



Laser Interferometer Gravitational Wave Observatory

**Optimization of thermal noise in multi-loop pendulum
suspensions for use in interferometric gravitational-wave
detectors**

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Abstract

We study the thermal-noise spectrum of multi-loop pendulum suspensions for test masses in interferometric gravitational-wave detectors. The dependence of the thermal noise on suspension parameters and on properties of the wire material is discussed for the situation in which the losses are dominated by the internal friction in the pendulum wires.

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1 Introduction

The thermal noise is expected to be one of the main limitations on the sensitivity of long-baseline interferometric gravitational-wave detectors like LIGO and VIRGO [1,2]. Thermal fluctuations of internal modes of the interferometer's test masses and of suspension modes will dominate the noise spectrum at the important frequency range between 50 and 200 Hz (seismic noise and photon shot noise dominate for lower and higher frequencies, respectively). The thermal fluctuations in pendulum suspensions were studied both theoretically and experimentally in a number of works (see, e.g. Refs. [3–15]). The predictions of the thermal-noise spectrum in interferometric gravitational-wave detectors combine theoretical models (with the fluctuation-dissipation theorem of statistical mechanics [16] serving as a basis) and experimental measurements of quality factors of systems and materials involved. It is usually assumed that losses in the suspensions will occur mainly due to the internal friction in the wires, which is related to anelasticity effects [17,18]. This assumption will be correct only provided that all the losses due to interactions with the external world (friction in the residual gas, dumping by eddy currents, recoil losses into the seismic isolation system, friction in the suspension clamps, etc.) are made insignificant by careful experimental design.

In the present work we consider a multi-loop pendulum suspension and study the dependence of the thermal-noise spectrum on properties of the wire material and on suspension

parameters. The thermal-noise spectral density $x^2(\omega)$ depends strongly on the type of the internal friction in the wires. We consider two possibilities: (i) the wire internal friction with a constant loss function and (ii) the thermoelastic damping mechanism [19]. The main conclusion is that the thermal noise can be reduced by increasing the number of suspension wires, especially in the case of the thermoelastic damping. This conclusion is valid as long as the dissipation due to the friction in the suspension clamps is insignificant.

2 Thermal-noise spectrum for a pendulum suspension

In interferometric gravitational-wave detectors, the test masses are suspended as pendulums by one or two loops of thin wires. We will consider a multi-loop suspension with the wires attached to the bob near the horizontal plane which cuts the bob through its center of mass. We will also assume that the mass of the wires is much smaller than the mass of the bob. In such a multi-loop suspension the rocking motion of the test mass is essentially suppressed and the main contribution to the thermal-noise spectrum is due to the pendulum mode and the violin modes. Then one can write the suspension thermal-noise spectral density as a sum,

$$x^2(\omega) = x_p^2(\omega) + x_v^2(\omega), \quad (1)$$

of the pendulum-mode contribution, $x_p^2(\omega)$, and of the violin-modes contribution, $x_v^2(\omega)$.

According to the fluctuation-dissipation theorem, the pendulum-mode contribution can be expressed as [3]

$$x_p^2(\omega) = \frac{4k_B\mathcal{T}}{\omega M} \frac{\omega_p^2 \phi_p(\omega)}{(\omega_p^2 - \omega^2)^2 + \omega_p^4 \phi_p^2}, \quad (2)$$

where k_B is Boltzmann's constant, \mathcal{T} is the temperature, M is the pendulum mass, $\phi_p(\omega)$ is the loss function, $\omega_p = (g/L)^{1/2}$ is the pendulum frequency, g is the acceleration due to the Earth gravity field, and L is the pendulum length. Note that the spectral density $x^2(\omega)$ is written explicitly in terms of the angular frequency ω , but in fact the density is with respect to the linear frequency $f = \omega/2\pi$ and $x^2(\omega)$ is measured in units of m^2/Hz .

The loss function ϕ is a measure of the energy dissipation. Let \mathcal{E} be the total energy of a dissipative oscillator (assuming that the losses are small) and $\Delta\mathcal{E}$ be the energy dissipated per cycle. Then

$$\phi = \frac{\Delta\mathcal{E}}{2\pi\mathcal{E}}. \quad (3)$$

The energy of the pendulum consists of two parts: the gravitational energy \mathcal{E}_{gr} and the elastic energy \mathcal{E}_{el} due to the bending of the wire. The gravitational energy is lossless; provided that

all the losses due to interactions with the external world are made insignificant by careful experimental design, the assumption is made that the losses are dominated by internal friction in the wire material. Consequently, $\Delta\mathcal{E} = \Delta\mathcal{E}_{\text{el}}$, and one obtains

$$\phi_{\text{p}} = \xi_{\text{p}}\phi_{\text{w}}, \quad (4)$$

where $\phi_{\text{w}} = \Delta\mathcal{E}_{\text{el}}/(2\pi\mathcal{E}_{\text{el}})$ is the loss function for the wire itself which occurs due to anelastic effects in the wire material, and $\xi_{\text{p}} = (\mathcal{E}_{\text{el}}/\mathcal{E}_{\text{gr}})_{\text{p}}$ is the ratio between the elastic energy and the gravitational energy for the pendulum mode. The elastic energy depends on how many wires are used and how they are attached to the pendulum bob. In the multi-loop configuration we consider, the wires bend both at the top and the bottom, so $\xi_{\text{p}} \simeq (k_e L)^{-1}$, where $k_e^{-1} \simeq (EI/T)^{1/2}$ is the characteristic distance scale over which the bending occurs. Here, T is the tension force in the wire, E is the Young modulus of the wire material, and I is the moment of inertia of the wire cross section ($I = \frac{1}{2}\pi r^4$ for a cylindrical wire of radius r). For a suspension with N wires (the number of wires is twice the number of loops), $T = Mg/N$, and one obtains

$$\xi_{\text{p}} \simeq \frac{N\sqrt{TEI}}{MgL} = \frac{1}{L}\sqrt{\frac{EIN}{Mg}}. \quad (5)$$

For LIGO suspensions, $f_{\text{p}} = \omega_{\text{p}}/2\pi$ is about 1 Hz. This is much below the working frequency range (near 100 Hz), so we may assume $\omega_{\text{p}}/\omega \ll 1$. Also, the loss function is very small, $\phi_{\text{p}} < 10^{-5}$. Then the pendulum-mode contribution to the thermal noise spectrum is

$$x_{\text{p}}^2(\omega) \simeq \frac{4k_B\mathcal{T}\omega_{\text{p}}^2\phi_{\text{p}}(\omega)}{M\omega^5} = \frac{4k_B\mathcal{T}}{L^2}\sqrt{\frac{gEIN}{M^3}}\frac{\phi_{\text{w}}(\omega)}{\omega^5}. \quad (6)$$

The contribution of the violin modes to the thermal noise spectrum is given by

$$x_{\text{v}}^2(\omega) = \frac{4k_B\mathcal{T}}{\omega} \sum_{n=1}^{\infty} \frac{\mu_n^{-1}\omega_n^2\phi_n(\omega)}{(\omega_n^2 - \omega^2)^2 + \omega_n^4\phi_n^2}, \quad (7)$$

where $n = 1, 2, 3, \dots$ is the mode number. The angular frequency of the n th mode is

$$\omega_n = \frac{n\pi}{L}\sqrt{\frac{T}{\rho}} \left[1 + \frac{2}{k_e L} + \frac{1}{2} \left(\frac{n\pi}{k_e L} \right)^2 \right], \quad (8)$$

where ρ is the linear mass density of the wire. For heavily loaded thin wires like in LIGO, $k_e^{-1} \ll L$, so

$$\omega_n \simeq \frac{n\pi}{L}\sqrt{\frac{T}{\rho}}. \quad (9)$$

This is just the angular frequency of the n th transverse vibrational mode of an ideal spring. The effective mass of the n th violin mode is

$$\mu_n = \frac{1}{2}NM \left(\frac{\omega_n}{\omega_p} \right)^2 \simeq \frac{\pi^2 M^2}{2\rho L} n^2, \quad (10)$$

where we took expression (9) for ω_n and $T = Mg/N$. This effective mass arises because the violin vibrations of the wire cause only a tiny recoil of the test mass M . The loss function for the n th violin mode is

$$\phi_n = \xi_n \phi_w, \quad (11)$$

where $\xi_n = (\mathcal{E}_{\text{el}}/\mathcal{E}_{\text{gr}})_n$ is the ratio between the elastic energy and the gravitational energy. This ratio is [4]

$$\xi_n = \frac{2}{k_e L} \left(1 + \frac{n^2 \pi^2}{2k_e L} \right). \quad (12)$$

Since $k_e L \gg 1$, for first several modes the energy ratio is approximately

$$\xi_n \simeq \xi_v = \frac{2}{L} \sqrt{\frac{EIN}{Mg}}. \quad (13)$$

This expression takes into account only the contribution to the elastic energy due to wire bending near the top and the bottom. For higher violin modes, one should also consider the contribution due to wire bending along its length, which leads to Eq. (12).

Typical values of $f_1 = \omega_1/2\pi$ are from 250 to 500 Hz. If we are interested in the thermal spectral density near 100 Hz, we can assume $\omega^2 \ll \omega_n^2$. Then we have approximately

$$x_v^2(\omega) \simeq \frac{8k_B \mathcal{T} \omega_p^2}{NM\omega} \sum_{n=1}^{\infty} \frac{\phi_n(\omega)}{\omega_n^4} \simeq \frac{8k_B \mathcal{T} N \rho^2 L^3}{\pi^4 g M^3 \omega} \sum_{n=1}^{\infty} \frac{\phi_n(\omega)}{n^4}. \quad (14)$$

One can see that the contributions of higher violin modes are very small due to the factor n^{-4} in the sum. Taking $\phi_n = \xi_n \phi_w$ and assuming $k_e L \gg 1$, we find the following expression for the violin-mode contribution to the thermal-noise spectrum,

$$x_v^2(\omega) \simeq \frac{8}{45} k_B \mathcal{T} \rho^2 L^2 \sqrt{\frac{EIN^3}{g^3 M^7}} \frac{\phi_w(\omega)}{\omega}. \quad (15)$$

3 Dependence of thermal noise on wire material and suspension parameters

It can be seen from Eqs. (6) and (15) that the thermal noise increases with the area A of the wire cross section. Therefore, it is desirable to use wires as thin as possible. However, the wire thickness may not be too small since the stress $\sigma = T/A$ in the wire may not exceed the breaking stress σ_{br} . In fact, the wires are always operated at a fixed fraction of their breaking stress,

$$\sigma = \sigma_0 = \kappa\sigma_{\text{br}}, \quad (16)$$

where κ is a numerical coefficient. Typical values of κ are from 0.3 to 0.5 (it is undesirable to have larger values of κ because then events of spontaneous stress release will contribute excess noise [20]). Thus for a given type of the wire material, the cross-section area A should be proportional to the pendulum mass M , according to the relation $\sigma_0 = Mg/(NA)$. For a cylindrical wire, one has $I = A^2/2\pi$. Then we obtain

$$x_p^2(\omega) = \frac{4k_B\mathcal{T}}{L^2} \left(\frac{g^3 E}{2\pi MN\sigma_0^2} \right)^{1/2} \frac{\phi_w}{\omega^5}, \quad (17)$$

$$x_v^2(\omega) = \frac{8}{45} k_B\mathcal{T} \rho_v^2 L^2 \left(\frac{g^3 E}{2\pi MN^3\sigma_0^6} \right)^{1/2} \frac{\phi_w}{\omega}, \quad (18)$$

where $\rho_v = \rho/A$ is the volume mass density of the wire which depends only on the material used.

All the parameters in Eqs. (17) and (18) are easily measured except for the wire loss function ϕ_w . A number of experiments were recently performed [7,14,15,21–24] to study internal losses of various wire materials (e.g., steel, tungsten, fused quartz, and some others). However, the exact frequency dependence of the wire loss function $\phi_w(\omega)$ is not yet completely understood. In many experiments ϕ_w was measured only at few frequencies and experimental uncertainty of results was often quite large. Moreover, there are discrepancies between results of different experiments. Therefore, it is sometimes difficult to make certain conclusions about the behavior of $\phi_w(\omega)$.

A well known dissipation mechanism for thin samples in flexure is the so-called thermoelastic damping [19]. As a wire bends, one side contracts and heats and the other expands and cools. The resulting thermal diffusion leads to the dissipation of energy. The corresponding loss function is

$$\phi_w(\omega) = \Delta \frac{\omega\bar{\tau}}{1 + \omega^2\bar{\tau}^2}, \quad (19)$$

where Δ is the relaxation strength and $\bar{\tau}$ is the relaxation time. The loss function has its maximum $\phi = \Delta/2$ at $\omega = \bar{\tau}^{-1}$ (this is called the Debye peak). This behavior is character-

istic for processes in which the relaxation of stress and strain is exponential and occurs via a diffusion mechanism. For the thermoelastic damping, one has [19]

$$\Delta = \frac{E\mathcal{T}\alpha^2}{C_v}, \quad \bar{\tau} \simeq \frac{d^2}{D}, \quad (20)$$

where α is the linear thermal expansion coefficient, C_v is the specific heat per unit volume, d is the characteristic distance heat must flow, and D is the thermal diffusion coefficient, $D = \varrho/C_v$, where ϱ is the thermal conductivity. For a cylindrical wire of diameter d , the frequency of the Debye peak is

$$\bar{f} = \frac{1}{2\pi\bar{\tau}} \simeq 2.6 \frac{D}{d^2}. \quad (21)$$

For thin metallic wires ($d \sim 100 \mu\text{m}$) at the room temperature, the Debye peak frequency is typically from few hundred Hz to few kHz. Therefore at the frequency range near 100 Hz, we are usually far below the Debye peak, and

$$\phi_w(\omega) \simeq \Delta\omega\bar{\tau} = \beta A\omega, \quad (22)$$

where $\beta \simeq \Delta/(1.3\pi^2 D)$.

According to a recent experiment by Huang and Saulson [15], internal losses in stainless steel wires are in good agreement with predictions of thermoelastic damping, with $\phi_w(\omega)$ exhibiting the characteristic frequency dependence of Eq. (19). On the other hand, the loss function for tungsten wires was nearly constant, increasing slightly at high frequencies (above 500 Hz). ϕ_w for tungsten wires increased with the wire cross-section area A , but the exact functional dependence of ϕ_w on A is unclear as only three different wire diameters were examined. In some other experiments, the loss functions for various materials were found to be nearly constant over a wide frequency range. In a recent experiment by Cagnoli et al. [24], internal damping of a variety of metallic wires was found to be well modelled by the loss function of the form

$$\phi_w(\omega) = \phi_0 + \phi_{\text{ted}}(\omega), \quad (23)$$

where $\phi_{\text{ted}}(\omega)$ is the thermoelastic-damping loss function of Eq. (19) and ϕ_0 is a frequency-independent term. Unfortunately, the dependence of ϕ_0 on the wire diameter was not examined. It can be assumed that the thermoelastic damping is a basic dissipation mechanism, but for some materials it is masked by other processes. When those additional losses (whose nature is still a matter of controversy) are small, the characteristic frequency dependence of Eq. (19) may be observed. However, when the losses due to the thermoelastic damping are very small (which happens, for example, in the case of thin Invar and tungsten wires), then additional losses prevail, leading to ϕ_w which is nearly constant far from the Debye peak.

In what follows we will consider two possibilities: (i) a constant loss function ϕ_w and (ii) the loss function of Eq. (22) which is characteristic for the thermoelastic damping at frequencies well below the Debye peak. We might assume that for some materials the true behavior is somewhere between these two extreme variants. For example, for tungsten wires, ϕ_w is nearly frequency-independent from 50 to 500 Hz, but still increases to some extent with the wire cross-section area A , as one should expect from Eq. (22).

3.1 A constant loss function

For a constant ϕ_w , the dependence of the thermal-noise spectrum on various parameters is given directly by Eqs. (17) and (18). For the pendulum-mode contribution, we find

$$\begin{aligned} \text{for constant } M \text{ and } \sigma_0, x_p^2 &\propto N^{-1/2}; \\ \text{for constant } M \text{ and } N, x_p^2 &\propto \sigma_0^{-1}; \\ \text{for constant } N \text{ and } \sigma_0, x_p^2 &\propto M^{-1/2}. \end{aligned}$$

For the violin-modes contribution, we find

$$\begin{aligned} \text{for constant } M \text{ and } \sigma_0, x_v^2 &\propto N^{-3/2}; \\ \text{for constant } M \text{ and } N, x_v^2 &\propto \sigma_0^{-3}; \\ \text{for constant } N \text{ and } \sigma_0, x_v^2 &\propto M^{-1/2}. \end{aligned}$$

The allowed stress σ_0 is a property of the wire material (which is also true for E , ρ_v , and ϕ_w), so changing σ_0 means taking wires made of different materials. Clearly, it is desirable to have a material with a large value of σ_0 , but what decides is the value of the factor $\Lambda_w = E^{1/2}\phi_w/\sigma_0$ for the pendulum mode and $\Lambda_w = \rho_v^2 E^{1/2}\phi_w/\sigma_0^3$ for the violin modes. The factor Λ_w comprises all the parameters in x^2 which characterize the wire material.

One may see that taking multi-loop suspensions with large numbers of wires may help to reduce the thermal noise. As an example, let us consider tungsten wires of the type examined by Huang and Saulson [15]. The relevant parameters are $E \simeq 3.4 \times 10^{11}$ Pa, $\sigma_{br} \simeq 1671$ MPa, $\rho_v \simeq 1.93 \times 10^4$ kg/m³. We also take $M = 10.8$ kg, $L = 45$ cm and $\kappa = 0.5$ (the wires are operated at one half of their breaking stress), like in suspensions of the LIGO test masses. According to the data by Huang and Saulson [15], the loss function is nearly frequency-independent from 50 to 500 Hz, but depends on the wire diameter. For a two-loop suspension ($N = 4$), the wire diameter should be $d \simeq 200$ μ m, and the corresponding quality factor $Q_w = \phi_w^{-1}$ can be estimated to be $Q_w \simeq 1.3 \times 10^3$. For an eight-loop suspension ($N = 16$), the wire diameter should be $d \simeq 100$ μ m, and the corresponding quality factor can be estimated to be $Q_w \simeq 4.0 \times 10^3$. In Fig. 1 we plot the thermal-noise displacement spectrum $\sqrt{x^2(\omega)}$ for the room temperature ($\mathcal{T} = 295$ K) for three possibilities: (a) $N = 4$, $Q_w = 1.3 \times 10^3$; (b) $N = 16$, $Q_w = 1.3 \times 10^3$; (c) $N = 16$, $Q_w = 4.0 \times 10^3$. We see that for a constant loss function, the thermal noise is reduced by increasing the number of wires. The spectral density $x^2(\omega)$ scales as $N^{-1/2}$ for frequencies near 100 Hz (where the pendulum

mode dominates), in accordance with our analysis. Also, if the decrease of ϕ_w with the wire diameter is taken into account, the increase in the number of wires is even more helpful.

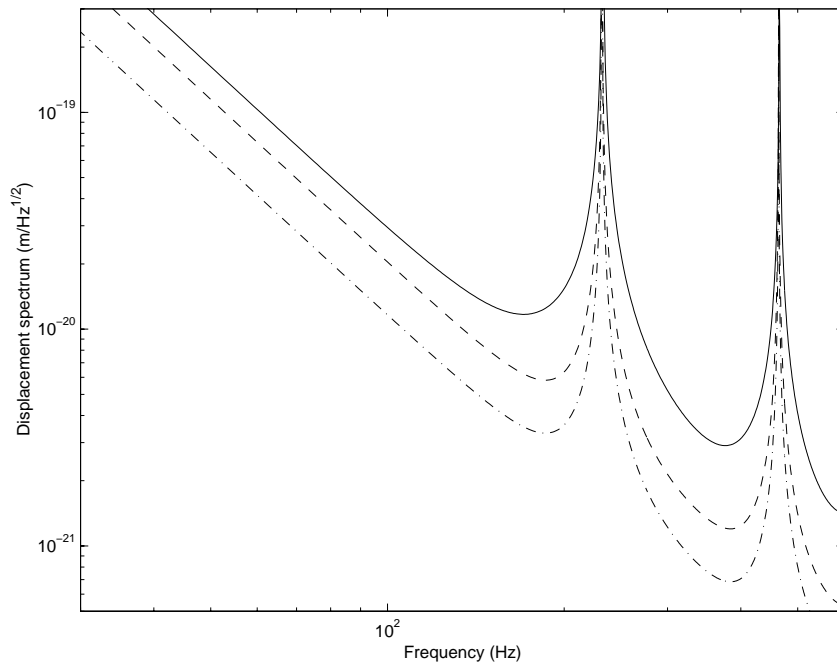


Fig. 1. The thermal-noise displacement spectrum $\sqrt{x^2(\omega)}$ for a multi-loop pendulum suspension with tungsten wires: $N = 4$, $Q_w = 1.3 \times 10^3$ (solid line); $N = 16$, $Q_w = 1.3 \times 10^3$ (dashed line); $N = 16$, $Q_w = 4.0 \times 10^3$ (dash-dot line).

3.2 Thermoelastic loss function

If we take the loss function of Eq. (22), then the thermal-noise spectrum is given by

$$x_p^2(\omega) = \frac{4k_B\mathcal{T}}{L^2}\beta \left(\frac{g^5 EM}{2\pi N^3 \sigma_0^4} \right)^{1/2} \frac{1}{\omega^4}, \quad (24)$$

$$x_v^2(\omega) = \frac{8}{45}k_B\mathcal{T}\beta\rho_v^2 L^2 \left(\frac{g^5 EM}{2\pi N^5 \sigma_0^8} \right)^{1/2}. \quad (25)$$

The dependence of the thermal-noise spectrum on various parameters can be characterized as follows. For the pendulum-mode contribution, we find

- for constant M and σ_0 , $x_p^2 \propto N^{-3/2}$;
- for constant M and N , $x_p^2 \propto \sigma_0^{-2}$;
- for constant N and σ_0 , $x_p^2 \propto M^{1/2}$.

For the violin-modes contribution, we find

for constant M and σ_0 , $x_v^2 \propto N^{-5/2}$;
for constant M and N , $x_v^2 \propto \sigma_0^{-4}$;
for constant N and σ_0 , $x_v^2 \propto M^{1/2}$.

Now, the dependence of x^2 on the wire material is given by the factor $\Lambda_w = \beta E^{1/2} / \sigma_0^2$ for the pendulum mode and $\Lambda_w = \beta \rho_v^2 E^{1/2} / \sigma_0^4$ for the violin modes. So, the value of the allowed stress σ_0 in this situation is more important than for the case of constant ϕ_w .

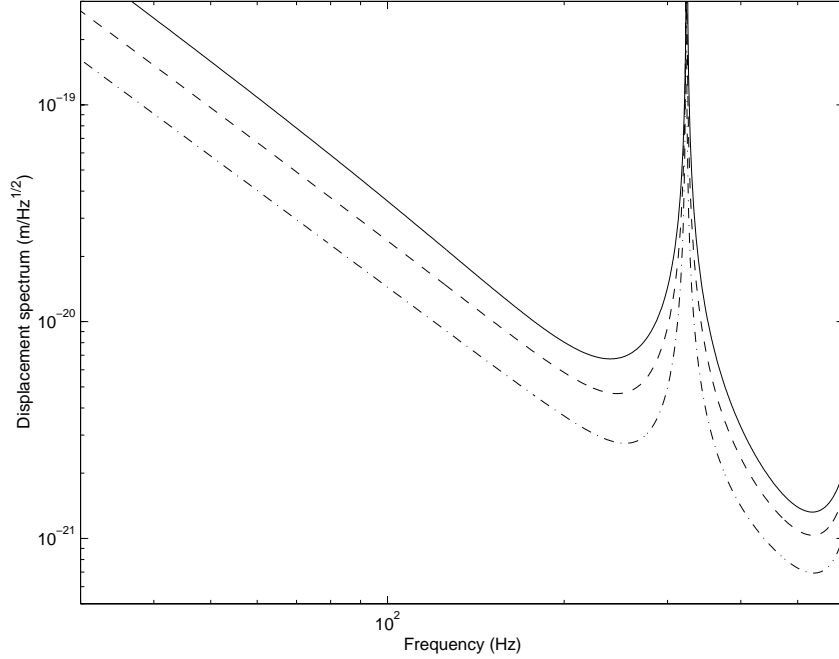


Fig. 2. The thermal-noise displacement spectrum $\sqrt{x^2(\omega)}$ for a multi-loop pendulum suspension with stainless steel wires: $N = 4$ (solid line); $N = 8$ (dashed line); $N = 16$ (dash-dot line).

One may see that in the case of the thermoelastic damping the thermal noise may be reduced to a larger extent by increasing the number of wires, as compared to the case of constant ϕ_w . As an example, let us consider wires made of stainless steel (AISI 302), which were examined by Huang and Saulson [15]. The relevant parameters are $E \simeq 1.9 \times 10^{11}$ Pa, $\sigma_{br} \simeq 1342$ MPa, $\rho_v \simeq 8.0 \times 10^3$ kg/m³. The losses are dominated by the thermoelastic damping mechanism. Taking $\alpha \simeq 1.6 \times 10^{-5}$ 1/K, $C_v \simeq 4.8 \times 10^6$ J/(K m³), $\varrho \simeq 16.3$ J/(K m s) and $\mathcal{T} = 295$ K, one obtains $\Delta \simeq 3.0 \times 10^{-3}$ and $\beta \simeq 68.6$ s/m². We also take $M = 10.8$ kg, $L = 45$ cm and $\kappa = 0.5$, like in suspensions of the LIGO test masses. The thermal-noise displacement spectrum $\sqrt{x^2(\omega)}$ is plotted in Fig. 2 for three possibilities: (a) $N = 4$ (then $d \simeq 224$ μ m and $\bar{f} \simeq 176$ Hz); (b) $N = 8$ (then $d \simeq 159$ μ m, and $\bar{f} \simeq 352$ Hz); (c) $N = 16$ (then $d \simeq 112$ μ m, and $\bar{f} \simeq 703$ Hz). The conclusion is that the thermal noise may be significantly reduced by increasing the number of wires. The numerical results confirm that the proportionalities $x_p^2 \propto N^{-3/2}$ and $x_v^2 \propto N^{-5/2}$ are valid for frequencies well below the Debye peak \bar{f} .

3.3 Comparison between different materials

We would like to compare the thermal-noise performance of a multi-loop suspension for different wire materials. For example, the tungsten wires examined by Huang and Saulson [15] have rather low breaking stress of 1671 MPa. There exist tungsten wires with higher breaking stress; for example, Dawid and Kawamura [14] experimented with tungsten wires for which they measured $\sigma_{\text{br}} = 2037$ MPa. It would be interesting to compare between tungsten wires with different breaking stress but with the same loss function. On the other hand, the comparison between wires made of tungsten and stainless steel will clarify how the difference in the loss mechanism (frequency-independent ϕ_w versus the thermoelastic damping) affects the thermal-noise spectrum. To this end, we also would like to consider a situation in which wires made of stainless steel have all properties as above except for the losses being dominated by a mechanism with frequency-independent ϕ_w , instead of the thermoelastic damping.

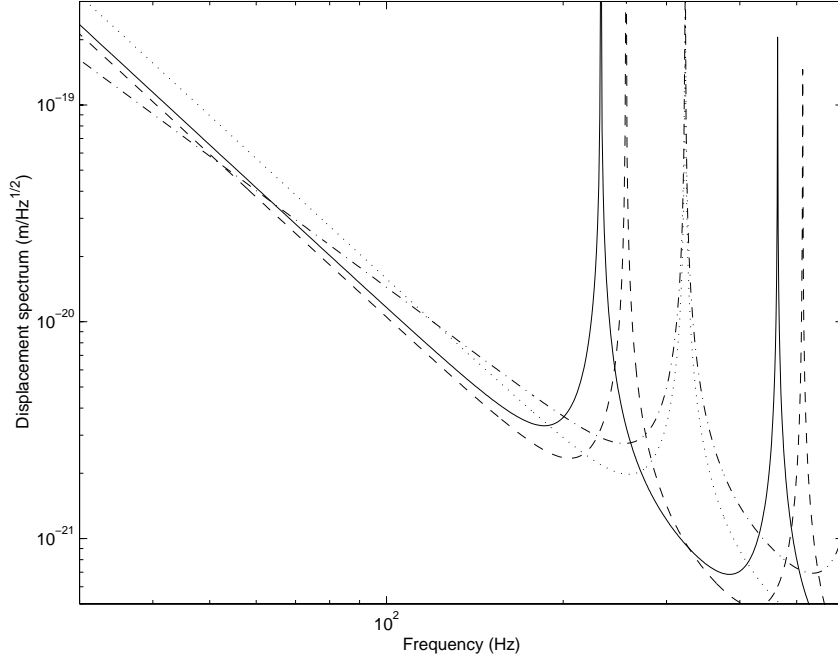


Fig. 3. The thermal-noise displacement spectrum $\sqrt{x^2(\omega)}$ for an eight-loop pendulum suspension ($N = 16$): tungsten wires with $\sigma_{\text{br}} = 1671$ MPa and $Q_w = 4.0 \times 10^3$ (solid line); tungsten wires with $\sigma_{\text{br}} = 2037$ MPa and $Q_w = 4.0 \times 10^3$ (dashed line); stainless steel wires with $\sigma_{\text{br}} = 1342$ MPa and thermoelastic damping (dash-dot line); stainless steel wires with $\sigma_{\text{br}} = 1342$ MPa and $Q = 2.0 \times 10^3$ (dotted line).

We consider an eight-loop suspension ($N = 16$), with $M = 10.8$ kg, $L = 45$ cm and $\kappa = 0.5$ (like in LIGO), and examine four possibilities: (a) tungsten wires as considered in see Sec 3.1, with $\sigma_{\text{br}} = 1671$ MPa (this gives $d \simeq 100$ μm) and $Q_w = 4.0 \times 10^3$; (b) tungsten wires with different breaking stress, $\sigma_{\text{br}} = 2037$ MPa (this gives $d \simeq 91$ μm) and the same quality factor, $Q_w = 4.0 \times 10^3$; (c) stainless steel wires as considered in Sec 3.2 ($\sigma_{\text{br}} = 1342$ MPa, $d \simeq 112$ μm), with the thermoelastic damping mechanism ($\beta \simeq 68.6$ s/m², $\bar{f} \simeq 703$

Hz); (d) stainless steel wires with the same parameters, but with a frequency-independent loss function, $Q = 2.0 \times 10^3$ (this value is close to the one given by the thermoelastic damping near 120 Hz). The resulting thermal-noise displacement spectra $\sqrt{x^2(\omega)}$ are shown in Fig. 3. One can see that the violin resonances of the stainless steel wires appear at higher frequencies (due to smaller density). On the other hand, the tungsten wires exhibit smaller thermal fluctuations at the frequency range between 50 and 200 Hz. The thermal noise is reduced by using wires with larger breaking stress, provided the other parameters remain the same.

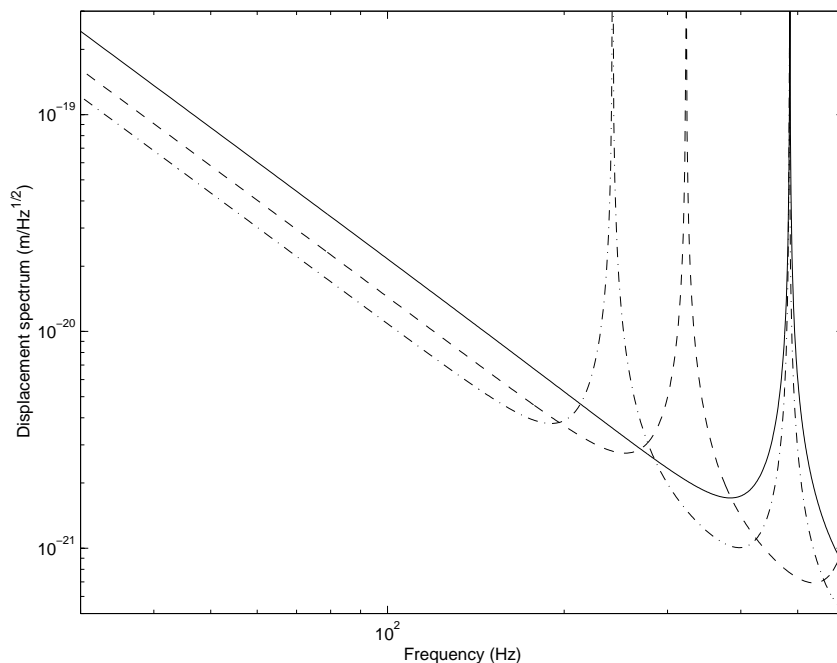


Fig. 4. The thermal-noise displacement spectrum $\sqrt{x^2(\omega)}$ for an eight-loop pendulum suspension ($N = 16$) with stainless steel wires of various length: $L = 30$ cm (solid line); $L = 45$ cm (dashed line); $L = 60$ cm (dash-dot line).

3.4 Optimization of the pendulum length

The thermal-noise spectrum depends on the pendulum length L . For frequencies well below the first violin resonance, $\omega^2 \ll \omega_1^2$, the pendulum-mode contribution dominates and the spectral density $x^2(\omega)$ is proportional to L^{-2} . However, by increasing L , one not only decreases the thermal fluctuations due to the pendulum mode, but also brings the violin resonances to lower frequencies, as $\omega_n \propto L^{-1}$. This effect is illustrated in Fig. 4, where the displacement spectrum $\sqrt{x^2(\omega)}$ is shown for an eight-loop suspension with stainless steel wires of various length. (We take $M = 10.8$ kg, $\kappa = 0.5$, and stainless steel wires with properties listed in Sec. 3.2.) Due to this competition between two opposite tendencies, the choice of the pendulum length is a delicate matter which depends on where in the spectrum the seismic perturbations and the photon shot noise prevail over the thermal fluctuations and on properties of expected gravitational-wave signals.

4 Discussion

Our analysis brings to an observation that the thermal noise in pendulum suspensions can be significantly reduced by using multi-loop configurations with a large number of wires. However, before implementing this conclusion one should consider a number of issues. First, our analysis is valid only if the losses are dominated by the internal friction in the pendulum wires and all other sources of dissipation are made negligible by careful experimental design. However, as was shown recently by Huang and Saulson [15], the sliding friction in the suspension clamps is often important as well. If this is the case, a large number of suspension loops will only sever the dissipation and thereby increase the thermal fluctuations. Therefore, if one wants to use multi-loop suspensions, a special care should be paid to the design of clamps. Another technical problem is to make a suspension in which all the loops will be equally loaded. One more issue which should be carefully studied is the effect which may have a large number of suspension wires on the internal resonances of the suspended test mass.

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