MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Laser Interferometer Gravitational Wave Observatory (LIGO) Project

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Subject: **Cavity length noise due to beam heating in LIGO II (sapphire)**

LIGO II designers-

At the recent LSC meeting Jordan Camp and Marty Fejer discussed a potential noise effect due to fluctuating thermal expansion driven by intensity noise on the “thermal figure correction” laser proposed for LIGO II. This is indeed a significant problem which I hadn’t thought of before. We haven’t tried to re-optimize with respect to this phenomenon, so I can’t give a verdict just yet . To be frank, it seems unlikely that our collective hope to correct arbitrary, spatially inhomogenious figure errors can be realized. So, core optics will probably need to be even better (and especially axisymmetric).

In any case, kicking it around, Ryan Lawrence, Peter Fritschel and I became concerned that the main laser beam itself might do similar mischief at a non-negligible level. It turns out this is essentially the second effect (“photo-thermal shot noise”) considered by Braginsky et al. in their paper discussing the thermoelastic problem which made the headlines. They estimated this other thing it to be about half the quantum limit, and set it aside. It looks to me like they may have underestimated the problem by neglecting the bulk absorption (which, at the current accepted bulk loss value of 40 ppm/cm, becomes comparable to surface absorption) and technical intensity noise (they calculate only the power fluctuation due to shot noise). Another effect, the fluctuating phase noise due to transmission through the heated substrate, was also left out but seems to be less important.

Fundamentally, any AC component of the thermal input drives an AC component in thermal index variation (which makes phase noise) and thermal expansion (which moves the surface). The mirror is a sluggish bolometer, but the time constants are finite, so some fluctuation remains at GW frequencies.

I haven’t repeated the Braginsky analysis, as it is very difficult to generalize to a spatially distributed heat input (bulk loss). For now I just took 2 simplifying limits to arrive at bounding time constants and displacements for the problem.

The simplest is the thermal expansion and radiative cooling of the whole mirror, taken as though it were in internal equilibrium. The radiative time constant is given by

$$\tau_R = \frac{MC}{4\eta A\sigma T_0^3} \sim 18,000\ \text{s}\ \ (5\ \text{h})$$

for 30 kg sapphire, 10 cm thick. Here M is the mass, C the specific heat, A the surface area, η the emissivity (assume $\eta \sim 1$), σ is the Stefan-Boltzmann constant, and T_0 is the temperature. It's independent of the conductivity because one assumes internal equilibrium, so it's sort of a lower bound to the actual relaxation time; but for sapphire it's pretty close.

Setting deposited power equal to radiated power gives the radiation-limited total temperature rise of the optic,

$$\Delta T_R = \frac{P_0(L_B h + 2\boldsymbol{F} L_S/\pi)}{4\eta A\sigma T_0^3} \sim 3.2\ \text{K (bulk)} + 1.0\ \text{K (surface)} = 4.2\ \text{K (total)}.$$

Here P_0 is the circulating power in the substrate (assume 10 kW), L_B is the bulk loss (assume 40 ppm/cm based on best measurement to date), L_S is the surface loss (take 1 ppm), h is the mirror thickness (10 cm), and $\boldsymbol{F}$ is the cavity finesse (take 200). The central temperature is only a few tenths of a degree above the mean (see below), so the sum of thermal expansion and index effects gives

$$\tilde{x}_R(f) \approx \frac{\tilde{P}_{abs}(f)}{P_{abs}} \cdot \frac{1}{2\pi f\tau_R} \cdot \Delta T_R \cdot \left(\frac{\alpha h}{2} + \frac{\pi h}{2\boldsymbol{F}}\frac{dn}{dT}\right)$$

where $\tilde{P}_{abs}(f)/P_{abs}$ is the spectral density of fractional fluctuations in absorbed power and α is the material expansion coefficient. The index effect in the second term turns out to be only about 10% of the expansion effect for $\boldsymbol{F} = 200$, so the measured displacement is mostly thermal expansion.Taking the shot noise in the absorbed power (the best one can do!), and the book values for sapphire properties, then gives what I would call a lower limit ,

$$\tilde{x}_R(f) \approx 3 \cdot 10^{-23} \frac{\text{m}}{\sqrt{\text{Hz}}} \cdot \left(\frac{100\ \text{Hz}}{f}\right) \cdot \left(\frac{\tilde{P}_{abs}(f)/P_{abs}}{2 \cdot 10^{-10}\ /\sqrt{\text{Hz}}}\right)$$

for *one* test mass.

The second case we consider is the differential heating of a smaller volume within the solid. This is the case applicable for thermal lensing, since only the gradients produce figure errors. Following Winkler et al., take a half-sphere (for the surface) or a cylinder (bulk) of radius equal to the beam waist ω. Deposition of heat principally within this volume leads to a temperature gradient over a spot radius . The central temperature rise with respect to the spot edge is approximately

$$\Delta T_{\omega} = \frac{P_0}{2\pi\kappa}\left(L_B + \frac{2\boldsymbol{F}L_S}{\pi\omega}\right) \sim 0.29\text{ K}$$

where κ is the thermal conductivity (46 W/m/K) and elsewhere we take the same values as above. The effective mass of the heated zone inside radius ω is of order $\rho \cdot 2\pi\omega^3/3$ where ρ is the density; the excess thermal energy stored in this zone divided by the absorbed power gives an approximate “local” thermal time constant

$$\tau_{\omega} \approx \frac{\rho\omega^2 C}{3\kappa} \sim 25\text{ s.}$$

which is much shorter than the full optic’s radiative equilibration time. This corresponds to the time it takes for the thermal lens to “set up”, which we know experimentally can be very fast for a small beam. We then estimate the surface displacement and index effects the same way,

$$\tilde{x}_{\omega}(f) \approx \frac{\tilde{P}_{abs}(f)}{P_{abs}} \cdot \frac{1}{2\pi f\tau_{\omega}} \cdot \Delta T_{\omega} \cdot \left(\frac{\alpha\omega}{2} + \frac{\pi h}{2\boldsymbol{F}}\frac{dn}{dT}\right).$$

Here we took the effective thermal expansion to be only $\alpha\omega/2$ rather than $\alpha h/2$ (as before the index effect is small). This roughly accounts for the shear constraint on axial expansion everywhere except near the surface. With the same assumptions taken above, this evaluates to

$$\tilde{x}_{\omega}(f) \approx 5 \cdot 10^{-22}\frac{\text{m}}{\sqrt{\text{Hz}}} \cdot \left(\frac{100\text{ Hz}}{f}\right) \cdot \left(\frac{\tilde{P}_{abs}(f)/P_{abs}}{2 \cdot 10^{-10}/\sqrt{\text{Hz}}}\right)$$

for one ITM. This looks to be the more serious mechanism, as at this point it falls just a decade below the LIGO II target displacement sensitivity (about $8 \cdot 10^{-21}\text{ m}/\sqrt{\text{Hz}}$ at 100 Hz).

If the fluctuation source is indeed shot noise in the dissipated power, the total effect for two ITM’s would be $\sqrt{2}$ higher. Each ETM will contribute about half as much noise power as an ITM (no bulk heating, but about the same surface heating). Given the various approximations used, the safety margin isn't comforting.

Technical noise on the laser amplitude well above the shot noise is essentially guaranteed. Here there are two factors in our favor. The recycling cavity storage time introduces another pole (at ~ 2 Hz) in the transfer function from laser intensity to internal circulating power. This is good for a factor of 50 at 100 Hz, but only 5 at 10 Hz. Second, there will be some cancellation between the arms since laser noise is common-mode. With careful matching of the two ITM boules and good beamsplitter & mirror coatings (to equalize the absorbed power in both ITM's) this could be 5%. Thus the incident power fluctuation spectral density could go as high as $2 \cdot 10^{-7} / \sqrt{\mathrm{Hz}}$ at the input, which is close to the required level required of the LIGO II laser (based on the proposed RMS dark-fringe error of 10^{-14} m).

However, there are at least three caveats:

1. The second pole due to the cavity storage means the effect rises as f^{-2} toward lower frequencies even for "white" intensity noise, whereas the target displacement spectrum rises more gently. As a result, technical intensity noise at the current specification level might violate the target noise budget between 10 Hz and 100 Hz. Even reaching $2 \cdot 10^{-7} / \sqrt{\mathrm{Hz}}$ at 10 Hz may be technically infeasible. (Note that the RMS fringe offset coupling, which set the AM spec up to this point, is ~ frequency-independent, and thus more forgiving of laser intensity noise at low frequencies).
2. If we choose a modulation scheme which resonates sidebands in the (low-finesse) front cavity, as in LIGO I, there is effectively no useful filtering of laser intensity noise on the circulating sidebands. This would only interact with the bulk loss, as the sidebands stay out of the arm cavities, but depending on the sideband power chosen (in turn depending on contrast defect, lensing, ...) the sideband contribution could significantly exceed that of the carrier.
3. There is evidence to suggest the true effective thermal time constant of the mirror surface over the spot size is significantly shorter than quoted above. The magnitude and frequency response for thermal expansion of the surface due to local bulk heating is hard to calculate .

Ryan is now using his finite-element thermal lensing code to try and a) recover Braginsky's surface result and b) come up with an accurate bulk-heating thermal expansion model.

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