

Demodulation of Intensity and Shot Noise in Optical Heterodyne Detection ¹

Malik Rakhmanov

LIGO Project
California Institute of Technology
Pasadena, CA 91125

Abstract

Demodulation of an arbitrary intensity noise in the optical heterodyne detection is analyzed. General formulae for the correlation function and the power spectral density of the demodulated intensity noise are derived. The analysis takes into account a band-pass filter at the photodiode and an arbitrary demodulation wave form. Shot noise is analyzed as a special case of intensity noise, and a general formula for the power spectral density of the demodulated nonstationary shot noise is derived. For the shot-noise-limited detection the signal-to-noise ratio is found in terms of Fourier amplitudes of the modulated photocurrent. The analysis is developed for applications to the emerging interferometric gravitational wave detectors.

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1 Introduction

The heterodyne technique is often used in high sensitivity measurements of the optical phase. The technique is based on the coherent detection of the RF-modulated intensity which results from superposition of beams with different frequencies. The RF-modulation shifts the frequency of the optical signal into the region where photodetection is quiet, and at best, is limited by the shot noise. The output of the photodetector, which is the RF-modulated photocurrent, is then rectified by the mixer to obtain a low frequency signal.

Much attention to the shot noise in the optical heterodyne detection has been given in the field of the laser interferometry for gravitational wave detection. This is because the sensitivity of these interferometers will ultimately be limited by the shot noise only. Due to the presence of the RF-modulation such shot noise is inherently nonstationary. The first account of the nonstationary shot noise in the prototype gravitational wave detectors was given by Niebauer et al [1]. They found that the nonstationary shot noise has frequency correlations, which may increase or reduce the power spectral density of the demodulated noise. At the same time, Meers and Strain [2] analyzed how the shot noise is generated in various modulation schemes. In the experiments performed by Mio and Tsubono [3], and by Gray et al [4] the frequency correlations of the nonstationary shot noise were studied indirectly through the phase dependence of the shot noise power spectral density. New calculations of the shot noise were developed by Lyons et al [5] in connection with the research on the 40m prototype interferometer at Caltech. Recently Winzer [6] discussed the effects of arbitrary filtering at the photodiode and showed that the variance of the nonstationary shot noise depends on the transfer function of the photodiode. However, his analysis did not address demodulation of the nonstationary shot noise.

Calculations of the nonstationary shot noise in the prototype interferometers usually do not take into account the effects of the band-pass filtering of the RF-photocurrent. Often the sine-wave demodulation is assumed instead of a more realistic square-wave demodulation. Moreover, the effects of the high or-

der harmonics of the RF-modulated photocurrent are usually neglected. Such a simplified approach can be sufficient for approximate calculations of the shot-noise. However, for accurate predictions and computer modeling of the shot-noise a rigorous approach is needed. Such a rigorous analysis is presented in this paper, which describes demodulation of an arbitrary intensity noise in a general optical heterodyne detector. The effects of the band-pass filtering of the RF-photocurrent are included, and the demodulation with an arbitrary wave-form is described. The higher order harmonics of the RF-photocurrent are taken into account and their effect on the noise power spectral density is analyzed. The main results of this paper are the general formulae for the correlation function and the power spectral density of an arbitrary intensity noise after demodulation. These formulae are used to describe the demodulation of the nonstationary shot noise. In particular, the phase dependence of the signal-to-noise ratio is analyzed, and the optimal demodulation phase is found in terms of the Fourier components of the photocurrent.

Although the calculations in this paper are developed for applications for interferometric gravitational wave detectors, the approach is general and can be used to analyze an arbitrary optical heterodyne detector.

2 Signal Extraction

2.1 Photodetection of Modulated Intensity

An optical heterodyne detection scheme which is adopted for the interferometric gravitational wave detectors is the Pound - Drever signal extraction [7]. It is shown schematically in Fig. 1 and can be briefly described as follows.

A single - frequency laser beam is phase-modulated by an electro- optic modulator (EOM), which takes its input from an RF-oscillator with frequency Ω . The light at the carrier frequency resonates in the cavity, acquires a round-trip phase shift $\phi(t)$ and leaks back to the laser. The light at the sideband frequencies is promptly reflected by the front mir-

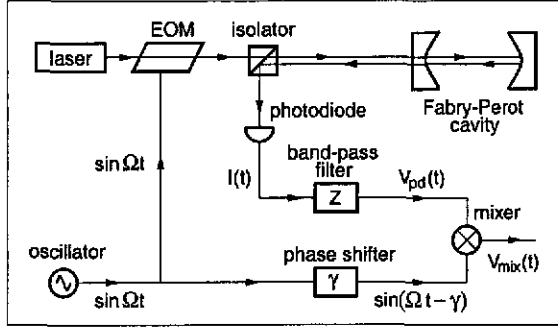


Figure 1: Signal extraction in the Pound-Drever locking scheme.

ror of the cavity. A superposition of the reflected light at the carrier and the sideband frequencies results in beats at the modulation frequency, which are detected by the photodiode. The output of the photodiode is then rectified by the mixer. Important elements of this signal extraction scheme are the phase shifter and the band-pass filter; their functions are described in detail in this paper. Essentially the same signal extraction scheme is utilized in a power recycling Michelson interferometer with Fabry-Perot arm cavities - an optical configuration currently adopted in several laser gravitational wave detectors [8], [9], [10].

The modulated intensity is usually described by its expansion in terms of harmonics of the modulation frequency:

$$P(t) = \sum_{n=-\infty}^{\infty} P_n e^{i\Omega_n t}, \quad (1)$$

where P_n are complex coefficients and Ω_n is the frequency of the n th harmonics:

$$\Omega_n = n\Omega. \quad (2)$$

Since the intensity is a real quantity the coefficients satisfy the condition:

$$P_{-n} = P_n^*, \quad (3)$$

where an asterisk * stands for complex conjugation.

The photodiode generates the electric current, $I(t)$, which is proportional to the intensity:

$$I(t) = \chi P(t), \quad (4)$$

where χ is the photodiode responsivity (typically about 0.5 – 0.7 A/W). If the intensity is modulated so is the photocurrent:

$$I(t) = \sum_{n=-\infty}^{\infty} I_n e^{i\Omega_n t}, \quad (5)$$

where $I_n = \chi P_n$.

Assume that the photodetection and the subsequent demodulation take place within a finite time T (observation time), which is much greater than the modulation period. Finite duration time-domain data can be analyzed using the finite-time Fourier transformation:

$$\bar{I}(\omega) = \int_{-T/2}^{T/2} I(t) e^{-i\omega t} dt. \quad (6)$$

In this approach the δ -function is defined as

$$2\pi\delta_T(\omega) = \int_{-T/2}^{T/2} e^{-i\omega t} dt \quad (7)$$

and therefore is finite: $2\pi\delta_T(0) = T$.

The finite-time Fourier transform of the photocurrent can be found as

$$\bar{I}(\omega) = 2\pi \sum_n I_n \delta_T(\omega - \Omega_n). \quad (8)$$

In general the coefficients P_n and also I_n are not constants but are slowly varying functions of time; that is, they change little during one modulation period. For example, the amplitude of the first harmonics of the modulated photocurrent, which is proportional to the error signal:

$$I_1(t) \propto \phi(t), \quad (9)$$

can be a time-dependent quantity. Therefore, in general, the finite-time Fourier transform of the photocurrent is given by

$$\bar{I}(\omega) = \sum_n \bar{I}_n(\omega - \Omega_n), \quad (10)$$

where $\tilde{I}_n(\omega)$ are the finite-time Fourier transforms of the coefficients $I_n(t)$. Since the coefficients $I_n(t)$ are slowly varying functions of time the Fourier transform of the photocurrent consists of narrow peaks at multiples of the modulation frequency. In the vicinity of Ω_n , the Fourier transform of the photocurrent is dominated by the n th harmonics:

$$\tilde{I}(\omega - \Omega_n) \approx \tilde{I}_n(\omega), \quad (11)$$

provided the frequency ω is sufficiently small:

$$|\omega| \ll \Omega. \quad (12)$$

2.2 Band-Pass Filtering of RF-Signal

Only the first harmonic of the photocurrent I_1 contributes to signal. The higher order harmonics I_2, I_3, \dots , as well as the dc-current I_0 contribute to noise. These harmonics can be suppressed by a band-pass filter shown in Figure 1, which often is built in the photodiode's pre-amplifier. Such a band-pass filter can be realized by a resonant LC-circuit with the resonance frequency close to the modulation frequency, Ω . An example of the transfer function of such a resonant circuit is shown in Fig. 2. The peak of the transfer function corresponds to the modulation frequency, and its width, $\Delta\Omega$, corresponds to the pass band of the filter.

Let $\tilde{Z}(\omega)$ be the impedance of the resonant circuit and $\tilde{V}_{pd}(\omega)$ be its voltage output. Then the result of the band-pass filtering is described by the formula:

$$\tilde{V}_{pd}(\omega) = \tilde{Z}(\omega)\tilde{I}(\omega). \quad (13)$$

The complex impedance satisfies the usual condition, $\tilde{Z}(-\omega) = \tilde{Z}(\omega)^*$, which guarantees that $V_{pd}(t)$ is a real quantity.

The magnitude of the signal is mostly defined by the value of the impedance at the peak:

$$\tilde{Z}(\Omega) = R e^{-i\beta}, \quad (14)$$

where R is the magnitude and β is the phase lag of the impedance at the resonance frequency. (The impedance shown in Figure 2 has the phase lag at the peak of approximately 4π .)

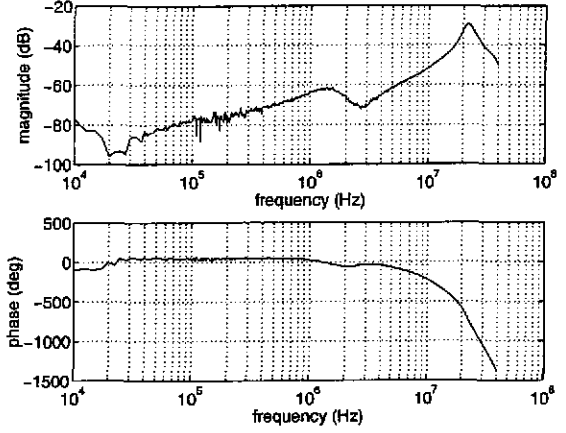


Figure 2: The transfer function of a typical LIGO RF-photodiode. The peak corresponds to the modulation frequency of approximately 22 MHz.

To avoid distortions of the signal, the width of the peak must be much greater than the band-width of the signal:

$$|\omega| \ll \Delta\Omega. \quad (15)$$

Therefore, for typical frequencies of the signal, the impedance can be approximated as constant:

$$\tilde{Z}(\Omega \pm \omega) \approx \tilde{Z}(\Omega). \quad (16)$$

When it comes to noise, no such approximation is made because the frequency of noise can be arbitrary.

2.3 Demodulation and Low-Pass Filtering

The filtered RF-signal is rectified by the mixer shown in Figure 1. The mixer output in time domain is given by the product:

$$V_{mix}(t) = D(t)V_{pd}(t), \quad (17)$$

where $D(t)$ is the demodulation wave. The mixer output in frequency domain is given by the convolution:

$$\tilde{V}_{mix}(\omega) = \int_{-\infty}^{\infty} \tilde{D}(\omega - \omega') \tilde{V}_{pd}(\omega') \frac{d\omega'}{2\pi}. \quad (18)$$

In the optical heterodyne detection the signal $\tilde{V}(\omega)$ is the low frequency component of the mixer output:

$$\tilde{V}(\omega) = \tilde{V}_{\text{mix}}(\omega) \Big|_{|\omega| \ll \Delta\Omega}. \quad (19)$$

In calculations the demodulation wave-form is often assumed to be sinusoidal:

$$D(t) = \sin(\Omega t - \gamma), \quad (20)$$

where γ is an adjustable phase set by the phase shifter, shown in Fig. 1. Such a choice of the demodulation wave-form greatly simplifies the calculations but is not usually realized in electronics.

The demodulation wave-form of a typical mixer (full-wave diode bridge) is a square wave with the amplitude very close to 1. Such a square-wave can be described by its Fourier series:

$$D(t) = \frac{4}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \sin n(\Omega t - \gamma). \quad (21)$$

To make the analysis general we assume that the demodulation wave is arbitrary:

$$D(t) = -i \sum_{n=-\infty}^{\infty} d_n e^{in(\Omega t - \gamma)}, \quad (22)$$

where d_n are arbitrary coefficients. Without loss of generality we can assume that d_1 is real and therefore

$$d_{-1} = -d_1. \quad (23)$$

The square wave demodulation can be obtained from the general form, eq.(22), by setting

$$d_n = \begin{cases} 0, & n = \text{even}, \\ 2/(\pi n), & n = \text{odd}. \end{cases} \quad (24)$$

The finite-time Fourier transform of the demodulation wave is

$$\tilde{D}(\omega) = -2\pi i \sum_n d_n e^{-in\gamma} \delta_T(\omega - \Omega_n). \quad (25)$$

Then the mixer output, eq.(18), becomes a series:

$$\tilde{V}_{\text{mix}}(\omega) = -i \sum_n d_n e^{-in\gamma} \tilde{Z}(\omega - \Omega_n) \tilde{I}(\omega - \Omega_n). \quad (26)$$

In calculating the integral, eq.(18), the finite-time δ -function was treated as the usual (Dirac) δ -function. This approximation is valid as long as the observation time is much greater than the modulation period, which is usually the case for the RF-modulation.

Explicit form of the signal can be obtained from eq.(26) by taking the low frequency limit, eq. (11) - (12). Such a signal is expressed in terms of the harmonics of the RF-photocurrent:

$$\tilde{V}(\omega) \approx -i \sum_n d_n e^{-in\gamma} \tilde{Z}(\omega - \Omega_n) \tilde{I}_{-n}(\omega). \quad (27)$$

Only the first ($n = \pm 1$) harmonics carries information about the signal, eq.(9). All other harmonics introduce an error in the signal extraction and therefore must be suppressed. The suppression can only be done before the mixer, and is the main function of the band-pass filter. To strongly suppress the high order harmonics of the RF-photocurrent, the filter must have a steep roll-off at high frequencies:

$$|\tilde{Z}(\omega - \Omega_n)| \ll R, \quad (28)$$

for $n = 2, 3, \dots$. In this case the signal becomes proportional to the first harmonics of the photocurrent:

$$\tilde{V}(\omega) \approx \begin{aligned} & id_1 e^{i\gamma} \tilde{Z}(\omega + \Omega) \tilde{I}_1(\omega) - \\ & id_1 e^{-i\gamma} \tilde{Z}(\omega - \Omega) \tilde{I}_{-1}(\omega). \end{aligned} \quad (29)$$

Note that such a signal is largely independent of the form of the demodulation wave. (The coefficient d_1 only sets the overall scale of the signal but does not affect its functional form.) This is a consequence of the narrow-band filtering.

The formula for the signal eq. (29) can be further simplified if we assume that the frequencies of interest are well within the band-width of the filter eq. (15). For these low frequencies the impedance can be approximated as constant, eq. (16), and the signal becomes

$$\tilde{V}(\omega) = \begin{aligned} & id_1 e^{i(\gamma-\beta)} R \tilde{I}_1(\omega) - \\ & id_1 e^{-i(\gamma-\beta)} R \tilde{I}_{-1}(\omega). \end{aligned} \quad (30)$$

The corresponding equation in time domain is

$$V(t) = -2d_1 R \text{Im} \left\{ e^{i(\gamma-\beta)} I_1(t) \right\}. \quad (31)$$

Often the initial phase of the modulation can be chosen so that $I_1(t)$ is a real function [11]. In this case

$$V(t) = -2d_1 R I_1(t) \sin(\gamma - \beta), \quad (32)$$

$$\tilde{V}(\omega) = -2d_1 R \tilde{I}_1(\omega) \sin(\gamma - \beta) \quad (33)$$

An optimal demodulation corresponds to the maximum of the absolute value of the signal and takes place if

$$\gamma - \beta = \frac{\pi}{2} + \pi N, \quad (34)$$

where N is integer. This condition can be met by adjusting the demodulation phase, γ , at the phase-shifter shown in Figure 1. Such an optimization is useful only if the signal is much greater than the noise. Otherwise optimal demodulation is achieved by maximizing the signal-to-noise ratio as described below.

3 Intensity and Shot Noise

3.1 Demodulation of Intensity Noise

A common noise source which affects measurements based on the photodetection of the laser intensity is the intensity noise. The demodulation of the intensity noise is similar to the demodulation of the signal. An arbitrary intensity noise can be described by the fluctuations of power $\delta P(t)$, which are detected at the photodiode. According to eq.(4) the intensity noise gives rise to the noise in the photocurrent:

$$\delta I(t) = \chi \delta P(t). \quad (35)$$

Let the noisy photocurrent be $J(t)$, which can be represented as the sum:

$$J(t) = I(t) + \delta I(t), \quad (36)$$

where $I(t)$ is the average photocurrent,

$$I(t) = \mathbf{E}\{J(t)\}, \quad (37)$$

and $\delta I(t)$ is the noise with zero mean. The expectation value $\mathbf{E}\{J(t)\}$ can be obtained by taking an ensemble average of the photocurrent. For example, this can be done by taking an average over many

traces of the photocurrent recorded with a storage oscilloscope. Another way of implementing an ensemble average is based on the ergodic theorem, according to which ensemble averaging is equivalent to time averaging. Note that the time averaging must be done over the time interval which is much less than the modulation period. Otherwise the time dependence of the modulated photocurrent will be lost.

The noise in the photocurrent is characterized by its correlation function:

$$C_I(t, t') \equiv \mathbf{E}\{\delta I(t) \delta I(t')\}, \quad (38)$$

which in frequency domain can be written as

$$\tilde{C}_I(\omega, \omega') = \mathbf{E}\{\delta \tilde{I}(\omega) \delta \tilde{I}(\omega')^*\}. \quad (39)$$

This correlation function is equal (up to a factor χ^2) to the correlation function of the intensity noise, which is mostly defined by the laser and the electro-optic devices on the beam path. Here we assume that it is known, and derive the correlation function of the filtered demodulated noise.

The noise is filtered by the photodiode's resonant circuit and is demodulated by the mixer. The noise at the output of the mixer is given by

$$\delta \tilde{V}(\omega) = -i \sum_n d_n e^{-in\gamma} \tilde{Z}(\omega - \Omega_n) \delta \tilde{I}(\omega - \Omega_n), \quad (40)$$

which is similar to eq. (27). Define the correlation function of the filtered demodulated noise as follows

$$\tilde{C}_V(\omega, \omega') = \mathbf{E}\{\delta \tilde{V}(\omega) \delta \tilde{V}(\omega')^*\}. \quad (41)$$

Using eq. (40) we obtain a general formula for this correlation function

$$\begin{aligned} \tilde{C}_V(\omega, \omega') = & \sum_m \sum_n d_m d_n^* e^{-i(m-n)\gamma} \times \\ & \tilde{Z}(\omega - \Omega_m) \tilde{Z}(\omega' - \Omega_n)^* \times \\ & \tilde{C}_I(\omega - \Omega_m, \omega' - \Omega_n). \end{aligned} \quad (42)$$

Therefore, the correlation function of the filtered demodulated noise depends on the transfer function of the photodiode, $\tilde{Z}(\omega)$, the Fourier coefficients of the demodulation wave, d_n , and the demodulation phase, γ . The power spectral density of the demodulated

noise can be derived from the correlation function as follows

$$S_V(\omega) = \lim_{T \rightarrow \infty} \left[\frac{\tilde{C}_V(\omega, \omega)}{T} \right], \quad (43)$$

and is calculated below for several different cases.

In the case of strong narrow-band filtering, eq.(28), the intensity noise outside the pass band is greatly suppressed. Then the formulae for the demodulated noise, eq. (40), and its correlation function, eq. (42), can be simplified. Namely, the demodulated noise is

$$\delta \tilde{V}(\omega) = id_1 e^{i\gamma} \tilde{Z}(\omega + \Omega) \delta \tilde{I}(\omega + \Omega) - id_1 e^{-i\gamma} \tilde{Z}(\omega - \Omega) \delta \tilde{I}(\omega - \Omega), \quad (44)$$

and there are only four terms ($m, n = \pm 1$) in the sum, eq. (42). Note that the same result can be obtained if the impedance $\tilde{Z}(\omega)$ is broad band but the demodulation wave is sinusoidal.

Thus the broad-band photodetection with the sine-wave demodulation is equivalent to the narrow-band photodetection with the square wave demodulation. Moreover, in the presence of the narrow band filter the demodulation with any wave form leads to the same result provided the coefficient d_1 is adjusted accordingly. For example, demodulation with the square wave of amplitude 1 is equivalent to the demodulation with the sine-wave of amplitude $4/\pi$.

3.2 Stationary Intensity Noise

Consider a situation when the intensity noise is stationary, and therefore its correlation function has the form:

$$C_I(t, t') = C_I(t - t'). \quad (45)$$

In frequency domain this correlation function can be written as

$$\tilde{C}_I(\omega, \omega') = 2\pi S_I(\omega) \delta_T(\omega - \omega'), \quad (46)$$

where $S_I(\omega)$ is the Fourier transform of $C_I(t)$ and also is the power spectral density of the noise according to the Wiener-Khinchin theorem. Note that the theorem cannot be applied for a nonstationary random process. In this case a more general definition, eq.(43), can be used to find the power spectral density.

The correlation function of the demodulated noise can be found from equations (46) and (42). The result is that the demodulated noise is also stationary,

$$\tilde{C}_V(\omega, \omega') = 2\pi S_V(\omega) \delta_T(\omega - \omega'), \quad (47)$$

and its power spectral density, $S_V(\omega)$, is given by

$$S_V(\omega) = \sum_n |d_n|^2 |\tilde{Z}(\omega - \Omega_n)|^2 S_I(\omega - \Omega_n). \quad (48)$$

This result shows that the demodulated noise acquires contributions from all frequencies. Moreover, all the terms in the sum, eq.(48), are positive and therefore add up.

An ideal narrow band filter greatly suppresses the noise at all frequencies except in the vicinity of the first harmonics:

$$S_V(\omega) \approx d_1^2 |\tilde{Z}(\omega - \Omega)|^2 S_I(\omega - \Omega) + d_1^2 |\tilde{Z}(\omega + \Omega)|^2 S_I(\omega + \Omega). \quad (49)$$

In this case the noise in the demodulated output of the mixer is mostly due to the intensity noise at the frequencies near the modulation frequency ($\omega \sim \Omega$). Since there is much less intensity noise at the RF-frequencies than at low frequencies,

$$S_I(\omega \pm \Omega) \ll S_I(\omega), \quad (50)$$

the effect of the intensity noise becomes greatly reduced by the heterodyne detection.

In practice, the band pass filter transmits some of the noise in its stop band. Since there are an infinite number of terms in the sum, eq. (48), a substantial difference between the anticipated power spectral density, eq.(49), and the real power spectral density, eq.(48), might occur. To avoid this the noise at the frequencies near harmonics of the modulation frequency must be strongly suppressed by the band pass filter. This is another reason for the band pass filter to have a steep roll-off at high frequencies.

3.3 Nonstationary Intensity Noise

Consider a nonstationary intensity noise and assume that the strong narrow band filtering takes place.

The correlation function of the demodulated noise is given by eq.(42) with $m, n = \pm 1$. In this case the power spectral density is

$$S_V(\omega) = d_1^2 |\bar{Z}(\omega - \Omega)|^2 S_I(\omega - \Omega) + d_1^2 |\bar{Z}(\omega + \Omega)|^2 S_I(\omega + \Omega) - 2d_1^2 \operatorname{Re} \left\{ e^{2i\gamma} \bar{Z}(\omega - \Omega)^* \bar{Z}(\omega + \Omega) Q_I(\omega) \right\}, \quad (51)$$

where we introduced a new quantity

$$Q_I(\omega) = \lim_{T \rightarrow \infty} \left[\frac{\bar{C}_I(\omega + \Omega, \omega - \Omega)}{T} \right], \quad (52)$$

which characterizes nonstationary properties of the noise. Note that $Q_I(\omega) = 0$ if the noise is stationary.

The first two terms on the right hand side of eq.(51) have a simple meaning: they represent the power spectral density of the intensity noise filtered by the band-pass filter and frequency-shifted by the mixer. The last term, which is proportional to $Q_I(\omega)$, is due to the frequency correlations of noise. It is not positive definite and therefore can increase or decrease the total noise power.

In the low frequency approximation, eq. (16), the demodulated noise becomes white:

$$S_V(\omega) = 2d_1^2 R^2 \left[S_I(\Omega) - \operatorname{Re} \left\{ e^{2i(\beta - \gamma)} Q_I(0) \right\} \right]. \quad (53)$$

Therefore, not only the power spectral density of the intensity noise, $S_I(\Omega)$, but also its frequency correlations, $Q_I(0)$, contribute to the demodulated noise.

3.4 Nonstationary Shot Noise

The heterodyne detection allows us to greatly reduce the effect of the laser intensity noise. By choosing a sufficiently high modulation frequency we can shift the detection to the frequencies where the intensity noise is minimal. At these frequencies the photodetection can be limited by shot noise only.

The shot noise is created during the photodetection and is largely defined by the average photocurrent, $I(t)$. In the optical heterodyne detection the average photocurrent varies with time, making the shot noise inherently nonstationary. The correlation function of

the nonstationary shot noise in frequency domain is given by

$$\bar{C}_I(\omega, \omega') = q \bar{I}(\omega - \omega'), \quad (54)$$

where q is the electron charge and $\bar{I}(\omega)$ is the Fourier transform of the average photocurrent. Derivation of this formula is given in Appendix.

Consider the noise in the absence of signal ($I_1 = 0$), and assume that all the coefficients I_n are constant. Then the correlation function of the nonstationary shot noise is given by

$$\bar{C}_I(\omega, \omega') = 2\pi q \sum_n I_n \delta_T(\omega - \omega' - \Omega_n). \quad (55)$$

Substituting this equation into the general formula, eq. (42), we obtain that the demodulated shot noise is stationary. Its Fourier-domain correlation function is diagonal:

$$\bar{C}_V(\omega, \omega') = 2\pi S_V(\omega) \delta_T(\omega - \omega'), \quad (56)$$

and its power spectral density is given by

$$S_V(\omega) = 2\pi q \sum_m \sum_n d_m d_n^* e^{-i(m-n)\gamma} \times I_{n-m} \bar{Z}(\omega - \Omega_m) \bar{Z}(\omega - \Omega_n)^*. \quad (57)$$

This is the general formula for the power spectral density of the demodulated shot noise in the optical heterodyne detection.

In the presence of the strong narrow band filtering the power spectral density is

$$S_V(\omega) = qd_1^2 I_0 \left[|\bar{Z}(\omega - \Omega)|^2 + |\bar{Z}(\omega + \Omega)|^2 \right] - 2qd_1^2 \operatorname{Re} \left\{ e^{2i\gamma} I_2 \bar{Z}(\omega - \Omega)^* \bar{Z}(\omega + \Omega) \right\}. \quad (58)$$

Such a power spectral density has a characteristic shape of the photodiode transfer function. This result can be used to distinguish the shot noise from other sources of noise.

Note that the power spectral density, eq. (58), can also be obtained directly from the corresponding formula for the demodulated intensity noise, eq. (51), by using the following observation:

$$S_I(\omega) = qI_0, \quad (59)$$

$$Q_I(\omega) = qI_2. \quad (60)$$