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Alignment of an Interferometric Gravitational Wave Detector

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Abstract

Interferometric gravitational wave detectors currently under construction are designed to detect small perturbations in the relative lengths of their kilometer-scale arms which would be induced by passing gravitational radiation. An analysis of the effects of imperfect optical alignment on the strain sensitivity of such an interferometer shows that in order to achieve maximum strain sensitivity in LIGO (Laser Interferometer Gravitational-wave Observatory) the angular orientations of the optics must be within 10^{-8} radian-rms of the optical axis. In addition, fluctuations in the input laser beam direction must be below 1.5×10^{-14} rad/$\sqrt{\mathrm{Hz}}$ in angle and 2.8×10^{-10} m/$\sqrt{\mathrm{Hz}}$ in transverse displacement for frequencies $f > 150$ Hz, in order that they not produce spurious noise in the gravitational wave readout channel. Thermally excited angular fluctuations of the optics produce unacceptably high longitudinal displacements when the laser beam axis has a lateral offset from the optics' centers-of-gravity larger than about 1 mm. We also show that seismic disturbances limit the use of local reference frames for angular alignment at a level about an order of magnitude worse than required. A wavefront sensing scheme using the input laser beam as the reference axis is presented which successfully discriminates between all angular degrees-of-freedom and allows the implementation of a closed-loop servo control to sufficiently suppress the environmentally driven angular fluctuations.

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1 Introduction

Currently several kilometer-scale gravitational wave interferometers are under construction around the world, all of which are anticipating start of operations near the end of the millennium. The Laser Interferometric Gravitational-wave Observatory (LIGO) [1] will consist of three interferometers operated simultaneously, two at a facility near Hanford, Washington and one at a facility in Livingston Parish, Louisiana. The VIRGO detector is a French-Italian project, building a single interferometer at Cascina, Italy [2]; a German-British group is constructing an interferome-

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ter near Hannover, Germany known as GEO 600 [3]; and a Japanese collaboration is building a 300 m system near Tokyo [4]. All of these are variants of the Michelson interferometer as a detector of gravitational radiation [5]; the geometry is well matched to the quadrupolar strain in space that would be produced by a passing gravitational wave. The mirrors in the interferometer act as test masses of the space-time geometry; the proper distances between the test masses change proportionally to the gravitational wave amplitude. The test masses are made 'free' by suspending them as pendula, typically having a period of 1 second. The differential change in proper distance between the interferometer's orthogonal arms produces a corresponding differential change in the phase of the laser light (i.e., a positive phase shift in one arm and a negative phase shift in the other). This differential phase shift is detected as an amplitude change in the light at the output of the interferometer.

A schematic of the *optical configuration* of the LIGO/VIRGO interferometers is shown in Figure 1. The interferometer optical design is an enhancement of the basic Michelson interferometer in several major respects. The arms contain Fabry-Perot cavities, each comprising a partially transmitting input mirror (which also serves as an inertially-free test mass) and a highly reflecting end mirror (and test mass), operated in resonance with the input laser light; the increased light storage time in these cavities serves to increase the phase shift of the reflected light produced by arm length changes [6]. This leads to the requirement that round-trip propagation phase in each cavity be $2n\pi$, where n is an integer. In addition, a partially transmitting 'recycling' mirror is placed between the laser source and the beamsplitter; this resonantly increases the optical power in the system, thereby enhancing the shot-noise-limited sensitivity [7]. The optical path difference between the two arms is held equal to $2m\pi$ (m an integer), the Michelson 'dark fringe' condition, where the power at the anti-symmetric port is a minimum; this minimizes coupling to some technical noise sources, and enables the recycling technique. The term 'recycling cavity' is often applied to the optical system comprising the recycling mirror, the two input test masses, and the beamsplitter.

Thus, the operating point of the interferometer is determined by the various light propagation phases (or resonance conditions) in the system. There are four independent length degrees-of-freedom in the system. To maintain the best sensitivity of the instrument, these lengths must be controlled to within small ranges — as tight as 10^{-13} m, or 10^{-7} of the light wavelength — of this operating point. It is equally critical to maintain the mirror angles close to the optimum point, so as to maximize the power coupled into the optical system, to maintain a good quality of interference at the antisymmetric interferometer port, and to limit coupling to technical noise sources.

The technique for detecting these length degrees-of-freedom is briefly described, since an extension of it is used to detect the alignment degrees-of-freedom; a more complete description is given in reference [8]. The input light is phase modulated at a radio-frequency f_m (typically 10MHz–50MHz), and with a relatively small amplitude, so that only the carrier, at ν_0, and the first order modulation sidebands, at $\nu_0 \pm f_m$, have significant amplitudes among the series of modulation sidebands developed. The frequency f_m is chosen so that the sidebands are also resonant in the recycling cavity, but not resonant in the arm cavities. The detected photocurrents at various ports of the interferometer are then demodulated to give zero-crossing error signals for the four lengths; the relative phases of the modulation and demodulation waveforms are additional degrees-of-freedom that can be adjusted to select an error signal. The in-phase signal derived from the reflected light is essentially equivalent to the traditional 'reflection locking' technique for a single cavity

[9] and is predominantly sensitive to the average length of the arm cavities, $(L_1 + L_2)/2$, but also has some sensitivity to the recycling cavity length, $(l_1 + l_2)/2$. This signal is typically used in a high-gain loop to stabilize the input laser frequency. The in-phase signal derived from a sample of the recycling cavity light is similar to, but not quite co-linear (in the $[L_1 + L_2, l_1 + l_2]$ space) with the reflected signal; it is also mostly sensitive to the average arm cavity length, but has a different dependence on the recycling cavity length (it is sensitive to the difference in the finesse of the recycling cavity for the carrier and sidebands). Furthermore, when the gain in the laser frequency stabilization loop is relatively high, this signal becomes sensitive only to the recycling cavity length.

The other two length degrees-of-freedom depend on the existence of a macroscopic asymmetry in the Michelson, $l_1 - l_2 \neq 0$, often called the Schnupp asymmetry [10]. This breaks the symmetry between the reflection and transmission sides of the beamsplitter, and allows signals proportional to length differences to be generated. Since the dark fringe condition is held for the carrier, the sidebands are not forcibly at a minimum at the anti-symmetric port, and in fact the parameters are chosen so that most of the input sideband light is transmitted to this port. This anti-symmetric port sideband light beats with any carrier light that is produced by a deviation from the dark fringe; this gives a signal that is proportional to $\{(L_1 - L_2) + (\pi/2F)(l_1 - l_2)\}$, where F is the finesse of the arm cavities. The final length error signal uses the carrier as the reference field, and derives from a change in the relative amplitudes of the sidebands, produced by the degree-of-freedom $\{(l_1 - l_2) + (\pi/2F)(L_1 - L_2)\}$; this mode has the effect of increasing the recycling cavity finesse for one of the sidebands and decreasing it for the other. This error signal appears in the quad-phase of the photocurrent from both the reflected light and the recycling cavity light.

The six interferometer optics contribute an additional twelve angular alignment degrees-of-freedom to the system. These are the pitch and yaw angles of the four arm cavity mirrors, the beamsplitter, and the recycling mirror; 'pitch' refers to the angle of motion around a horizontal axis, and 'yaw' to the angle of motion around a vertical axis. The orientation of the beamsplitter is not an independent variable, however, since the effect of its orientation on the angular alignment can be described in terms of the angles of ITM_2 and ETM_2. Similarly, the input beam direction is described by four additional degrees-of-freedom (displacement and tilt in each plane), but the effect of these on the angular alignment can be expressed in terms of the angles of the five interferometer optics. We thus have ten independent angular alignment degrees-of-freedom. (The orientations of the beamsplitter and input beam are relevant later, when we consider transverse alignment, i.e., the transverse position of the beam on the finite-sized mirrors.) The sign convention for these angles is defined in Figure 1. The origins for the angles are defined at the point of optimal angular alignment, which in turn is defined as the point where the beam axis is colinear with the optic axes of the two arm cavities and the recycling cavity. A cavity optic axis is defined as the line through the centers of curvature of the cavity mirrors.

The pitch angle of an optic is named θ, and the yaw angle φ. Normalized optic angles are also used; these are the physical angles divided by the beam divergence angle in the arm cavities. The individual mirror angles present one basis for expressing the alignment; for pitch: $[\theta_{ITM1}\ \theta_{ETM1}\ \theta_{ITM2}\ \theta_{ETM2}\ \theta_{RM}]$. Another useful basis uses common and differential angles of the test masses; this is obtained by a rotation of the above basis:

$$\begin{bmatrix} \Delta\theta_{ETM} \\ \Delta\theta_{ITM} \\ \overline{\theta_{ETM}} \\ \overline{\theta_{ITM}} \\ RM \end{bmatrix} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} \theta_{ITM1} \\ \theta_{ETM1} \\ \theta_{ITM2} \\ \theta_{ETM2} \\ \theta_{RM} \end{bmatrix} \tag{1}$$

Having defined the relevant degrees-of-freedom in the system, we proceed to discuss alignment effects in LIGO. Section 2 analyses the effects of alignment on the interferometer sensitivity, and the resulting implications for alignment requirements. Section 3 presents a scheme for detecting all the alignment angles, using an extension of the phase modulation/demodulation technique used for length detection. Section 4 looks at the alignment fluctuations due to environmental disturbances at the LIGO sites, and discusses the planned servo control system of the mirror angles. The quantitative results given below will apply to the initial LIGO interferometers, but the same treatment applies qualitatively to any power recycled, Fabry-Perot Michelson interferometer (such as VIRGO). Relevant design parameters of the LIGO interferometers are given in Appendix 1.

The formalism for calculating the effects of misalignment (as well as other types of optical beam distortion) in the interferometer has been developed in detail in reference [11]. The approach is to expand the electromagnetic field into Hermite-Gaussian modes, which are the eigenmodes of an ideal optical resonator with mirrors of infinite extent. Free space propagators and misaligned optical components are represented by matrix operators which act on the state vectors in this basis. This allows calculation of the carrier and sideband fields at any plane in the interferometer, expressed in the Hermite-Gaussian basis; from the field values, the relevant detectable quantities are calculated. For small misalignments (when the normalized angles are much less than unity) the only modes that are significant in the expansion are the fundamental TEM_{00} mode and the lowest order transverse modes, TEM_{10} and TEM_{01}. The pitch and yaw angles in the system are separable and are treated in the same way, so in much of the discussion to follow we treat explicitly misalignments in one plane only, and carry just the TEM_{00} and TEM_{10} modes.

2 Alignment Effects on Interferometer Sensitivity

2.1. Reduction of Sensitivity

We look first at how deviations from perfect alignment reduce the sensitivity of the instrument. In particular, it is the *shot-noise limited sensitivity* that is degraded, since misalignments change the effective optical power in the interferometer; for the initial LIGO design, the sensitivity spectrum will be shot-noise limited for frequencies $f > 150\,\text{Hz}$, so at issue is the degradation in this spectral region.

The shot-noise limited sensitivity is made up of two parts: the *signal sensitivity*, which is the magnitude of the signal produced at the output by a strain, or differential arm length change; and the *shot noise* due to the detected optical power. The shot noise limited sensitivity is the ratio of the shot noise to the signal sensitivity. The output signal is generated by the interference of the carrier and sideband fields at the output, and so is proportional to the carrier and sideband field coupling into the interferometer. At the point of optimal alignment this coupling is a maximum and is first-order insensitive to the alignment. Thus if $S_{sens}(\vec{\theta})$ is the gravitational wave signal sensitivity as a function of the misalignment angles $\vec{\theta}$ (a 5-component vector when considering only the horizontal or vertical plane), it can be approximated at small angles by:

$$S_{sens}(\vec{\theta}) = S_{sens}(0)\left[1 - \frac{1}{2}\vec{\theta}H\vec{\theta}\right]. \tag{2}$$

The Hessian matrix H is defined by

$$H_{ij} = \frac{d^2}{d\theta_i d\theta_j} S_{sens}(\vec{\theta}) \tag{3}$$

and is (up to a constant) the inverse of the covariance matrix $C = 2H^{-1}$. Since the matrix H is symmetric, in general there are 15 coefficients to specify the sensitivity loss. However, noting that Eqn. (2) describes a 5-dimensional ellipsoid, the problem can be reduced to specifying 5 coefficients by using the principal axes of the (variance) ellipsoid. This is done by diagonalizing the covariance matrix C; the eigenvectors u_i are the axes of the variance ellipsoid, and the corresponding eigenvalues σ_i^2 are the squares of the axes lengths (i.e., the variances). Using the new basis u_i to express the mirror angles, the relative loss of sensitivity ε is then simply expressed as:

$$\varepsilon = 2\sum_{i=1}^{5}\left(\frac{\psi_i}{\sigma_i}\right)^2 \tag{4}$$

where ψ_i is the misalignment amplitude in the u_i direction, and the factor of 2 accounts for misalignments in both pitch and yaw (assumed to be the same for a given u_i).

A two mirror cavity will serve as a simple example of the above description. Consider a stable, two mirror Fabry-Perot cavity, with a flat input mirror and a rear mirror with a radius of curvature R, spaced a distance L apart; the cavity is also characterized by its g-parameter, $g = 1 - L/R$. The power build-up of a TEM_{00} gaussian beam incident on the resonant cavity is given by the spatial overlap of the incident mode on the mode sustained by the cavity. If the only mode mismatch is due to misalignment, so that the cavity optic axis is misaligned with respect to the input beam axis, the overlap for small angles is:

$$\begin{aligned} P(\theta_1, \theta_2)/P_{\max} &\approx e^{-R^2(g\theta_1 + \theta_2)^2/\omega_0^2} e^{-LRg(\theta_1/\omega_0)^2} \\ &\approx 1 - \left(\frac{R}{\omega_0}\right)^2 [g\theta_1^2 + 2g\theta_1\theta_2 + \theta_2^2] \end{aligned} \tag{5}$$

where $P_{\max}$ is the power build-up with perfect alignment, θ_1 and θ_2 are the (pitch) angles of the front and back mirrors, respectively, and ω_0 is the cavity waist size — the $1/e$ radius of the field distribution.

For a given level of power coupling, $P(\theta_1, \theta_2)/P_{max} = \text{const.}$, Eqn. (5) describes an ellipse as shown in Figure 1. The orientation of the ellipse and the ratio between the major and minor axes are determined completely by the g-parameter. The most sensitive misalignment direction is $u_2 = \{\theta_1 \cdot \sin\alpha, \theta_2 \cdot \cos\alpha\}$, with α defined in Figure 1. The ratio of the variances also depends only on g,

$$\sigma_1^2/\sigma_2^2 = (1 + g + \sqrt{5g^2 - 2g + 1})/(1 + g - \sqrt{5g^2 - 2g + 1}) \qquad (6)$$

Returning to the problem of the interferometer sensitivity loss, the first step is to compute the elements of the matrix H, as defined in Eqn. (4). This is done numerically using the modal model described in the first section. The eigenvectors and eigenvalues of the covariance matrix $C = 2H^{-1}$ then yield the u_i and σ_i^2, respectively.

This procedure is applied to obtain three useful pieces of information:

i. signal strength of the gravitational wave output, S_{sens}
ii. noise level at the gravitational wave output, N
iii. signal-to-noise ratio of the gravitational wave output

The interferometer signal-to-noise sensitivity is simply the ratio (S_{sens}/N). The noise amplitude due to shot-noise is proportional to the square-root of the power detected:

$$N \sim \sqrt{P_c + \frac{3}{2} P_{sb}} \qquad (7)$$

where P_c and P_{sb} are the power levels of the carrier and sidebands, respectively, at the output port (most of the light power is from the sidebands – in this model the carrier interference at the beamsplitter is perfectly destructive in the absence of misalignments). The factor of $3/2$ in front of P_{sb} is due to the non-stationary characteristic of the noise [12]. The total power at the detector output may actually increase or decrease, depending on the type of misalignment; for small misalignments the change in output power, and thus the change in noise amplitude, is also quadratic in the misalignment. The signal and the noise depend differently on the angles, so these cases give three distinct covariance matrices.

The results of these calculations for the signal-to-noise ratio, using the LIGO parameters given in Appendix A, are shown in Table 1. The eigenvalues σ_i^2 are given in normalized angle units — i.e., in units of the beam divergence angle in the arm cavities, $\theta_D = \lambda/\pi\omega_0 = 9.65\times10^{-6}$ rad with $\omega_0 = 35.1$ mm.

The most sensitive misalignment mode for (S_{sens}/N) involves a differential misalignment of the ETMs against an ITM differential misalignment in the opposite direction (the signs are the same in Table 1 because of the sign convention given in Figure 1). Nearly as sensitive is a common rotation of the ITMs against an opposite rotation of the recycling mirror. These two modes, u_2 and u_1, are shown pictorially in Figure 1. Separate calculations of the variance ellipsoids for the signal and noise indicate the source of the degradation in each case. In mode u_2, the sensitivity degradation is dominated by an increase in the shot noise; the TEM_{10} fields produced by the differential misalignment appear directly at the anti-symmetric output and increase the noise-producing power [13].