ELSEVIER

17 March 1997

PHYSICS LETTERS A

Physics Letters A 227 (1997) 153-158

The quality factor of natural fused quartz ribbons over a frequency range from 6 to 160 Hz

S. Rowan [1], R. Hutchins, A. McLaren, N.A. Robertson, S.M. Twyford, J. Hough

Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK

Received 21 October 1996; accepted for publication 2 January 1997
Communicated by P.R. Holland

Abstract

Measurements of the Q factor of oven pulled ribbons of standard fused quartz over a frequency range of 6 to 160 Hz give results close to those predicted by thermo-elastic damping. This material loss, maintained after breaking and rewelding, is consistent with suspension requirements of advanced gravitational wave detectors.

1. Introduction

Several large interferometric gravitational wave detectors are currently under construction in various countries around the world [1-3]. Each of these detectors relies on using laser interferometry to sense the differential strain, caused by the passage of gravitational waves, between mirrors suspended as pendulums.

It appears that thermal noise associated with the internal modes and suspensions of the test masses is likely to be the most important limitation to the sensitivity of such instruments and for the thermal noise to be of an acceptable level it is essential that the loss factors associated with the relevant modes and resonances are very low.

In the case of the GEO 600 detector the sensitivity model is based on thermal noise above 50 Hz being dominated by losses in the internal modes of the test masses. Using a reported value [4,5] for the material loss factor of fused silica of 2×10^{-7} thus sets a limit to the detector sensitivity of $7 \times 10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 50 Hz.

To achieve this sensitivity the contribution to the motion of each test mass at 50 Hz from the thermal noise of the pendulum mode of its suspension must be significantly less than the internal mode contribution. Thermal noise driven motion at a level of $1 \times 10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ from the pendulum mode thus seems desirable.

The power spectral density of the motion of the test mass, $\tilde{x}^2(\omega)$, is related to the total loss tangent, $\phi(\omega)$, of the suspension by [6]

$$\tilde{x}^2(\omega) = \frac{4k_{\mathrm{B}}Tw_0^2\phi(\omega)}{\omega m[(\omega_0^2-\omega^2)^2+\omega_0^4\phi^2(\omega)]}, \qquad (1)$$

where k_{B} is Boltzmann's constant, T is the temperature and m is the mass of the oscillator. Thus a loss tangent for the pendulum mode of approximately 4×10^{-8} is required. It should be noted that while $\tilde{x}^2(\omega)$ is expressed as a function of angular frequency, ω (units rads^{-1}) the density is with respect to linear frequency (Hz).

[1] E-mail: s.rowan@physics.gla.ac.uk.

PII S0375-9601(97)00042-X

Experiments [7] suggest that the most suitable suspension fibre material to allow such performance to be achieved is fused silica. A typical suspension design consists of a test mass of 16 kg suspended on two fibre loops or on four fibres. Following the discussion in the Appendix and assuming that there are no excess losses at the joints where the fibres leave the mass, the loss tangent of the pendulum mode of the suspension, $\phi_{\rm pend}(\omega)$, is related to the loss tangent of the material of the suspension fibres, $\phi_{\rm mat}(\omega)$, by

$$\frac{1}{\phi_{\rm pend}(\omega)} = \frac{1}{\phi_{\rm mat}(\omega)} \frac{mgl}{4\sqrt{TEI}}, \tag{2}$$

where m is the mass of the pendulum, l is the length of pendulum, T is the tension in each wire, E is the Young's modulus and I is the moment of each wire. If the fibres are operated at 33% of their breaking stress (assuming a breaking stress of approximately 800 MPa for fused silica) the above formula for the 16 kg test masses of GEO 600 suggests that a material loss factor of less than 5.6×10^{-6} is required. The experiments of Braginskii and colleagues [7] have demonstrated that better loss factors can be achieved in very carefully prepared fused silica fibres over a range of frequencies. Similarly Kovalik and Saulson [8] have achieved encouraging results with cylindrical fused quartz fibres.

In this paper we demonstrate that with standard normal grade fused quartz ribbon fibres drawn in a simple radio frequency (rf) oven, loss factors may be achieved which are more than adequate for the needs of GEO 600 and other gravitational wave detectors. We also show that welding of these fibres may be carried out with no observable increase in the losses of the material.

2. Experimental philosophy

At a resonance of a horizontally suspended fused quartz ribbon the loss tangent of the ribbon material, $\phi_{\rm mat}(\omega_0)$ is given by the inverse of the quality factor, $Q_{\rm mat}$, of the resonance of the ribbon [9]. The quality factors of the first four resonances of a ribbon fibre were measured under high vacuum conditions by measuring the ring down of the fibre resonances with time. For a resonating fibre, with no drive, the energy, E, decays as

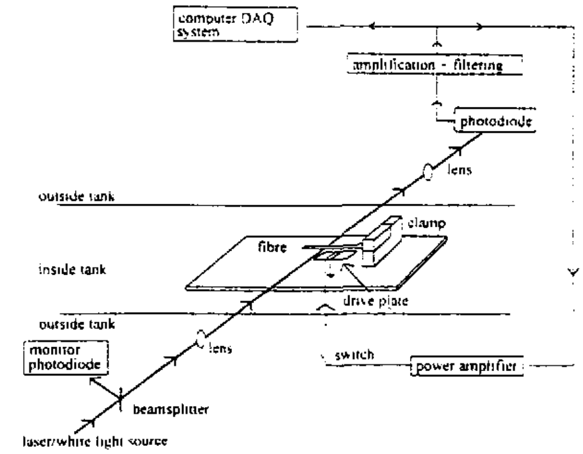

Fig. 1. Schematic diagram of experimental apparatus for measuring amplitude decay of fibre resonances.

$$E = (E_0 - E_b)e^{-t\omega_0/Q} + E_b, \tag{3}$$

where ω_0 is the angular resonant frequency of the fibre, E_0 is the initial energy of the fibre resonance and E_b is the background value of energy to which the resonance decays. The Q of the fibre material at angular frequency ω_0 is thus obtained from the gradient of the line $\ln(E - E_b)$ versus t.

The first ribbon tested, of length 12.5 cm, width 3 mm, and thickness approximately 54 μm, was drawn from a fused quartz slide in an rf induction furnace with a graphite susceptor. The end of the slide, of thickness 1 mm, was left attached to the ribbon to allow robust clamping between aluminium plates. This clamping arrangement should avoid problems associated with stick slip effects at the clamping line [10]. The clamp was rigidly attached to an aluminium table of thickness 25 mm supported by four aluminium legs of diameter 50 mm pressed rigidly against the wall of the stainless steel vacuum tank used. A vacuum level of approximately 2×10^{-7} mbar was achieved using a turbo molecular pump and an ion pump operating in parallel. The oscillatory motion was sensed by illuminating the fibre with the expanded beam from a diode laser or white light source and monitoring the diffraction/shadow pattern on a split photodiode, see Fig. 1.

The outputs from the two halves of the diode were subtracted, amplified and filtered before being recorded on a PC based data logging system. The fibre was excited by means of a two-strip capacitor plate placed underneath its end. The capacitor was biased to 750 V dc and the exciting signal was added on top of this. The bias and drive were disconnected and the plate grounded when ring down measurements were being made. The first four modes of the fibre were excited in turn by a positive feedback method in which the amplified and suitably filtered output from the sensor was applied in the correct phase to the drive capacitor. The mode frequencies of this fibre were found to be 6.06 Hz, 22.8 Hz, 59.6 Hz and 106 Hz.

3. Initial loss measurements

In order to check that the measured Q factors of the fibre were not being significantly limited by damping from the residual gas in the system the Q factor of the 22.8 Hz resonance was measured as a function of gas pressure in the system. The pressure was varied by switching off the pumps and allowing the system to slowly outgas. Our theoretical analysis suggests that, at an angular resonant frequency ω_0

$$\frac{1}{Q_{gas}} \propto \frac{1}{\omega_0}\sqrt{\frac{M}{RT}}P, \tag{4}$$

where M is the molecular weight of the gas of pressure P, T is temperature and R is the gas constant. We expect

$$\frac{1}{Q_{meas}} = \frac{1}{Q_{mat}} + \frac{1}{Q_{gas}} + \frac{1}{Q_{other}}, \tag{5}$$

where $1/Q_{meas}$ is the loss tangent of the system obtained experimentally, $1/Q_{gas}$ is the contribution to the measured loss tangent from gas damping of the resonance, $1/Q_{mat}$ is the intrinsic loss tangent of the material and $1/Q_{other}$ represents any other sources of loss present. A plot of $1/Q_{meas}$ as a function of P should thus intersect the y axis at a value equal to $(1/Q_{mat} + 1/Q_{other})$ allowing an upper limit to the material loss at 22.8 Hz to be found and thus a lower limit to the material Q of the ribbon fibre at that frequency to be estimated. A plot of $1/Q_{meas}$ as a function of P obtained as the system outgassed is shown in Fig. 2.

From this figure it can be deduced that for pressures below approximately 10^{-6} mbar gas damping is not significant and that the material Q at this frequency is of the order of 10^6.

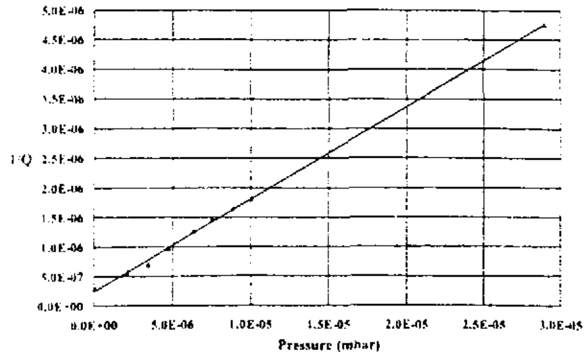

Fig. 2. Variation of $1/Q_{meas}$ for the 22.8 Hz resonance of a quartz ribbon fibre as the pressure in the tank was varied by turning off the vacuum pumps and allowing the system to outgas.

3.1. Gas damping observations

It is interesting to note that when Eqs. (4) and (5) are combined the gradient, m_{gas}, of a graph of $1/Q_{meas}$ against P is directly proportional to the square root of the molecular weight, $\sqrt{M}$, of the gas in the system such that

$$m_{gas} = \beta\sqrt{M}, \tag{6}$$

where β is a constant. If the constant in Eq. (6) is known the molecular weight of the gas contributing to the damping may then be found. In order to evaluate β the system was backfilled with nitrogen and the gradient of a plot of $1/Q$ against P calculated. A value for the gradient was obtained giving $\beta = 0.013 \pm 0.002$. To check this value the system was then backfilled with hydrogen. However in order to obtain an appropriate value for β for a case where the gas in the system is something other than nitrogen a correction factor, η, must be applied to the pressure readings as shown on the ion gauge for the system [11]. The results obtained using hydrogen suggested a value for β of 0.011 ± 0.003, consistent with the previous value within experimental errors.

For the case where the system was allowed to outgas we are thus left with a relationship between the molecular weight of the gas in the system and the appropriate gas correction factor given by

$$\frac{\sqrt{M}}{\eta} \sim 13 \pm 2. \tag{7}$$

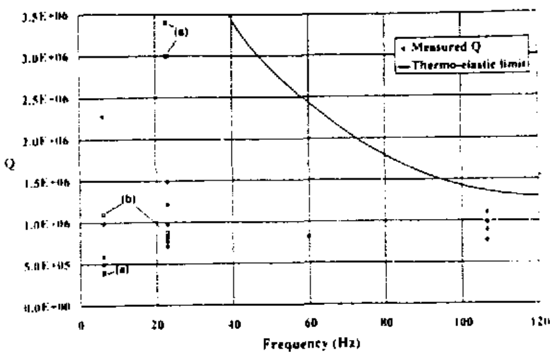

Fig. 3. Spread of measured Q values for the first four modes of a fused quartz ribbon and the limit to measurable Q set by thermo-elastic damping. The temperature in the laboratory when measurements labelled (a) were taken was significantly lower than when measurements labelled (b) were taken, approximately one month later.

A residual gas analyser sensitive to molecular weights up to a maximum of 80 was used to analyse the gas content of the system; however none of the gases found had correction factors consistent with the above relation. This leads us to suspect that when the system outgasses the gas responsible for damping of the fibre resonances contains molecules of very high molecular weight, perhaps produced by an epoxy used.

4. Results

A series of measurements at the best achievable vacuum, where gas damping effects were negligible, were made and the results are shown in Fig. 3.

The measurements were made over a period of several weeks during which the temperature of the laboratory rose by several degrees. Coupling between modes was an observed feature of the measurements and as the temperature rose it became impossible to obtain an exponential ringdown for the mode at 59.6 Hz. Instead the 59.6 Hz mode died very quickly and the energy was seen to transfer into the 6.06 Hz mode. Thus there is only one point recorded at 59.6 Hz. A similar effect was observed at 106 Hz. Also there is a significant spread of Q values for 6.06 Hz and 22.8 Hz. It should be noted that the best Q at 22.8 Hz corresponded to a relatively poor Q at 6.06 Hz when the laboratory temperature was particularly low and that as the laboratory temperature rose the Q at 6.06 Hz improved while the Q at 22.8 Hz reduced. The existence of mode coupling makes the results shown in Fig. 3 somewhat difficult to interpret in detail. However it can be clearly seen that the material used and method of fibre production adopted allow material Q factors of approximately 5×10^5 to a few 10^6 to be achieved which are significantly better than is required for the gravitational wave detectors under construction.

5. Thermo-elastic damping

It is expected that thermo-elastic damping [12] should be significant for the fused quartz ribbons under test. In particular the limit, Q_{te}, to the measured Q of the material set by thermo-elastic damping is given by

$$\frac{1}{Q_{te}} = \Delta \frac{\omega\tau}{1+\omega^2\tau^2}, \tag{8}$$

where

$$\Delta = \frac{E\alpha^2 T}{\rho C} \tag{9}$$

and $\tau = 1/2\pi f_{char}$ with the characteristic frequency, f_{char} of the fibre given by

$$f_{char} = \frac{\pi}{2}\frac{K}{\rho C t^2}, \tag{10}$$

with α being the coefficient of linear expansion, C being the specific heat capacity, ρ being the density, K the thermal conductivity, and t the thickness of the ribbon.

The thermo-elastic limit for the ribbon under test is shown in Fig. 3 and it can be seen that while the thermo-elastic effect is close to limiting the fibre at frequencies above approximately 100 Hz, there must be some other limiting mechanism, perhaps intrinsic to the ribbon production method, at lower frequency.

6. Further experiments

When using fused quartz as a suspension material it is very useful if quartz ribbons can be welded to each other or to fused quartz substrates for jointing purposes and it is important to determine whether such welding can be carried out without increasing the losses in the

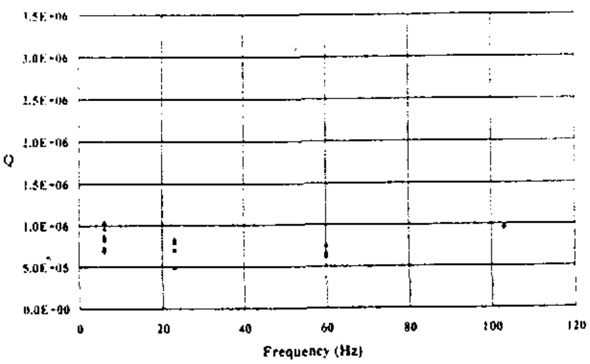

Fig. 4. Spread of measured Q values for the first four modes of quartz ribbon after having been broken and welded.

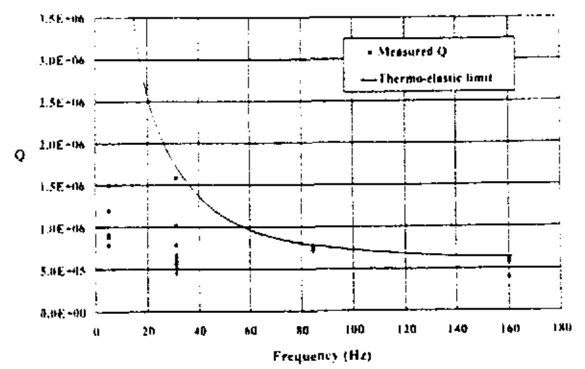

Fig. 5. Spread of measured Q values plus thermoelastic damping limit for second sample of fused quartz ribbon.

material. To investigate this the ribbon tested above was broken and the two pieces of the ribbon welded close to the neck at which the ribbon expands into the thicker slide. The Q values for the first four modes of the ribbon were remeasured, the mode frequencies being only slightly changed. The results are shown in Fig. 4 from which it can be seen that the welding procedure has not caused any apparent increase in loss.

To investigate the variation of material Q values between samples a second fused quartz ribbon was tested with approximate dimensions of 14 cm length, 1 mm width and 90 μm thickness. The measured Q values for the first four ribbon resonances along with the appropriate thermoelastic damping limit for this ribbon are shown in Fig. 5.

The measured Q factors for this ribbon can be seen to be of the order of 10^6 in agreement with the results for the previous measurements.

To check that the geometry of the ribbon was not significantly affecting the values of Q factor obtained, a cylindrical fused quartz fibre was tested. The measured Q factors were found to be of the same order as for the quartz ribbons.

7. Conclusions

In the frequency range 6 to 160 Hz the material loss factor of standard fused quartz ribbons, drawn from a slide in an rf induction furnace, is reproducibly found to be of the order of 2×10^{-6} to 10^{-6}, a level where this fused quartz should be an excellent material for the suspension fibres for the test masses of gravitational wave detectors such as GEO 600. Welding of a ribbon appeared to have a negligible effect on the material losses. Measurements of the material loss of a cylindrical fused quartz fibre gave values of the same order as for the ribbons.

Acknowledgement

The authors wish to thank our colleagues in the gravitational waves group at Glasgow, the University of Hannover and the Max-Planck-Institut für Quantenoptik, Garching for their interest in this work. We also wish to thank J. Faller of JILA, Boulder for experimental help and J. Logan of Caltech, Pasadena for useful discussions. Financial support for this work was provided by a research grant from PPARC and the University of Glasgow.

Appendix A. Brief discussion of the relationship between the loss tangent of a pendulum and the material loss tangent of the pendulum suspension fibres

There have been several publications on the relationships of the loss tangent of a pendulum, $\phi_{\text{pend}}(\omega)$, to that of the violin modes of the pendulum suspension wires, $\phi_{\text{violin}}(\omega)$, [13,14], and to that of the material of the suspension wires, $\phi_{\text{mat}}(\omega)$ [6]. In the latter case, for a pendulum suspended by n wires the following relationship is derived,

$$\phi_{\rm pend}(\omega) = \phi_{\rm mat}(\omega)\frac{n\sqrt{TEI}}{2mgl}, \qquad (A.1)$$

where m is the mass of the pendulum, l is the length of pendulum, T is the tension in each wire, E is the Young's modulus and I is the moment of each wire. Study of Refs. [13–15] suggests that this formula is a description of the situation where the wires are in one plane, a plane through the center of mass of the pendulum whose normal is parallel to the direction of swing of the pendulum. In this arrangement the suspension wires bend only at the top when the pendulum swings. If the wires are not all in this plane the pendulum mass is constrained to move essentially without rocking such that bending takes place both at the top and bottom of the suspension wires; thus the contribution of the material loss to that of the pendulum is increased by a factor of two. For the GEO 600 suspension design discussed in Section 1 the relationship between the loss factor of the pendulum and the loss factor of the material becomes

$$\phi_{\rm pend}(\omega) = \phi_{\rm mat}(\omega)\frac{n\sqrt{TEI}}{mgl}, \qquad (A.2)$$

where $n = 4$.

References

[1] K. Danzmann et al., GEO 600: Proposal for a 600 m laser interferometric gravitational wave antenna, Max-Planck-Institut für Quantenoptik Report 190, Garching, Germany (1994).

[2] R.E. Vogt, R.W.P. Drever, K.S. Thorne, F.J. Raab and R. Weiss, A laser interferometer gravitational-wave observatory (LIGO), Proposal to The National Science Foundation (1989).

[3] A. Brillet et al., VIRGO final conceptual design (1992).

[4] A. Gillespie, Thermal noise in the initial LIGO interferometers, Ph.D. thesis (1995).

[5] S. Traeger, private communication.

[6] P.R. Saulson, Phys. Rev. D 42 (1990) 2437.

[7] V.B. Braginskii, V.P. Mitrofanov and S.P. Vyatchanin, Rev. Sci. Instrum. 65 (1994) 12 3771.

[8] J. Kovalik and P.R. Saulson, Rev. Sci. Instrum. 64 (1993) 10

[9] A.S. Nowick and B.S. Berry, Anelastic relaxation in crystalline solids (Academic Press, New York, 1972).

[10] T.J. Quinn, C.C. Speake, R.S. Davis and W. Tew, Phys. Lett. A 197 (1995) 197.

[11] Edwards High Vacuum International, Instruction manual, Active inverted magnetron gauge.

[12] C. Zener, Phys. Rev. 52 (1937) 230.

[13] G.I. Gonzlez and P.R. Saulson, J. Acoust. Soc. Am. 96 (1994) 207

[14] J.E. Logan, J. Hough and N.A. Robertson, Phys. Lett. A 183 (1993) 145-152

[15] A. Gillespie and F. Raab, Phys. Lett A 178 (1993) 357-363