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Detecting free-mass common-mode motion induced by incident gravitational waves

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In this paper we show that information on both the differential and common mode free-mass response to a gravitational wave can provide important information on discriminating the direction of the gravitational wave source and between different theories of gravitation. The conventional Michelson interferometer scheme only measures the differential free-mass response. By changing the orientation of the beam splitter, it is possible to configure the detector so it is sensitive to the common-mode of the free-mass motion. The proposed interferometer is an adaptation of the Fox-Smith interferometer. A major limitation to the new scheme is its enhanced sensitivity to laser frequency fluctuations over the conventional, and we propose a method of cancelling these fluctuations. The configuration could be used in parallel to the conventional differential detection scheme with a significant sensitivity and bandwidth. [S0556-2821(99)03308-1]

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I. INTRODUCTION

Interferometric gravitational wave detectors are now poised to detect gravitation waves from astrophysical sources over a large detection bandwidth. Large detectors of a few kilometers are currently under construction in Europe and the USA (VIRGO and LIGO) [1–3]. Other current projects include detectors of order a few hundred meters (GEO, TAMA, and ACIGA) [4] as well as cryogenically cooled detectors (LCGT) [5]. Standard configurations of these detectors will be sensitive to the quadrupole component of radiation predicted by Einstein's theory of general relativity (GR). Much work has been done in regards to these types of detectors, and for a good description see [6] and references therein.

It is widely acknowledged that GR may not necessarily describe gravity in the strong-field regime, and alternative scalar-tensor theories exist that cannot be disproved from experimental evidence to date. Also, it has been shown that these theories may be important in describing inflation models of the universe [7] as well as unified theories such as string theory [8–10]. In particular, it has been shown that Brans-Dicke theory [11] will produce significant amounts of scalar radiation in collapsing astrophysical systems [12–16], especially in spherical symmetric collapse.

It is well known that spherical resonant-mass detectors can determine the direction of the incoming signal; this principle was first shown in 1971 by Forward [17] and later revived by Merkowitz and Johnson in 1993 [18]. Also, it has been known since 1971 that a spherical antenna could be used to distinguish between different polarizations and metric theories of gravitation [17]. Recently a more detailed analysis was performed which has fully revived the concept of the spherical detector [19]. A disadvantage of a spherical detector is that the frequency of detection of any scalar component is limited to the monopole modes of the sphere which are different to the frequency of the spherical quadrupole modes. Thus to determine the quadrupole and scalar content, the radiation itself must be sufficiently broadband to cover both frequencies, which necessarily limits the detection to burst wave forms. Also, a different set of resonant transducers is required at the monopole frequency, and would add to the complexity of the detector.

In this paper the antenna beam patterns were calculated for both the common mode and differential motion of the interferometer test masses. Beam patterns were calculated for the six possible polarizations available in metric theories of gravitation. We show from the calculated beam patterns that important information is acquired that can discriminate the direction of the source and between the metric theories of gravitation. *Specifically we highlight the example of discriminating* between Brans-Dicke scalar waves and Einstein quadrupole radiation. Following this we present a practical scheme based on a Fox-Smith interferometer that may be configured to measure the common-mode response of gravitational radiation at a similar sensitivity and bandwidth to the conventional differential interferometer schemes. The Fox-Smith configuration is not limited to detecting burst sources and is generally broadband. Also, it could be added as one of the beams in the LIGO detector so a simultaneous detection of the differential and common-mode response to gravitational radiation can be achieved.

II. INTERFEROMETER RESPONSE TO INCIDENT GRAVITATIONAL WAVES

Incident gravitational radiation will cause relative motion of the two mirror test masses with respect to the beam splitter of the interferometer. In general this motion will have a differential and common-mode component. Michelson type interferometers are only sensitive to the differential component so past analysis of interferometer response has mainly dealt with the quadrupole radiation of general relativity causing differential motion of the two test masses [20,21]. In this section we assume both the common-mode and differential responses may be detected and we calculate the antenna patterns with respect to the six independent possible polarizations [22].

Gravitational waves are believed to propagate as a tensor wave given by

$$\frac{\partial^2 h_{\alpha\beta}}{\partial x_\alpha^2}-\frac{1}{c^2}\frac{\partial^2 h_{\alpha\beta}}{\partial t^2}=0. \tag{1}$$

The general form of a gravitational wave in the z direction can be written as

$$h_{\alpha\beta}=\begin{pmatrix} h_{\mathrm{Re}[\Psi_4]}e^{i\phi_{\mathrm{Re}[\Psi_4]}}t_{\alpha\beta}+h_{\mathrm{Im}[\Psi_4]}e^{i\phi_{\mathrm{Im}[\Psi_4]}}s_{\alpha\beta}+h_{\mathrm{Re}[\Psi_3]}e^{i\phi_{\mathrm{Re}[\Psi_3]}}q_{\alpha\beta} \\ +h_{\mathrm{Im}[\Psi_3]}e^{i\phi_{\mathrm{Im}[\Psi_3]}}p_{\alpha\beta}+h_{\Phi_{22}}e^{i\phi_{\Phi_{22}}}n_{\alpha\beta}+h_{\Psi_2}e^{i\phi_{\Psi_2}}m_{\alpha\beta} \end{pmatrix} e^{-i(\omega t-kz)}. \tag{2}$$

Here the subscripts follow the Newman-Penrose parameters; $\mathrm{Re}[\Psi_4]$ is the plus (or in phase) quadrupole polarization; $\mathrm{Im}[\Psi_4]$ is the cross (or quadrature) quadrupole polarization; $\mathrm{Re}[\Psi_3]$ is the in phase vector polarization; $\mathrm{Im}[\Psi_3]$ is the quadrature vector polarization; Φ_{22} is the transverse scalar polarization; Ψ_2 is the longitudinal scalar polarization. Each polarization in Eq. (2) is represented by a scalar amplitude, h_i, and phase shift, ϕ_i, followed by a second order tensor that describes the pattern of the polarization. The pattern tensors have the form

$$t_{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{bmatrix},\quad s_{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{bmatrix},\quad q_{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix},$$

$$p_{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix},\quad n_{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},\quad m_{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}. \tag{3}$$

To calculate the output response we follow closely the method used by Forward where a tensor format for the combined response of the two interferometer arms was assumed [20]. Forward showed the response could be written as

$$\Delta\xi=\frac{1}{2}h_{\alpha\beta}A^{\alpha\beta}, \tag{4}$$

where $A^{\alpha\beta}$ is the tensor format of the response of the two arms. Assuming that the arms of the interferometer are along the x and y axis and l is equal to the arm length, the differential format can be written as

$$A_{diff}^{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{bmatrix}l \tag{5}$$

and the common-mode format may be written as

$$A_{cm}^{\alpha\beta}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}l. \tag{6}$$

To calculate the sensitivity pattern we assume the gravitational wave is incident on the interferometer from an arbitrary direction defined by angles θ and ϕ, but with the polarization angle ψ assumed to be zero, as shown in Fig. 1. Before the response given by Eq. (4) can be calculated $h_{\alpha\beta}$ must be converted to the coordinate system of the interferometer given that $A^{\alpha\beta}$ is in the interferometer frame. To do

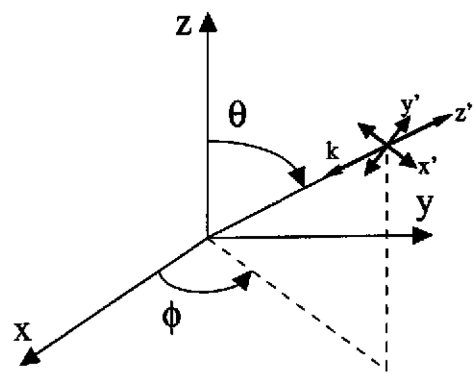

FIG. 1. Co-ordinate system where the x and y axis represent the directions of the interferometer arms, and the dashed system represents the coordinates of the gravitational radiation.

TABLE I. Normalized differential and common mode response of a free-mass interferometer detector per unit arm length to the six possible metric polarizations.

Radiation type	NP parameter	$\Delta\xi_{\rm int}$ differential	$\Delta\xi_{\rm int}$ common mode
Quadrupole plus (in phase)	$\mathrm{Re}[\Psi_4]$	$\frac{1}{2}\cos 2\phi(1+\cos^2\theta)$	$\frac{1}{4}(1-\cos 2\theta)$
Quadrupole cross (quadrature)	$\mathrm{Im}[\Psi_4]$	$-\cos\theta\sin 2\phi$	0
Vector (in phase)	$\mathrm{Re}[\Psi_3]$	$\sin\theta\sin 2\phi$	0
Vector (quadrature)	$\mathrm{Im}[\Psi_3]$	$\frac{1}{2}\cos 2\phi\sin 2\theta$	$-\frac{1}{2}\sin 2\theta$
Scalar transverse	Φ_{22}	$\frac{1}{2}\cos 2\phi\sin^2\theta$	$\frac{1}{2}(1+\cos^2\theta)$
Scalar longitudinal	Ψ_2	$-\frac{1}{2}\cos 2\phi\sin^2\theta$	$\frac{1}{2}\sin^2\theta$

this we use the general form of the rotation matrix, R^α_β, with the Euler angle Ψ equal to zero [23]:

$$R^\alpha_\beta=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta\cos\phi & \cos\theta\sin\phi & \sin\theta \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & -\sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta \end{bmatrix}. \quad (7)$$

Thus, Eq. (4) can be rewritten in the interferometer frame as

$$\Delta\xi_{\rm int}=\frac{1}{2}R^{\gamma^{-1}}_{\alpha}h_{\gamma\delta}R^\delta_\beta A^{\alpha\beta}. \quad (8)$$

We implement this equation by considering the six polarizations of Eqs. (2) and (3) independently, i.e., by assuming the amplitudes of all the polarizations except for the one under consideration are zero. The normalized (assuming the amplitude is unity) common-mode and differential response per unit arm length of the six polarizations are summarized in Table I, and plotted in Figs. 2(a)–2(l).

In the past only the differential response to the quadrupole radiation has been considered. In general quadrupole radiation can consist of two polarizations. In our calculations it was assumed that the polarization angle (or Euler angle) was zero. There is no particular angle that is special, as the convention for choosing the zero of the polarization angle is arbitrary. Thus, a polarized gravitational wave will be in fact a linear combination of the plus and cross polarizations and the antenna pattern will depend on the angle of polarization. However, if we assume the radiation is unpolarized and consists of many gravitons of random polarization, the antenna pattern may be calculated by taking the square root of the sum of the squares of $\mathrm{Re}[\psi_4]$ and $\mathrm{Im}[\psi_4]$. Figure 2(d) shows the response to the unpolarized case; this is the same response calculated by Saulson [6].

The broad angular response of the interferometer to differential motion has been described by Saulson as both a "blessing and a curse." This is because it is very nondirectional and behaves more like an ear on the ground than a telescope pointed towards the sky. The blessing is that it is very easy to survey the sky, the curse is that it is very hard to determine the position in the sky without an extreme amount of effort. If only the differential motion is monitored, the direction can be determined from difference in arrival times of signals from detectors at widely separated locations. To uniquely define the position four detectors are needed.

The common-mode response to the plus quadrupole polarization is shown in Fig. 2(b). The response to the cross polarization is zero, thus the response to unpolarized quadrupole radiation will be the same as Fig. 2(b). The striking feature of the antenna pattern of the common-mode response is that it is very directional and mainly responds to signals in the x-y plane of the interferometer. Thus, if the common-mode response can be detected a comparison with the differential response will give information on the direction of the gravitational wave source.

There is no definite experimental proof that quadrupole radiation is the only type of gravitation radiation. In particular Brans-Dicke theory predicts the existence of a transverse scalar wave. The common-mode response to the transverse scalar wave is shown in Fig. 2(j). The response is very broad, and thus a sensitive detection scheme for scalar waves can be created by monitoring the common-mode motion. Comparing the differential response of the transverse scalar wave, we note that it is very directional and information regarding the direction of a scalar wave could be determined by monitoring both the common-mode and differential responses.

It is evident that the scalar radiation mainly induces a common-mode signal while the quadrupole radiation mainly induces a differential signal. Thus, by monitoring the relative amounts of each, a test of gravitational theories could be undertaken. There are many combinations of different polarizations that could be looked at. It is not our intention to go through all these possibilities. In the next section we will restrict ourself to Brans-Dicke theory which includes the quadrupole and transverse scalar polarizations.

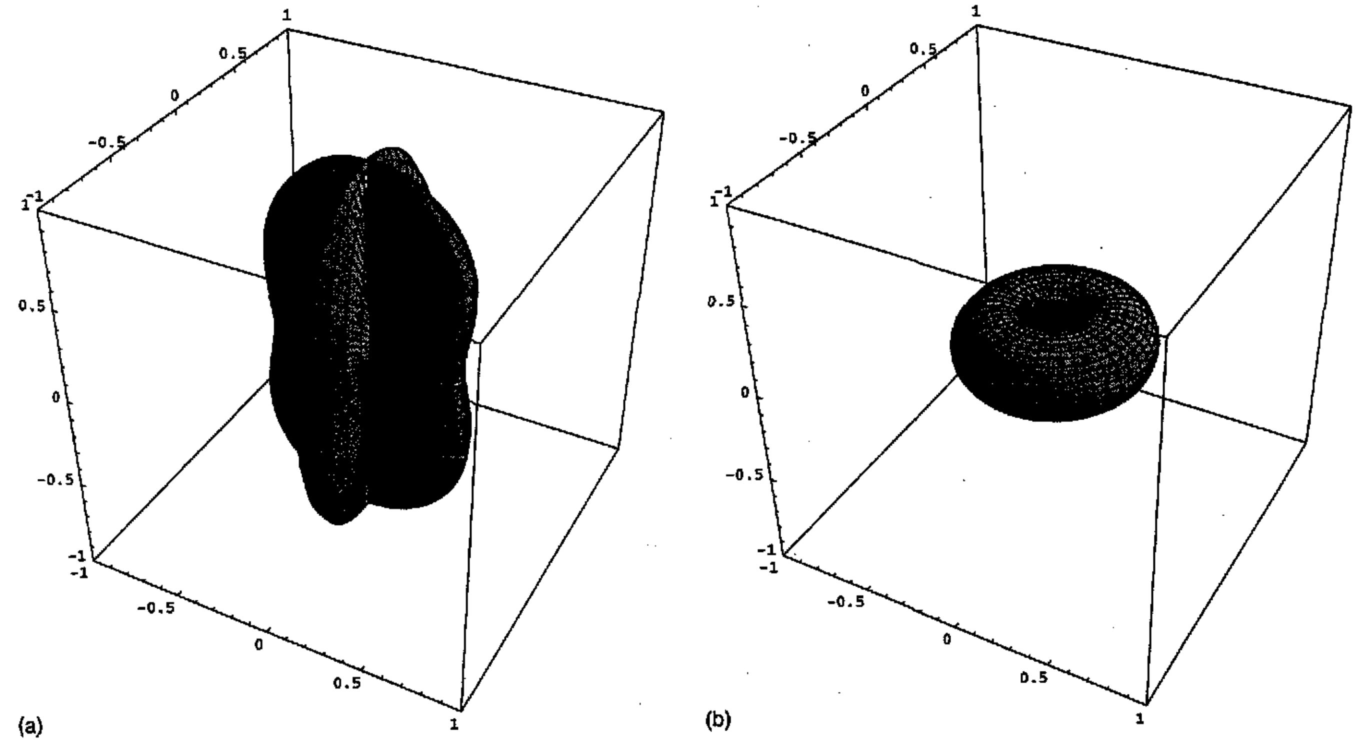

FIG. 2. (a) Antenna sensitivity pattern for the differential response to the plus quadrupole polarization $\mathrm{Re}[\Psi_4]$, with the x-axis labeled on the bottom and the y-axis labeled on the top. (b) Antenna sensitivity pattern for the common-mode response to the plus quadrupole polarization $\mathrm{Re}[\Psi_4]$, with the x-axis labeled on the bottom and the y-axis labeled on the top. (c) Antenna sensitivity pattern for the differential response to the cross quadrupole polarization $\mathrm{Im}[\Psi_4]$, with the x-axis labeled on the bottom and the y-axis labeled on the top. The common-mode response to this polarization is zero for all θ and ϕ. (d) Antenna sensitivity pattern for the differential response to unpolarized quadrupole radiation $(\mathrm{Re}[\Psi_4]^2+\mathrm{Re}[\Psi_4]^2)^{1/2}$, with the x-axis labeled on the bottom and the y-axis labeled on the top. The common-mode response to unpolarized radiation is the same as in (b) because the common-mode response to the cross polarization is zero. (e) Antenna sensitivity pattern for the differential response to the in phase vector polarization $\mathrm{Re}[\Psi_3]$, with the x-axis labeled on the bottom and the y-axis labeled on the top. The common-mode response to this polarization is zero for all θ and ϕ. (f) Antenna sensitivity pattern for the differential response to the quadrature vector polarization $\mathrm{Im}[\Psi_3]$, with the x-axis labeled on the bottom and the y-axis labeled on the top. (g) Antenna sensitivity pattern for the common-mode response to the quadrature vector polarization $\mathrm{Im}[\Psi_3]$, with the x-axis labeled on the bottom and the y-axis labeled on the top. (h) Antenna sensitivity pattern for the differential response to unpolarized vector radiation $(\mathrm{Re}[\Psi_3]^2+\mathrm{Re}[\Psi_3]^2)^{1/2}$, with the x-axis labeled on the bottom and the y-axis labeled on the top. The common-mode response to unpolarized radiation is the same as in (g) because the common-mode response to the in-phase polarization is zero. (i) Antenna sensitivity pattern for the differential response to the transverse scalar polarization Φ_{22}, with the x-axis labeled on the bottom and the y-axis labeled on the top. (j) Antenna sensitivity pattern for the common-mode response to the transverse scalar polarization Φ_{22}, with the x-axis labeled on the bottom and the y-axis labeled on the top. (k) Antenna sensitivity pattern for the differential response to the longitudinal scalar polarization Ψ_{22}, with the x-axis labeled on the bottom and the y-axis labeled on the top. (l) Antenna sensitivity pattern for the common-mode response to the longitudinal scalar polarization Ψ_{22}, with the x-axis labeled on the bottom and the y-axis labeled on the top.

III. DETERMINING DIRECTION AND THE SCALAR CONTENT IN EINSTEIN AND BRANS-DICKE THEORY

First we assume the theory of general relativity is correct, and that 100% unpolarized quadrupole radiation is incident on the detector. The ratio of the differential to common-mode response is given by

$$\gamma_{quad} = \sqrt{\frac{(\frac{1}{2}\cos 2\phi(1+\cos^2\theta))^2+(-\cos\theta\sin 2\phi)^2}{(\frac{1}{4}(1-\cos 2\theta))^2}}. \quad (9)$$

This function is plotted as a two-dimensional contour plot in Fig. 3, with θ as the vertical axis and ϕ as the horizontal. The distinguishing feature is that most of the time the differential response is greater than the common-mode response ($\gamma_{quad} > 0$ dB); the exception is at the bisectors of the interferometer arms in the x-y plane at $\theta=\pi/2$ and $\phi=\pm\pi/4$ or $\pm 3\pi/4$. At these points the differential response is zero. The common-mode signal remains greater than the differential at all angles in the x-y plane ($\theta=\pi/2$) except along the interferometer arms where the response is equal ($\gamma_{quad}=0$ dB). If the differential response is greater than the common mode response by at least 10 dB this means that $\theta<\pi/4$ or $\theta > 3\pi/4$, and as $\theta\rightarrow 0$ or $\theta\rightarrow\pi$, the common mode response decreases very rapidly to zero. Clearly if we know that the radiation is quadrupole we can determine information about the direction. For example, if γ_{quad} was measured to be 10 dB, from Fig. 3 it could be determined that θ must be $\pi/4$ or

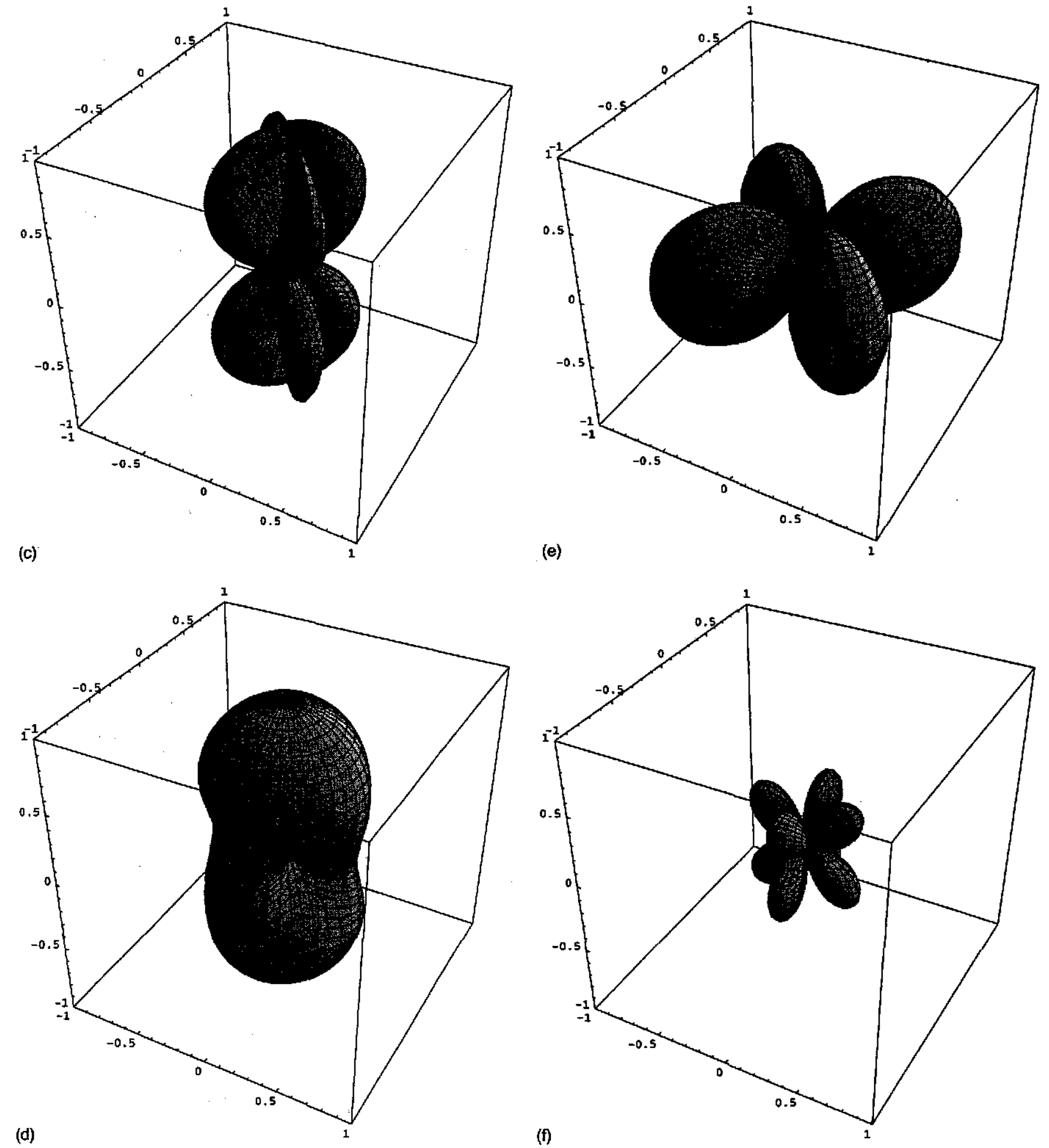

FIG. 2. *(Continued)*.

$3\pi/4$. This will describe two circles in the celestial sphere and will reduce the number of necessary detectors to pinpoint the position. If a second detector that uses both common mode and differential detection was used, the intersection of the four circles on the celestial sphere could be used to pin point *at most 8 possible patches* on the sky (could be less depending on the orientation of the two detectors with respect to the gravitational wave). If we then used the time difference of arrival between the two detectors, the patch from which the radiation came from could be determined as long as the time difference circle on the celestial sphere lined up with only one of the patches. Thus to determine the direction only two detectors are necessary rather than four. In effect still four detectors are being used, i.e., two common

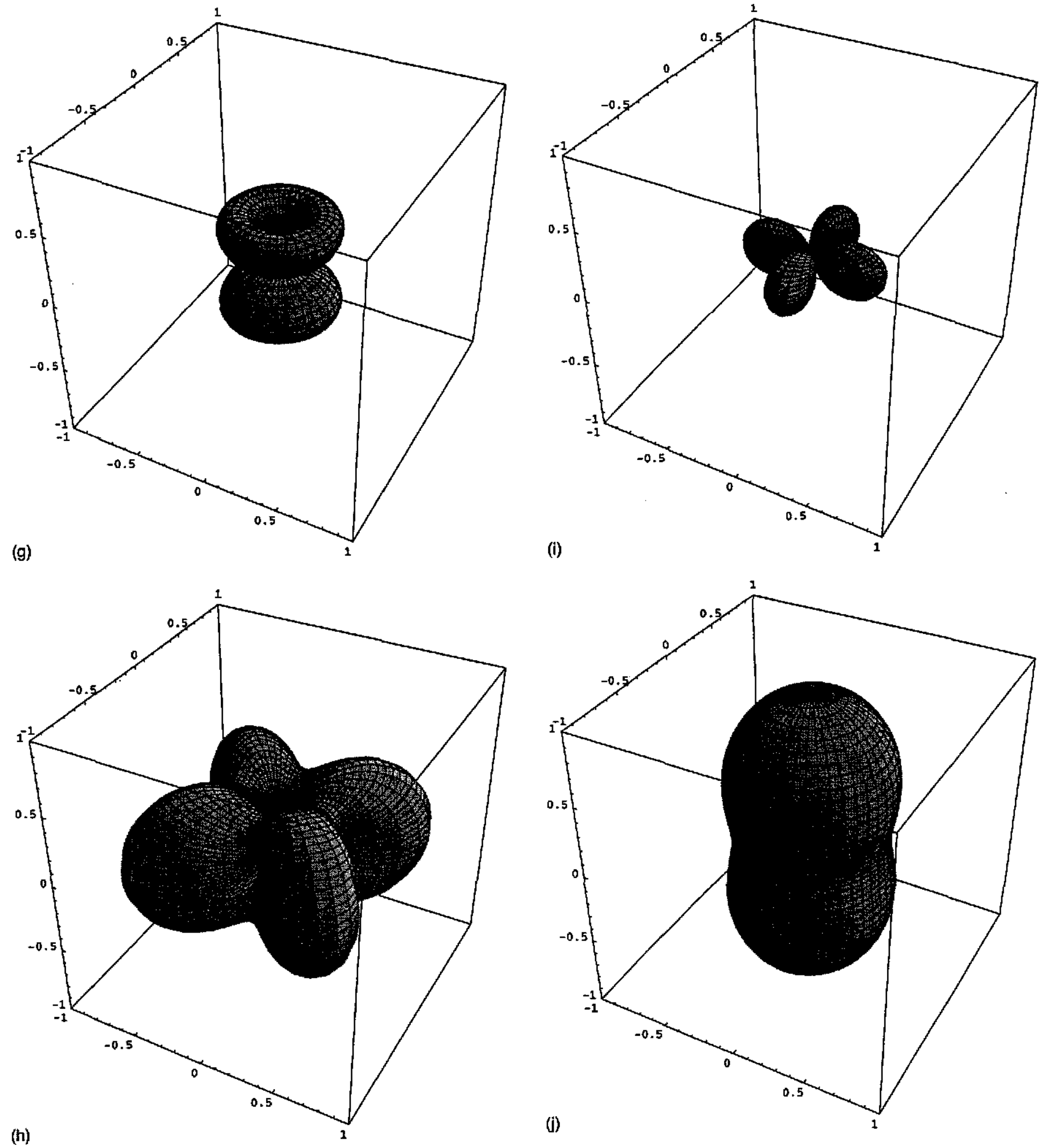

FIG. 2. (*Continued*).

mode and two differential. For more details on determining direction and waveform using the conventional interferometer, see Saulson (p. 255 of [6]) and Gürsel and Tinto [24].

In Brans-Dicke theory a large amount of transverse scalar radiation can exist as well as the quadrupole component, especially in the case of symmetrical collapse. Suppose now that a pure transverse scalar wave was incident on the detector, the ratio of the differential to common-mode response is then given by

$$\gamma_{scal}=\sqrt{\frac{(\frac{1}{2}\cos 2\phi \sin^2\theta)^2}{(\frac{1}{2}(1+\cos^2\theta))^2}}. \tag{10}$$

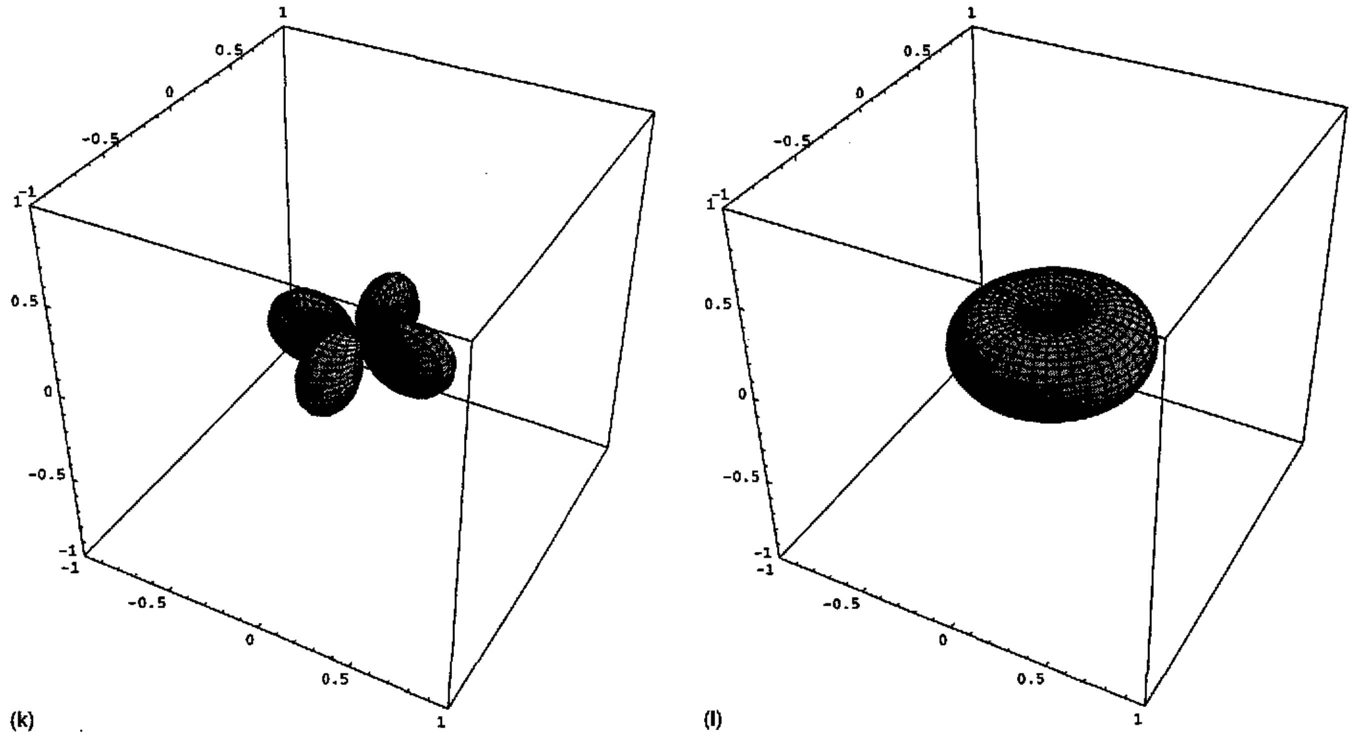

FIG. 2. *(Continued).*

A contour plot of this function is shown in Fig. 4. For this case the common-mode response is always greater than the differential response, except for when the radiation is incident along the x or y axis of the interferometer, at these points they are equal.

For a specific value of γ_{scal} four possible contours must be considered. In general the contour plot is different to the quadrupole plot of Fig. 3. The only plane where they are equal is in the x-y plane. Thus, if the direction of the gravitational wave source can be determined using the time difference technique of four detectors, then the amount of differential to common-mode response could uniquely determine the scalar and quadrupole content of the radiation. The exception is in the x-y plane where a transverse unpo-

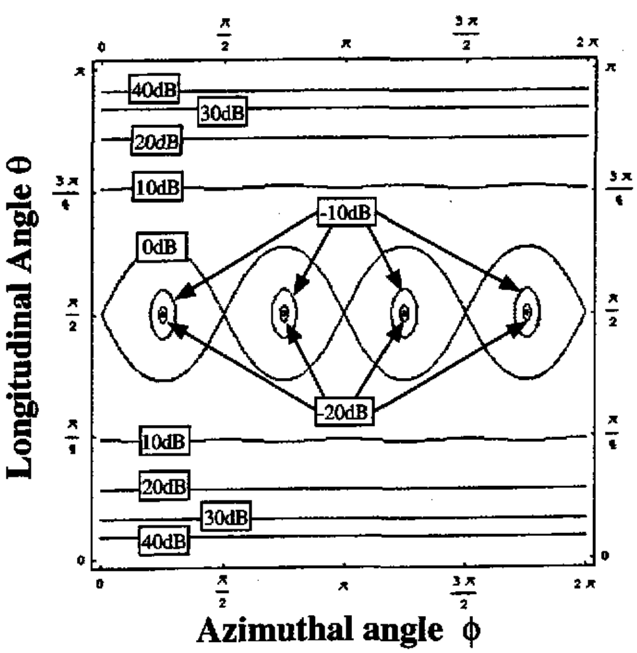

FIG. 3. Contour plot of γ_{quad} as a function of azimutal angle ϕ and longitudinal angle θ.

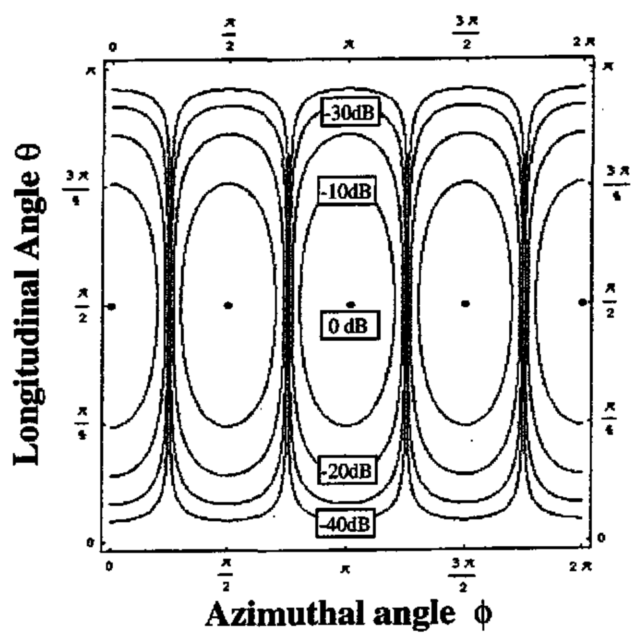

FIG. 4. Contour plot of γ_{scal} as a function of azimutal angle ϕ and longitudinal angle θ.