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Low noise rigidity in quantum measurements

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Abstract

It is shown that to obtain higher sensitivity in quantum experiments with test masses an oscillator is more promising than a free mass. New methods of realization of mechanical rigidity with very low quantum noise are described. Possible schemes of quantum measurements based of these method are discussed.

1 Introduction

The creation of new or the development of existing theoretical models are forcing the experimentalists to invent more sensitive methods of measurements. This trend, in particular, is evident in experiments with test masses in which the task of detecting the signal is the detection of a small force $F(t)$ (an acceleration, a gradient of acceleration) acting on a macroscopic mass m. For example, at the second stage of the project LIGO-II (Laser Interferometer Gravitational Wave Observatory [1]) the experimentalists will be confronted with the so-called Standard Quantum Limit of the sensitivity (SQL) for the force $F(t)$ [2]. It is very likely that even tougher conditions for the smallness of $F(t)$ will be finally formulated for the detection of the space-time foam action on macroscopic mass m (see original paper by S.Hawking [3] and also [4, 5, 6]).

The experimentalist has a choice in this type of experiments: to detect $F(t)$ acting on a free mass m or on a mass coupled with rigidity $K = m\omega_m^2$, in other words on an oscillator with eigen frequency ω_m which is close to ω_F - characteristic frequency of $F(t)$. At first glimpse the choice of free mass does look more attractive because it is easier to isolate the mass from the heat bath. The free mass and the oscillator "behave" as quantum objects if the following conditions for them are satisfied:

$$\frac{2kT\tau^2}{\tau^*_{H.B.}} \leq \hbar, \tag{1}$$

$$\frac{2kT\tau}{Q_{H.B.}} \leq \hbar,, \tag{2}$$

where τ is the averaging time, k is Boltsmann constant, T is the temperature of the heatbath, $\tau^*_{H.B.}$ is the relaxation time for the free mass, $Q_{H.B.}$ is the quality factor of the oscillator (see e.g. [7]). For $T = 300K$ and $\tau = 10^{-2}sec$ it is necessary to reach $\tau^*_{H.B.} \simeq 10^{10}sec.$ to satisfy condition (1). The value $\tau^*_{H.B.} \simeq 10^{8}sec$ was already reached [8], new methods of the "substraction" of the heatbath action on the free mass are also proposed. These methods permit to reach the equivalent $\tau^*_{H.B.} \simeq 10^{10}sec$ [9]. For the same values of T and τ it is necessary to have $Q_{H.B.} \geq 10^{12}$ (note that at room temperature the highest obtained $Q \simeq 4 \cdot 10^8$ [10]) to satisfy the condition (2) . At the same time if conditions (1) and (2) are satisfied and the experimentalist has meters which continuously monitor the coordinate $x(t)$ at his disposal then the sensitivity will be limited by SQL (see e.g. [7, 11]) which for the variances of the coordinate have simple forms:

$$(\Delta x_{SQL})_{freemass} \simeq \sqrt{\frac{\hbar\tau}{2m}}, \tag{3}$$

$$(\Delta x_{SQL})_{oscillator} \simeq \sqrt{\frac{\hbar}{2m\omega_m}}. \tag{4}$$

To circumvent these limits it is necessary to have other methods of quantum measurements, for example Quantum-Non-Demolition methods (see [12]) or methods based on continuous monitoring of coordinate with the use of spectral or time domain features of noises of the meter [13, 14]. All proposed schemes for a free mass based on these last two methods have two common features: 1) all meters are in essence parametrical converters (the $x(t)$ is transferred into the modulation of some parameter of an e.m. resonator coupled to the mass m) and 2) all of them require correlation between back action noise and output noise of the meter. Due to the second requirement all these schemes depend on the level of intrinsic losses of the e.m. resonator because fluctuations corresponding to these losses can't correlate with the output noise of the meter. It can be shown that in order to detect signal ξ times smaller than the SQL it is necessary to satisfy the condition

$$\xi \geq \left(\frac{\tau^*_{load}}{\tau^*_{intr}}\right)^{1/4}, \tag{5}$$

where τ^*_{intr} is the relaxation time which characterize intrinsic losses in the e.m. resonator and τ^*_{load} is the relaxation time corresponding to coupling of the resonator with e.m. detector (we omit here and below all intermediate cumbersome calculations). This condition together with the energetic quantum limit [15] leads to very severe requirement for the power of e.m. pumping W. For example, if $\tau_{load} < \tau$, where τ is the averaging time, then the power

$$W \geq \frac{m\omega_F^2 l^2}{64\omega_e(\tau^*_{load})^2\xi^2} \geq \frac{m\omega_F^2 l^2}{64\omega_e(\tau^*_{intr})^2\xi^{10}}, \tag{6}$$

or, in the case of optical Fabry-Perot resonator,

$$W \geq \frac{\pi^2 m c^2 \omega_F^2}{64\omega_e \mathcal{F}^2_{intr}\xi^{10}}, \tag{7}$$

where l is the length of the resonator, c is the speed of light, $\mathcal{F}_{intr}$ is the finess of the resonator corresponding to intrinsic losses.

In some specific cases of this class of meter even more strict limitations can exist. For example, in the case of QND speed meter [16] in addition to the limit (6) it is necessary to have

$$\xi \geq \left(\frac{\tau}{\tau^*_{intr}}\right)^{1/4}, \tag{8}$$

so if $\xi = 0.1$ and $\tau = 10^{-2}s$ then must be $\tau_{intr} \geq 10^2 s$ (which corresponds to the quality factor $Q_{intr} \geq 10^{12}$ even in the case of microwave resonator).

In contrast with the free mass in the case of harmonic oscillator it is possible to overcome the SQL without using the correlation of the meter noises because during free unitary evolution quantum state of the oscillator is periodic. This feature permits to evade the confrontations with conditions (5-8).

The goal of this article is to show that it is possible to realize a relatively large mechanical rigidity which at the same time is a source of very small quantum noise and to discuss the implementation of new schemes of quantum measurements.

2 Low noise rigidity produced by pondermotive force

For many years it has been known that pondermotive force which acts on two mirrors of Fabry-Perot resonator (in microwave or optical bandwidth) creates mechanical rigidity K_{pond} which may be large if the reflectivity of the mirrors is high and its losses are low and if this resonator is pumped with the frequency ω which is detuned from the eigen frequency $\omega_e = \omega - \gamma$ (see e.g. [17]). If the detuning is large (this is necessary to get a low level of noises, see below) and the following inequality is satisfied:

$$\gamma\tau_e^* \gg 1, \tag{9}$$

then this rigidity is equal to

$$K_{pond} = m\omega_m^2 = \frac{2\omega_e \mathcal{E}}{l^2\gamma} = \frac{2\omega_e W}{l^2\gamma^3\tau_e^*}. \tag{10}$$

Here

$$\tau_e^* = \frac{l\mathcal{F}}{\pi c} \tag{11}$$

- is the relaxation time of the resonator, l is the distance between the mirrors, c is the speed of light, $\mathcal{E}$ -is the energy between the mirrors.

On the other hand the finite value of $\mathcal{F}$ will produce a fluctuating force which will act on the masses coupled to the mirrors. The spectral density of this "back action" force is equal to

$$S_{B.A.} = \frac{\hbar\omega_e\mathcal{E}}{l^2\gamma^2\tau_e^*} = \frac{\hbar\omega_e W}{l^2\gamma^4(\tau_e^*)^2}. \tag{12}$$

The "quality" of rigidity may be characterized by the minimal detectable force in units of SQL. We may use the same parameter ξ as in formula (5-8), but now we take into account only the noise from the rigidity. In this case

$$\xi^2 = \frac{S_{B.A.}}{\hbar m\omega_m^2} = \frac{1}{2\gamma\tau_e^*}. \tag{13}$$

Substituting this value of ξ into the formula (10) results in

$$K_{pond} = \frac{16\omega_e W\mathcal{F}^2\xi^6}{\pi^2c^2} \simeq 10^{10} dyn/cm \times \left(\frac{W}{10^4 erg/s}\right) \times \left(\frac{\mathcal{F}}{5\cdot 10^5}\right)^2 \times \left(\frac{\omega_e}{2\cdot 10^{15}}\right) \times \xi^6. \tag{14}$$

Thus for $\xi = 0.3$ and $W = 10^7 erg/s$ one may obtain the rigidity $K_{pond} \simeq 10^{10} dyn/cm$ which will "convert" a 10^4 gram mass into a mechanical oscillator with eigen frequency $\omega_m \simeq 10^3 s^{-1}$. For $\xi < 0.3$ the value of W will be inaccessible high. However for smaller masses (e.g. $m \simeq 10 gr$) this type of artifical rigidity seems attractive. It is worth noting that this method may be realized only because very high values of $\mathcal{F}$ were recently obtained [18].

Another scheme with the pondermotive rigidity has been considered in the article [2]. If mirror with transmittance $\mathcal{T}$ is situated inside the Fabri-Perot resonator equidistant from two end mirrors with high finess $\mathcal{F}$ then this internal mirror splits the eigen modes of the resonator into doublets whose frequencies are apart from each other by

$$\Omega = \frac{c\mathcal{T}}{l}. \tag{15}$$

where l is the distance between the internal mirror and the end ones.

If the upper frequency component of the doublet is excited then the pondermotive force which acts on the internal mirror will strongly depend on the displacement of the mirrors due to the redistribution of the e.m. energy $\mathcal{E}$ in the two parts of the resonator. In other words mechanical rigidity K_{pond} between the internal mirror and the end mirrors will be created. The value of K_{pond} may be presented as

$$K_{pond} = \frac{4\omega_e \mathcal{E}}{l^2\Omega} = \frac{4\omega_e W\mathcal{F}}{\pi c^2 \mathcal{T}} \simeq 10^{10}\, dyn/cm \times \left(\frac{W}{3\cdot 10^7 erg/s}\right) \times \left(\frac{\omega_e}{2\cdot 10^{15}}\right) \times \left(\frac{\mathcal{F}}{10^6}\right) \times \left(\frac{\mathcal{T}}{10^{-2}}\right)^{-1} \tag{16}$$

Here $\mathcal{E}$ is the energy stored in each half of the resonator. This estimate show that for the mass of the mirror $m = 10^4 g$ the mechanical eigen frequency of the internal mirror will be equal to $10^3 s^{-1}$ if the pumping power equals to $W = 3\cdot 10^7 erg/s$. This relatively modest value of W for such a large K_{pond} is the first substantial advantage of this type of artifical rigidity.

Spectral density of the back action force in this case is equal to

$$S_{B.A.} = \frac{2\hbar\omega_e W}{l^2\Omega^2}, \tag{17}$$

hence

$$\xi^2 = \frac{S_{B.A.}}{\hbar m\omega^2} = \frac{\pi}{2\mathcal{F}\mathcal{T}} = 1.5\cdot 10^{-4} \times \left(\frac{\mathcal{T}}{10^{-2}}\right)^{-1} \times \left(\frac{\mathcal{F}}{10^6}\right)^{-1}. \tag{18}$$

Thus the second advantage is a very low level of intrinsic quantum noise. This level is approximately equal to the level of noise of mechanical oscillator with quality factor $Q_m \simeq 2\cdot 10^{16}$ in the heatbath with temperature 300K. The "fee" which experimentalist has to pay for such a large value of K_{pond} and small value of $S_{B.A.}$ is the level of symmetry in the positioning of inside mirror $\delta l/l$ and the phase difference in the two arms of the resonator $\delta\phi$. which may be quite severe if the value of $\xi < 0.1$ is required.

This method which permits to obtain a relatively large rigidity with low level of intrinsic noise may be used in the scheme of gravitational wave antenna with optical bars [2] (see Fig.1). In this scheme the same type of rigidity is transferring the displacement $hL/2$ of the end mirrors produced by the gravitational wave with amplitude h to the same displacement of the inside mirror relatively to the reference mass (L is the length of the antenna's arms, in the LIGO project $L = 4 \cdot 10^5 cm$). If the inside mirror has an appropriate "bump" and the reference mirror an appropriate "trench" it is possible to use the "trench" and the "bump" to create a second Fabry-Perot resonator with another inside mirror. In Fig.1 the dashed lines indicate the beams of light inside the main resonator (1) and the small one (2). The rigidity K_{pond} between the big inside mirror and the reference mass (created by the small resonator and the beams) convert this pair of masses into a mechanical oscillator whose eigen frequency ω_m may be of the same order as the frequency of gravitational radiation. Thus instead of the using QND speedmeter or similar scheme it is possible to use the meter which monitors QND observable of the oscillator.

3 The methods of measurement

During the last twenty years several schemes of the QND measurement of observables of the oscillator were proposed (see [15]). One of the oldest of them and probably the simpliest for realization is the stroboscopic meter [19]. This scheme uses short measurements of coordinate repeated periodically with frequency $2\omega_m$. Two-time uncertainty relation for the coordinate of the oscillator has the following form:

$$\Delta x(t) \cdot \Delta x(t') \geq \frac{\hbar}{2m\omega_m} |\sin \omega_m (t - t')|. \tag{19}$$

If $t - t' = \pi n/\omega_m$ (n is any integer) then can be

$$\Delta x(t) = \Delta x(t') \to 0. \tag{20}$$

Hence the sensitivity in such a procedure is not limited by the SQL.

Now suppose that in addition to the resonators which create K_{pond} we may add another Fabry-Perot resonator between the probe mass (in the case of bar antenna - the internal mirror) and the reference mass. If this resonator is pumped periodically by short (with duration $\tau_1 \ll \omega_m^{-1}$) resonant pulses divided by time interval π/ω_m and the phase of the output power will be registered then such a setup will work as a stroboscopic meter.

Simple calculations (without rigorous optimization of the procedure of the signal filtering) show that in order to obtain sensitivity ξ^{-1} times higher than the SQL it is necessary to have

$$\overline{W} \sim \frac{m\omega_m^2 l^2}{\omega_e (\tau_{intr}^*)^2 \xi^2} \sim \frac{mc^2 \omega_m^2}{\omega_e \mathcal{F}_{intr}^2 \xi^2}, \tag{21}$$

where $\overline{W}$ - is averaged over the mechanical period power of the pumping, all other notations being the same as in the formula (7). It is important to note that this limitation looks very modest in comparision with limits (5-8) for the schemes based on free masses.

Unfortunately in the case of large masses $m \simeq 10^4 gr$ the value of τ_1 will be close to periods of the internal mechanical modes of m which have relatively high level of dissipation and thermal noise. Estimates show that for large masses this factor limits the sensitivity of stroboscopic method at the level of $\xi \simeq 0.3$. But for the small masses (of the order of $10 gr$) the stroboscopic procedure may allow to get ξ substantionally smaller than 0.3.

It is possible that the following procedure will allow to obtain a better sensitivity. This procedure relies on another important feature of the proposed artifical rigidity. The value of K_{pond} is proportional to the pumping power W. Thus by the modulation of the power with frequency $2\omega_m$ one may realize the parametrical action on the oscillator which will not perturb irreversibly the wave function of the relative movement of the the two masses (due to the very small value of $S_{B.A.}$). It is well known that parametrical action produces the affine transformation of plane of two quadrature amplitudes X_1 and X_2. Thus if one applies the periodical parametrical action with certain phase then one quadrature amplitude is amplificated with some factor $1/\zeta$ and correspondingly another one is deamplificated with the factor ζ. It is important that if the oscillator is a quantum one then the wave function may be completely restored by shifting the phase of the parametrical modulation by π.

The Fig.2 illustrates this process. Let us assume that the oscillator was initially in the coherent quantum state with quadrature amplitudes equal to X_{1in} and X_{2in}, see the position A in Fig.2 (in Fig. 2 values $X_{1in} = 2.5$ and $X_{2in} = 1$ in the units of the width of the coherent state are used). If the value of K_{pond} will be modulated during certain time interval then one of the outcome of the parametrical action will be the deamplification of the X_1 and correspondingly the uncertainty ΔX_1 by some factor ζ. One will obtain a squeezed quantum state (in Fig.2 the value $\zeta = 5$ is used and the described evolution corresponds to the transition from state A to the state B). If the phase of the modulation is changed by π then during the same time interval the oscillator will return back from B to A. This procedure may be repeated many times (as long as one neglects $S_{B.A.}$).

Now suppose that external force $F(t)$ has displaced the mean value of X_1 by x_F. If the oscillator was in coherent state during the action of force then this action will be detected if

$$x_F \geq (\Delta x_{SQL})_{oscillator} \tag{22}$$

(see formula (4)). But if before the action of $F(t)$ the oscillator was in the state B then after the action of $F(t)$ it will be in the state C and after the second cycle of modulation of K_{pond} the oscillator will be in the coherent state but now with $X_1 = X_{1in} + \zeta x_F$ (state D). So it is evident that this procedure permits to detect the action

$$x_F = \frac{1}{\zeta}(\Delta x_{SQL})_{oscillator} \tag{23}$$

using a simple coordinate meter with the resolution at the level of the SQL and so obtain the value of $\xi = 1/\zeta$.

The procedure qualitively described above has one disadvantage: if applied to the gravitational wave antenna then the antenna will have "blind intervals". But it is possible to evade this disadvantage at the expence of reducing the sensitivity: $\xi = 1/\sqrt{\zeta}$.

4 Conclusion

The aim of the authors of this article is to present the key features of a new concept of quantum measurement based on low noise rigidity. That is why all cumbersome calculations were omitted and the numerical examples were emphasized to illustrate the concept. In the same time we think that much more indepth rigorous calculations must be done to clarify many technical details before starting the implementation. On the other hand we also think that for the searches of decoherence of ordinary matter produced by spacetime foam the rigidity created in a simple two mirrors resonator and stroboscopic procedure of measurement is sufficient. For the gravitational wave antenna (where the value of mirror mass must much larger) the rigidity from the resonators with the internal mirror is preferable.

It is also important to note that, probably, better low noise rigidity (with smaller value of $S_{B.A.}$ and higher K_{pond}) may be proposed. One of the possibility is the rigidity produced by electrostatic force between two metal plane gratings consisting of regular rows of "bumps" and "trenches". If the size of the "bumps" and "trenches" will be $\simeq 10^{-3} cm$ and the electrical field strength $\simeq 2 \cdot 10^3 esu$ then with the surfaces $\simeq 10^2 cm^2$ one may obtain $K_{pond} \simeq 10^{10} dyn/cm$. Unfortunately it is very difficult to precalculate the value of $S_{B.A.}$ for this case because of unknown features of quantum surface states in few monolayers of oxides and water which may be a source of losses [20]. In this case only direct experiments may give a reliable answer.

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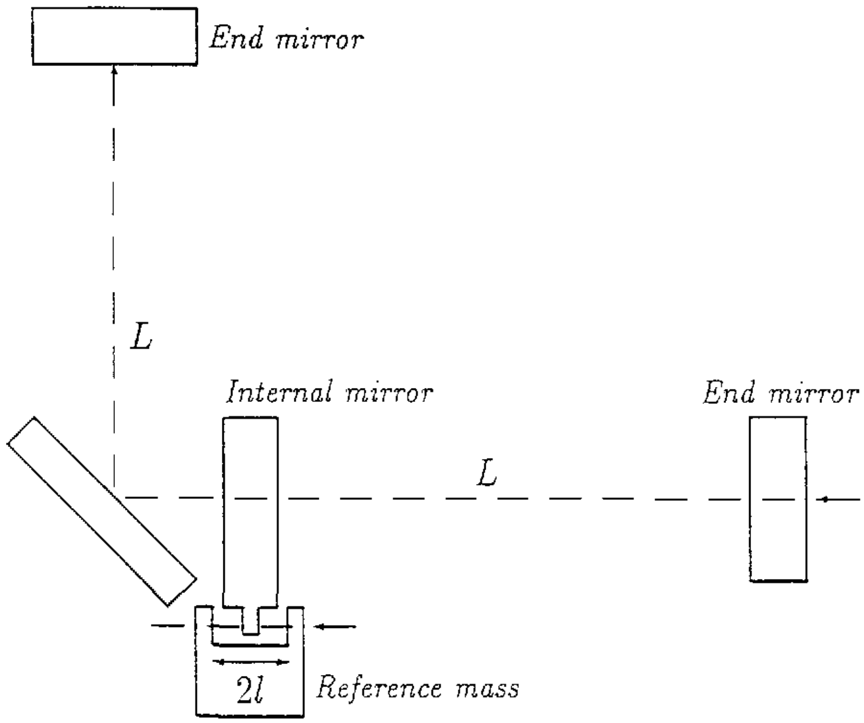
End mirror
L
Internal mirror
End mirror
L
2l
Reference mass

Fig. 1

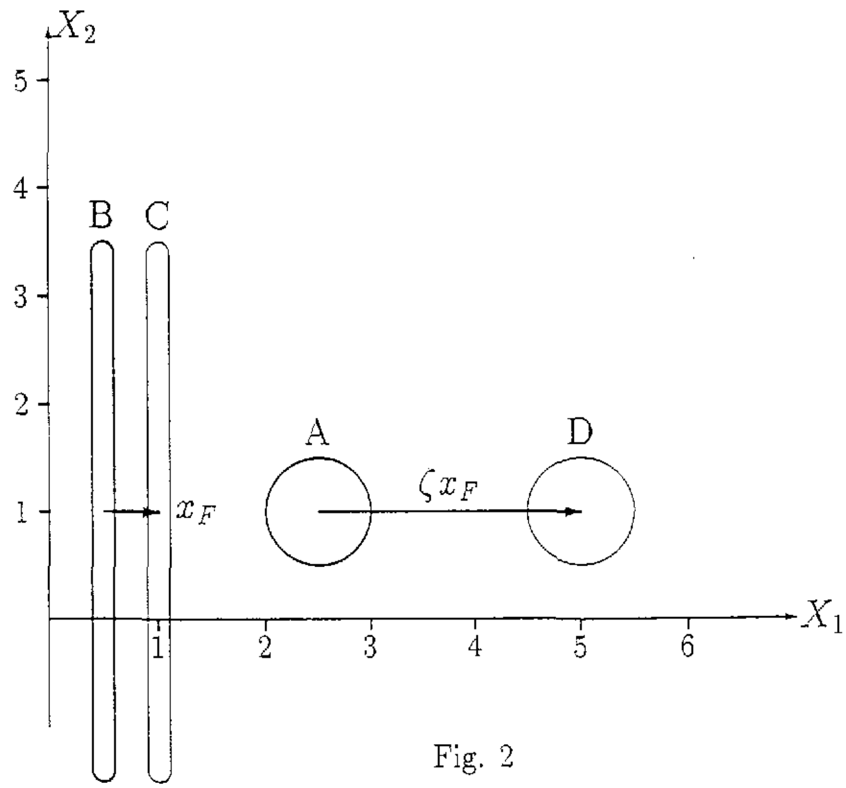
X_2
5
4
3
2
1
B
C
A
D
x_F
ζx_F
X_1
1
2
3
4
5
6

Fig. 2