0	0	0
47	13.7493	0	0	0	0	0	0
48	13.7493	0	0	0	0	0	0
49	13.7493	0	0	0	0	0	0
50	13.7493	0	0	0	0	0	0
51	13.7493	0	0	0	0	0	0
52	13.8265	0	0	0	0	0	0.026256
53	13.9546	0.030418	0	0	0	-0.0302272	0
54	14.0044	0	0.0334438	0	0.0476902	0	0
55	26.7777	0.0000222536	0	0	0	-0.0000388689	0
56	31.7892	0	0	-0.0000908093	0	0	0
57	32.9103	0.0000164738	0	0	0	0	0
58	54.0856	0	0	0	0	0	0
59	58.628	0	0	0	0	0	0
60	76.5252	0	0	0	0	0	0
61	99.7179	0	0	0	0	0	0

62	116.098	0	0	0	0	0	0	0
63	129.793	0	0	0	0	0	0	0
64	144.168	0	0	0	0	0	0	0
65	372.012	0	0	0	0	0	0	0
66	375.778	0	0	0	0	0	0	0

```
Min[Table[Max[Abs[vecs[[i]]]], {i, 1, nDOFs}]]
```

```
0.304202
```

#### ■ factorization of M

#### ■ factorization of K

### ■ Stack Geometry

### ■ Modal Plots

```
massdof = Drop[massdof, 1];
```

```
nElements = Dimensions[massdof][[1]]
```

```
11
```

```
nPlots = Dimensions[opticstableModes][[1]];
```

```
transdof = Flatten[Transpose[Drop[Transpose[massdof], -3]]];
```

```
rotatedof = Flatten[Transpose[Drop[Transpose[massdof], 3]]];
```

```
range = 40;
```

```
factorT = 40;
```

```
factorR = 10;
```

#### ■ Animated Mode Plots

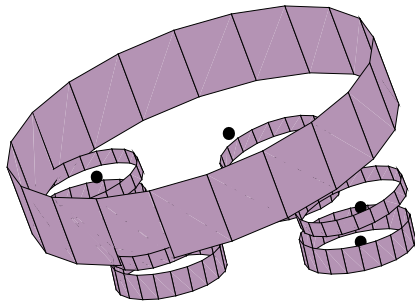
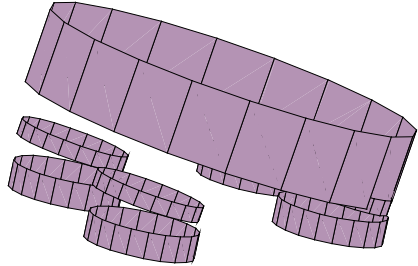
## ■ Static Mode Plots

```

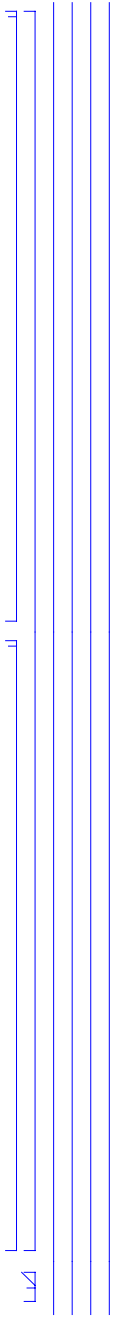
modePlot[k_, factorT_, factorR_] := Block[{j},
  transvec = Partition[Table[vecs[[k, transdof[[j]]]], {j, 1, Length[transdof]}, 3];
  rotatevec = Partition[Table[vecs[[k, rotatedof[[j]]]], {j, 1, Length[rotatedof]}, 3];
  jsin = Table[Sin[j Pi], {j, 0.5, 1.5, 1}];
  Table[
    Show[
      Table[
        TranslateShape[
          RotateShape[
            RotateShape[
              Graphics3D[Cylinder[legMassRadius, massCylinderHalfHeights[[i - 2]], 20]],
              Pi / 2, -jsin[[j]] factorR rotatevec[[i, 2]], -Pi / 2 - jsin[[j]] factorR rotatevec[[i, 3]],
              0, -jsin[[j]] factorR rotatevec[[i, 1]], 0],
            massPositions[[i - 2]] + jsin[[j]] factorT transvec[[i]],
            {i, 3, nElements - 1}],
        TranslateShape[
          RotateShape[
            RotateShape[
              Graphics3D[Cylinder[opticstableRadius, opticstableMassHeight / 2, 20]],
              Pi / 2, -jsin[[j]] factorR rotatevec[[nElements, 2]], -Pi / 2 - jsin[[j]] factorR rotatevec[[nElements, 3]],
              0, -jsin[[j]] factorR rotatevec[[nElements, 1]], 0],
            massPositions[[nElements - 2]] + jsin[[j]] factorT transvec[[nElements]],
            Graphics3D[{PointSize[0.02], pts}],
            BoxRatios → {1, 1, 1}, PlotRange → {{-range, range}, {-range, range}, {-range / 2, 3 range / 2}},
            ViewPoint → {40000, 100000, 30000}, Boxed → False],
          {j, 1, 2}];
    Print["Mode # ", k, "      ", freqs[[k]], " Hz"];
  ];

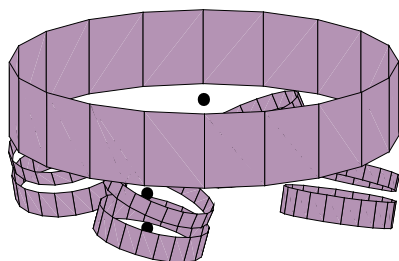
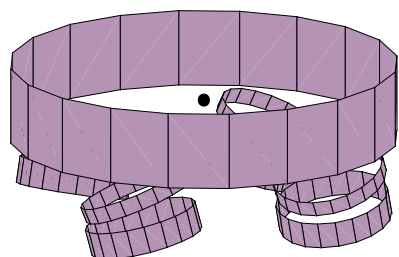
Do[modePlot[opticstableModes[[k]], 20, 1], {k, 1, nPlots}];

```

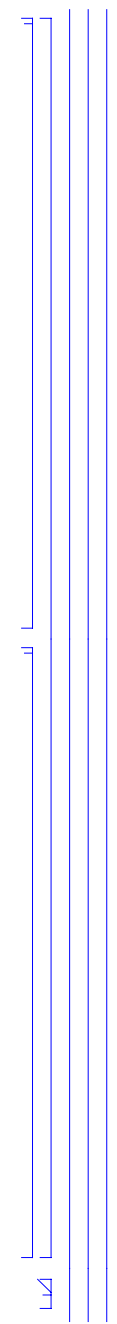


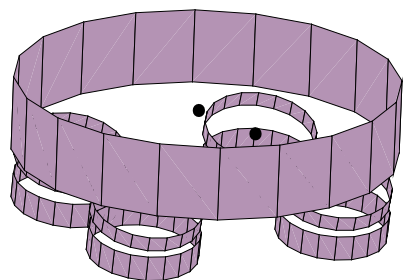
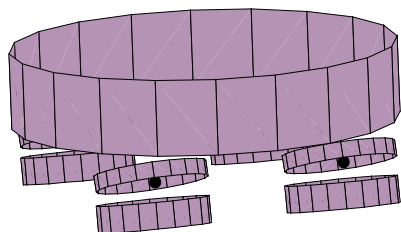
Mode # 1      1.52298 Hz



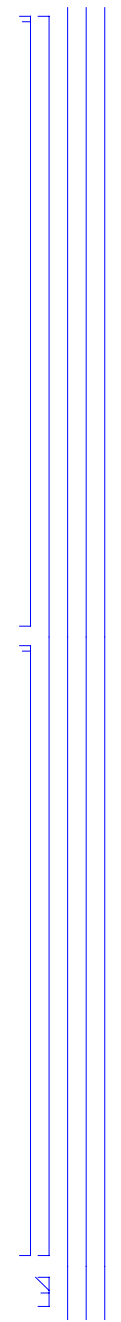


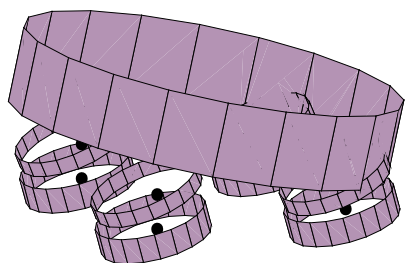
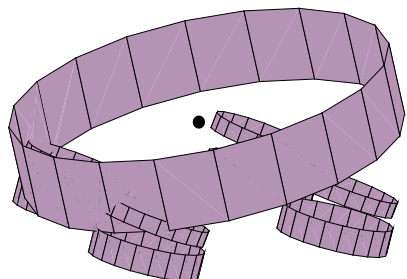
Mode # 2      1.65443 Hz



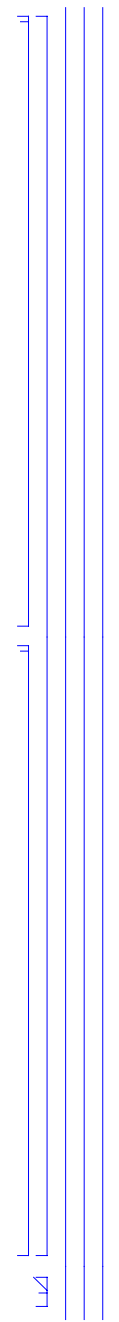


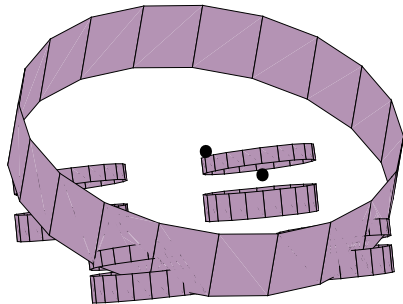
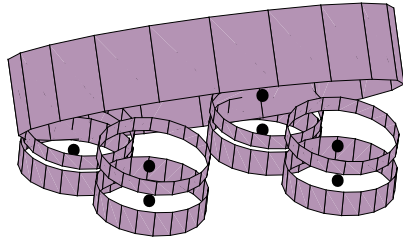
Mode # 3      1.82706 Hz



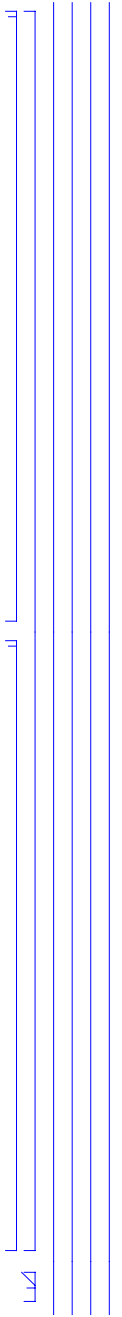


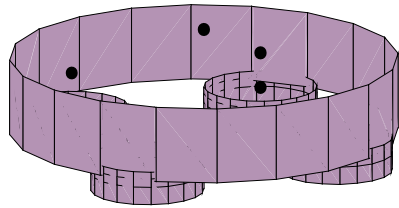
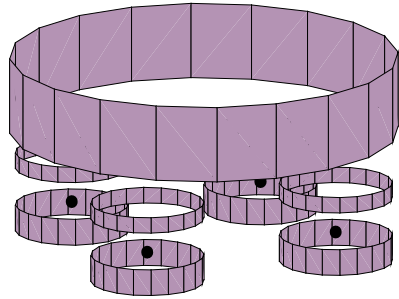
Mode # 4      2.36539 Hz



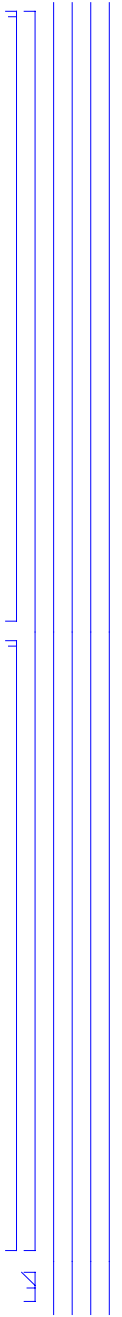


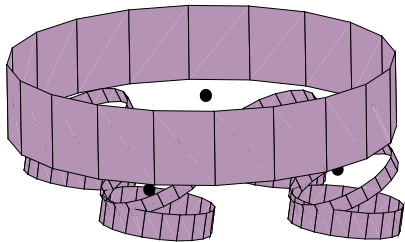
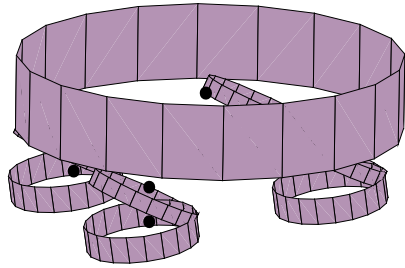
Mode # 5      3.11095 Hz



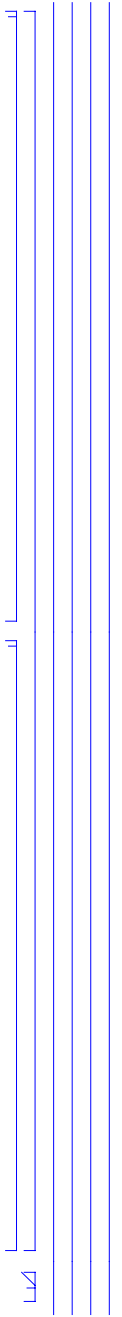


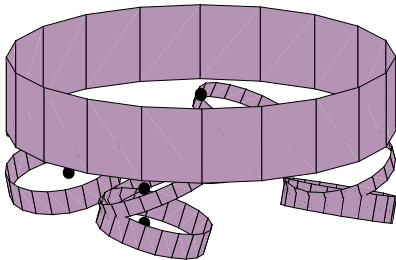
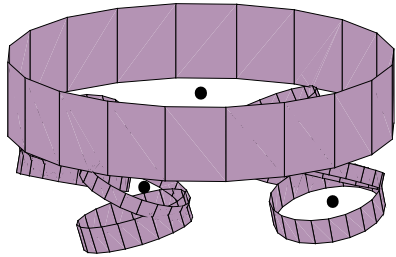
Mode # 6      3.18398 Hz



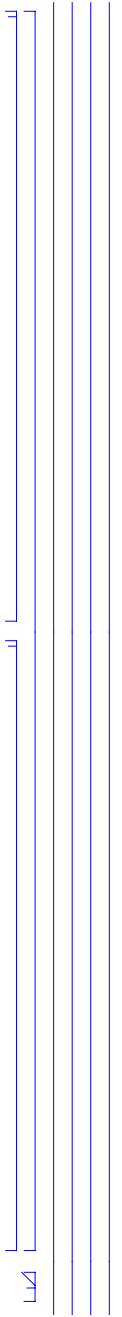


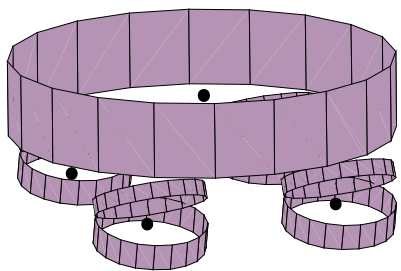
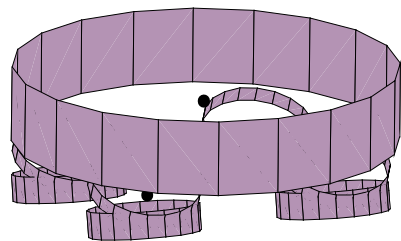
Mode # 16      7.12138 Hz



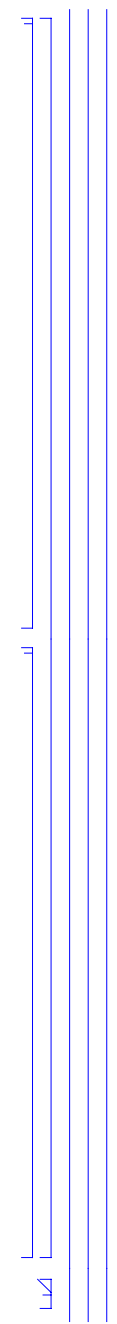


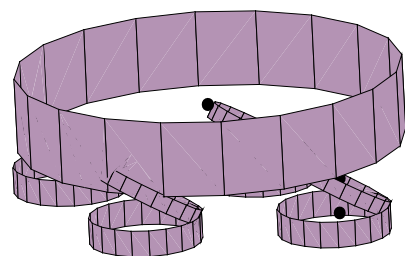
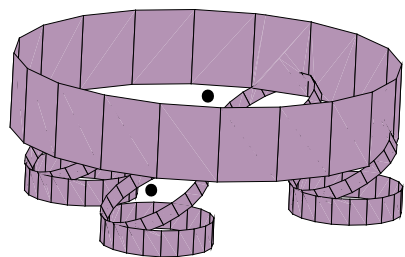
Mode # 18      7.22233 Hz



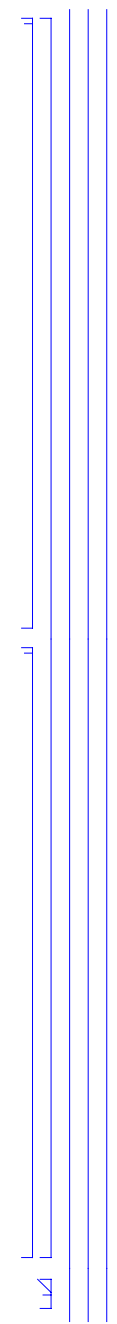


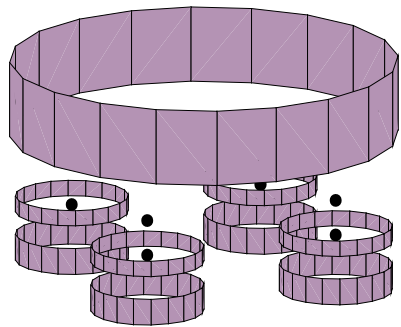
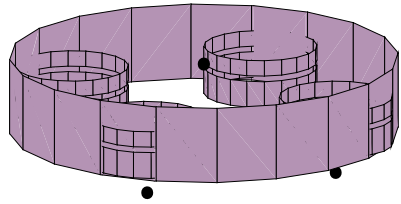
Mode # 19      7.26137 Hz



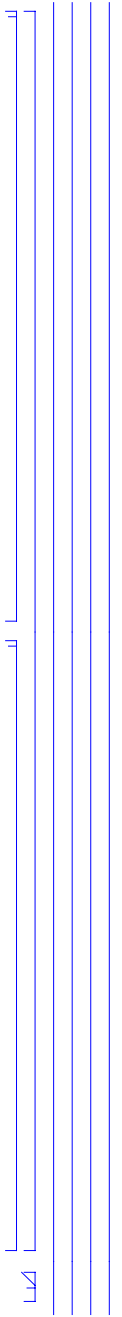


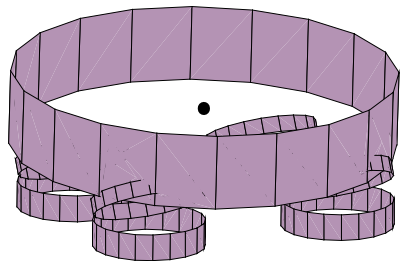
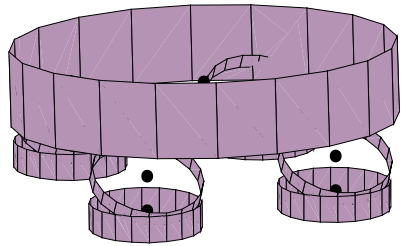
Mode # 20      7.48743 Hz



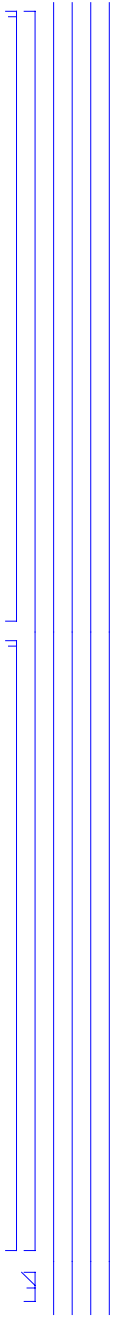


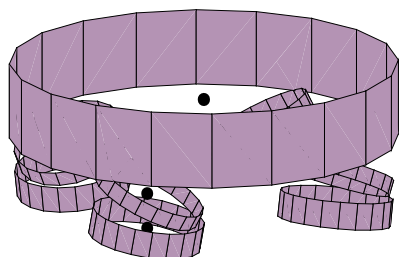
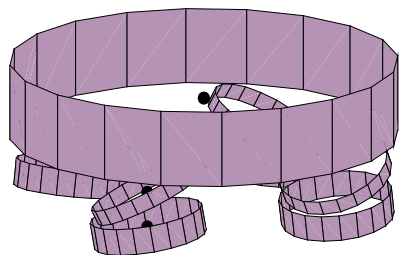
Mode # 21      7.73173 Hz



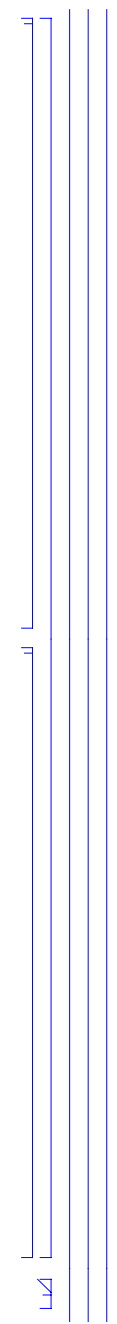


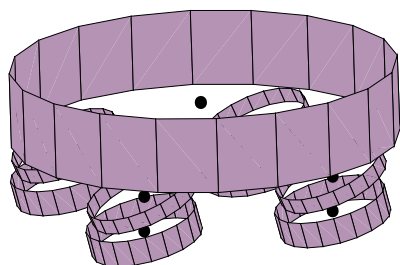
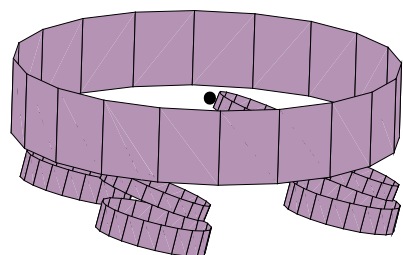
Mode # 22      7.89605 Hz



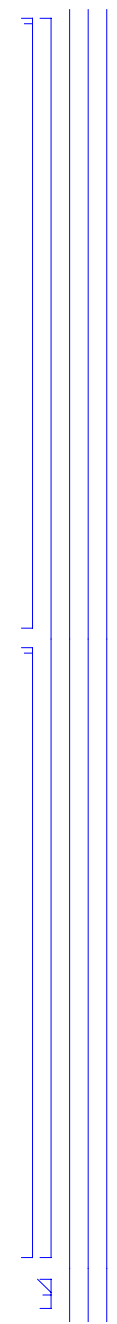


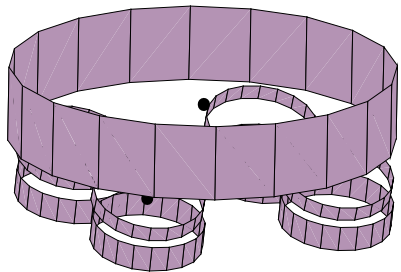
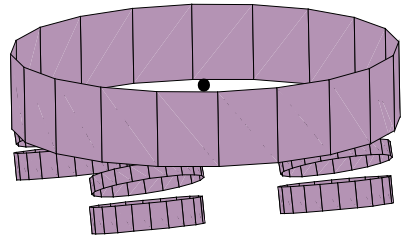
Mode # 28      9.10424 Hz



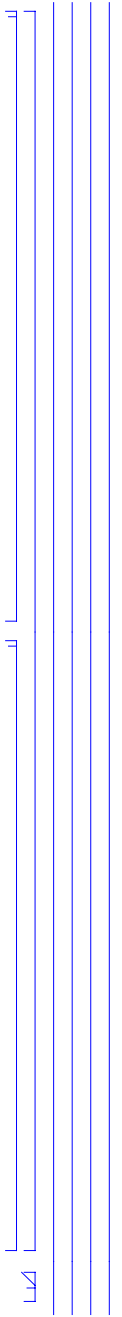


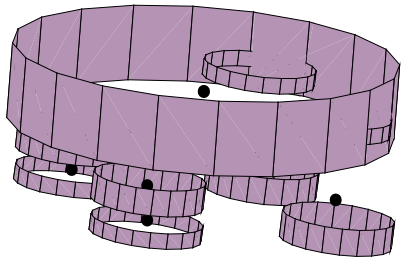
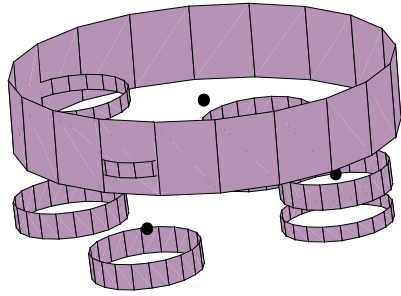
Mode # 29      9.73015 Hz



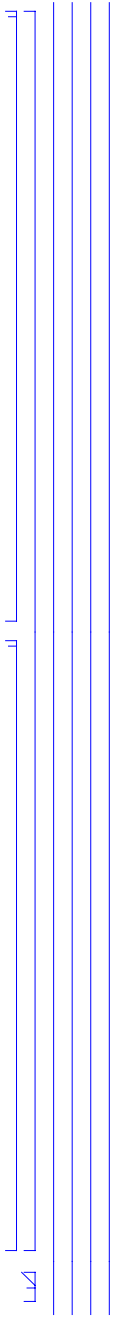


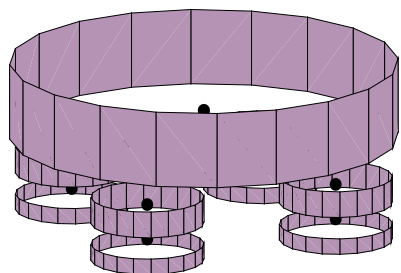
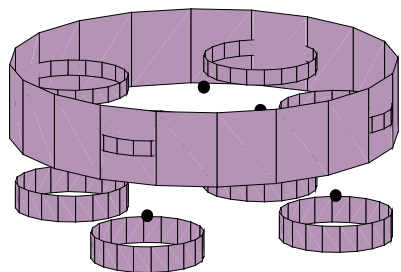
Mode # 30      9.91126 Hz



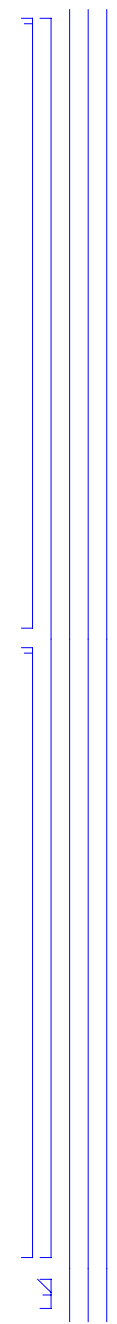


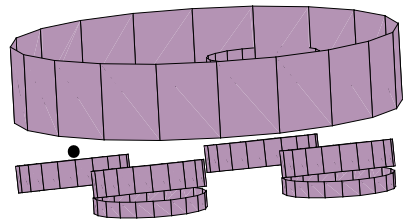
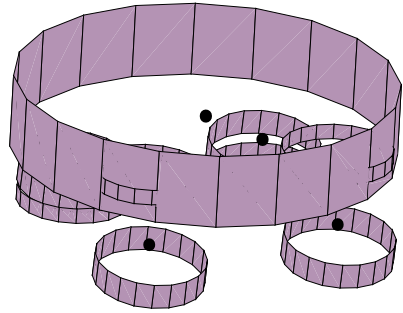
Mode # 32      11.9921 Hz



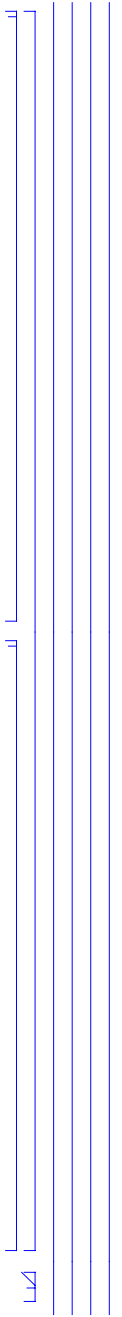


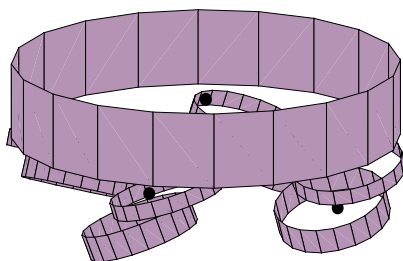
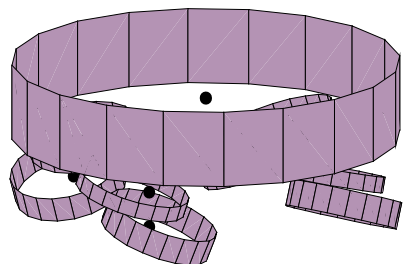
Mode # 33 12.0558 Hz



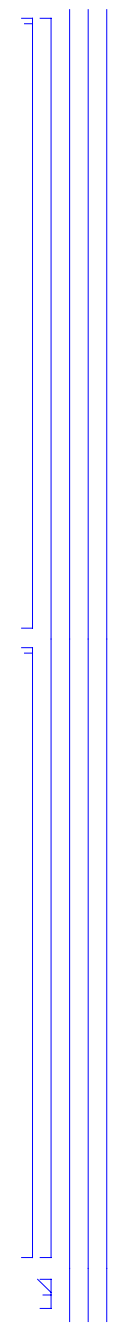


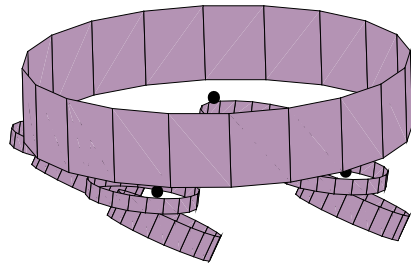
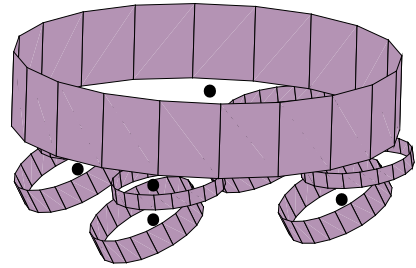
Mode # 34      12.1277 Hz



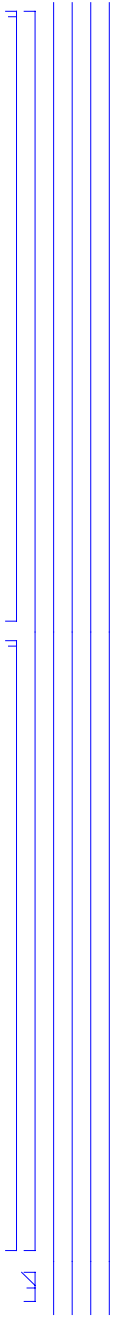


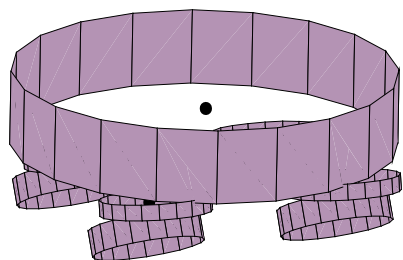
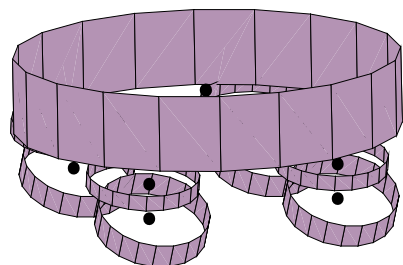
Mode # 52      13.8265 Hz





Mode # 53      13.9546 Hz





Mode # 54      14.0044 Hz

