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Measuring the anelasticity of Cu-Be film under tensile stress using the X-pendulum

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Abstract

We measured the anelasticity of Cu-Be foil at very low frequencies by incorporating it in an X-pendulum. The imaginary part of the spring constant was constant from 0.03 Hz to 0.5 Hz under tensile stresses of 12-20 MPa. The X-pendulum technique proved convenient for low frequency use.

Keywords: Anelasticity; Gravity wave detectors; Mechanical instruments

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1. Introduction

Although the characteristic of anelasticity has long been known in the field of material science [1], it has recently received particular attention in the fields of gravitational wave detection [2] and precise measurement [3]. In an interferometric gravitational wave detector (IGWD), the mirrors, which serve as test masses for gravitational waves, are suspended, in order to realize inertial motion (at least above the suspension pendulum mode frequency) and to suppress vibrations from seismic noise in the ground. However, thermal noise of the pendulum and internal to the mirror limit the sensitivity to gravitational waves at intermediate frequencies, 100-300 Hz.

The thermal noise is directly related to the damping via the fluctuation-dissipation theorem. Previous analyses of thermal noise have applied a model for the damping which is proportional to velocity. However, some research suggests that this description is not correct for the damping from internal friction and that a better description is an extended Hooke's law with a frequency independent imaginary term in the spring constant [4-8]. This is a generalization of a phenomenon called anelasticity and reflects damping inside material from dislocations, domain walls, impurities, grain boundaries and point defects.

If the complex spring constant model is correct, then given the design parameters in current IGWD projects, the dominant source of thermal noise in the intermediate frequency region is that from the internal modes of the mirror, whereas thermal noise of the suspension pendulum had been dominant in the case of velocity damping.

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However, there is not so much experimental data from tests of the extended Hooke's model, particularly for materials likely to be used in suspensions of the IGWD, or for such materials under tensile stress. A key advantage of using a pendulum as a suspension is that all but a tiny proportion of the restoring force is gravitational and therefore lossless, and this reduces the damping and thermal noise. Of course, this also makes it difficult to measure. Anelasticity is not easily seen above very low frequencies because it is very difficult to exclude unidentified sources of damping in high frequency measurements (especially ≥ 100 Hz). Therefore we need to make observations at much lower frequencies and risk extrapolation.

An ordinary simple pendulum with a very low frequency is inconveniently tall, e.g., 10 s requires 25 m. A much better technique is that of the "X-pendulum" [9,10], which is composed of crossed wires or flexure hinges and can easily give such a long period within a size of some 20 cm. Thus we used the X-pendulum to observe the anelasticity of hinge material at long periods.

2. Theory

It is convenient to present both the elasticity and damping in terms of a complex spring constant expressed as $k[1 + i\phi(\omega)]$, where k is the usual spring constant and $\phi(\omega)$ represents losses due to internal friction as a function of angular frequency. Specifically it gives, at least for small ϕ , the phase angle by which the force transmitted by the spring lags behind the driving displacement. Using this, the equation of motion for a general simple harmonic oscillator in the frequency domain under an external force $f(\omega)$ becomes

$$m\{-\omega^2 + \omega_0^2[1 + i\phi(\omega)]\}x(\omega) = f(\omega), \quad (1)$$

where m is the mass of the system, $x(\omega)$ is the position of the mass and ω_0 is its resonant frequency. The quality factor Q of the resonance is $1/\phi(\omega)$. We can then derive the spectrum of the thermal noise due to this complex spring constant using the fluctuation-dissipation theorem:

$$\langle x^2(\omega) \rangle = \frac{2k_B T}{\pi\omega} \frac{\omega_0^2 \phi(\omega)}{m[(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi^2(\omega)]}, \quad (2)$$

where k_B is Boltzmann's constant and T is temperature. Note that the units are $\text{m}^2 \text{s}/\text{rad}$, not m^2/Hz .

Convenient approximations to the spectrum away from resonance are

$$\begin{aligned} \langle x^2(\omega) \rangle &\approx (2k_B T/\pi\omega)\phi(\omega)/\omega_0^2 & (\omega \ll \omega_0), \\ &\approx (2k_B T/\pi\omega)\omega_0^2\phi(\omega)/\omega^4 & (\omega \gg \omega_0). \end{aligned} \quad (3)$$

If ϕ is proportional to ω , we have the ordinary velocity proportional damping model. In the case of $\phi(\omega) = \text{const}$, the thermal power spectrum above the resonant frequency is proportional to $\omega^{-5/2}$, and $\omega^{-1/2}$ below. Therefore if we observe $\phi(\omega) = \text{constant}$ in our experimental frequency range, it may very well also be constant in the intermediate frequency region of an IGWD which would force us to re-estimate the sensitivity. The numerical value obtained would serve as a base for extrapolation.

In a pendulum using a particular flexure material, the flexure provides only part of the elasticity. By analogy with the complex spring constant model for the material itself we can define a damping factor $\phi_p = 1/Q_p$ for the pendulum as a whole. Since the gravitational contribution k_g to the restoring force is lossless,

$$\phi_p = \phi_e k_e / (k_e + k_g), \quad (4)$$

where k_e is the real part of the purely elastic spring constant by the bending of the suspension and ϕ_e is the associated damping factor as above. Eq. (2) says that small ϕ_p is needed to reduce the thermal noise of the pendulum mode of the suspension for the mirror system. Therefore we envisage using very fine suspension wires to reduce the relative effect of internal friction. However, if ϕ_e depends strongly on the stress, this approach will fail to reduce the thermal noise. This is why we need to measure the dependence of ϕ_e on the stress as well.

3. Apparatus description

To test this complex spring constant model, we designed a X-pendulum with a fairly long period, adjustable from 2 s to 30 s. Although the anelasticity model may be applicable to a wide range of materials, we were interested to make comparisons with the result of Quinn et al. [12,13]. We thus tested the same

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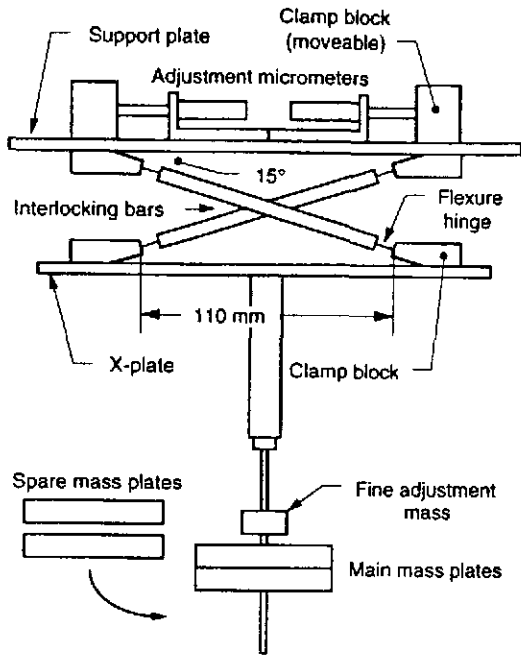


Fig. 1. Schematic diagram of X-pendulum.

material, Cu-Be foil, in a similar range of frequencies and tensile stresses. The design of the X-pendulum is shown in Fig. 1.

Instead of the usual four X-wires, we used two solid bars with Cu-Be flexure hinges at the ends. The solidity of the bars gave better dimensional stability and the extra weight was not relevant in this application. A two wire system would not be stable but the width of the foil gives transverse stability. One bar passes through a hole in the another. The material Cu-Be has suitable characteristics for use as a spring material: high toughness, high resistance to fatigue and low frictional hysteresis. We used Cu-Be foil made by Nilaco Co., Japan, composed of 98% Cu and 2% Be. The foil was in a partially precipitated state due to having been rolled to 0.05 mm in thickness, but was not further heat treated. The flexure hinges were each 30 mm in width and 8 mm in working length (24 mm in total length). Slots of a thickness a little greater than that of the foil were made with an electric wire cutter in the ends of the bars and in blocks attached to the support plate and X-plate. The foil was inserted in the slots and secured tightly with screws.

The effective potential becomes flat (to second order) at a point on the vertical axis around

$z \approx -136$ mm. The center of mass is contrived to be just below this point. As the center of mass approaches the critical point from below, the restoring force become very weak. Thus to second order, as far as motion of the center of mass is concerned, the X-pendulum is equivalent to a much taller simple pendulum. The period of the pendulum is adjustable by moving weight plates vertically using an adjustment screw of pitch 0.35 mm/turn. The tensile stress on the Cu-Be flexure hinges can be varied by increasing the number of the plates: each plate has a mass of 0.24 kg.

In this experiment, we measured the Q of the pendulum, Q_p , and from it calculated ϕ_e for the Cu-Be flexure hinge as follows. Ignoring damping for a moment, the resonant frequency of the X-pendulum is given by

$$\omega_r^2 = (k_g + k_e)/I, \tag{5}$$

where I is the effective moment of inertia of the suspended part about the top, k_g is the gravitational restoring force, and $k_e = k_{e(4,\theta)}$ is the total elastic constant due to all the flexure hinges together, referred to the pendulum angle θ .

The gravitational restoring force has two components: $k_g = k_x + k_b$. The first, $k_x = Mge$, which is standard for any X-pendulum, represents the center of mass of the pendulum moving in the flat potential well due to the X geometry. Here M is the mass of the pendulum, $g = 9.8 \text{ m s}^{-2}$ is the acceleration due to gravity, and e is the vertical distance between the center of mass and the critical point. The second, $k_b = mgl \sin \alpha_0 / (\cos \alpha_0)^2$ is a small correction for the finite weight of the X-bars. Here $m = 0.105$ kg is the combined weight of the bars, $l = 116$ mm is the net length of the X-bar and both flexure hinges and $\alpha_0 \approx 17.2^\circ$ is the angle between the bar and the support or between the bar and the (centered) X-plate.

The net elasticity must be related to the elasticity of a single hinge referred to the bending angle of the hinge $\Delta\alpha$, i.e., $k_{e(1,\alpha)}$, which is the desired measure of the intrinsic damping. Although each hinge has two ends, the end joining the bar does not bend and so does not contribute to the elasticity or damping. Thus if $N = 2$ is the number of bars, the number of flexure points is $2N = 4$. These are effectively in parallel and both the real and imaginary components of elasticity add. The mechanical advantage is $a = \Delta\alpha/\theta = 0.5$ so that we have the deceptively simple relation

$$k_e = k_{e(4,\theta)} = 2Na^2 k_{e(1,\alpha)} = k_{e(1,\alpha)}. \quad (6)$$

Since the working length of the flexure hinge is large compared to its thickness its angular elasticity under load is

$$k_{e(1,\alpha)} = \left(\frac{mgE_H I_H}{N \sin \alpha_0} \right)^{1/2}, \quad (7)$$

where E_H and I_H are the Young's modulus and moment of area of the hinge material (Cu-Be) and $\sin \alpha_0$ is the wire angle in the center position. The imaginary parts are proportional to the real parts throughout so that the net ϕ_e is the average of the individual values for the hinges. Then combining Eqs. (4) and (5), we get

$$Q_p = \frac{1}{\phi_p} \approx \frac{I}{k_e \phi_e(\omega_f)} \omega_f^2. \quad (8)$$

Hence from the relation between Q_p and angular frequency we can obtain the frequency dependence of the complex spring constant. We excited the pendulum swing about 4 mm and recorded the decay of the swing. We obtained Q_p graphically from the chart recorder output. The apparatus was put in a quiet basement, belonging to a building on our campus in the suburbs of Tokyo. The basement is very stable in temperature (less than ± 1 K per day) and has almost no circulation of wind.

4. Results and discussion

We made Q_p measurements for pendulums with 2, 3, 4 and 5 mass plates.

The parameters of each pendulum are given in Table 1. The upper and lower flexure hinges are loaded by a slightly different amount, due to the weight of the X-bars, and so the average stress on the flexure hinges has been calculated from the "corrected mass", $M + m/2$. The moment of inertia of the pendulum has been calculated about the effective pivot point in the middle of the gap. Similarly, to allow for the mass of the X-bars, the "corrected MOI" adds a term $a^2 I_b$ where I_b is twice the moment of inertia of a single X-bar about one end.

We set the period of the X-pendulum to values ranging from 2 s to 30 s. In this region, Q_p should be proportional to angular frequency according to Eq. (8).

Table 1

The parameters of 4 different X-pendulums, labelled A to D. The "corrected" values of mass and moment of inertia have been adjusted to allow for the effects of the non-zero mass of the X-bars

Parameter	A	B	C	D
mass plates	2	3	4	5
gap size (mm)	27.2	27.5	27.8	28.1
mass (kg)	0.77	1.01	1.25	1.49
corrected mass (kg)	0.82	1.06	1.30	1.54
MOI (kg m ²)	0.0154	0.0184	0.215	0.246
corrected MOI (kg m ²)	0.0155	0.0185	0.0216	0.0247
average stress (MPa)	11.5	14.7	17.8	20.9

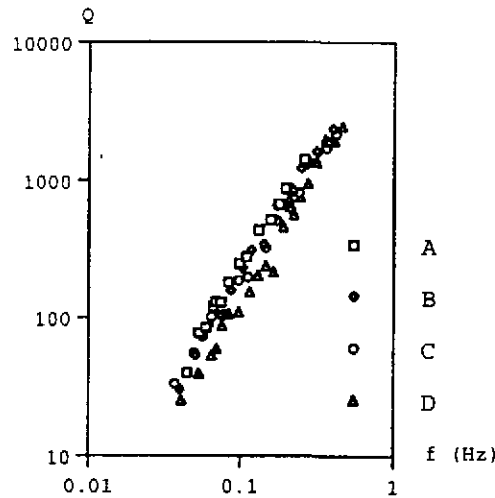


Fig. 2. The relation between Q_p and frequency. Four sets are plotted. Most errors are within markers.

The observed Q_p is plotted as a function of frequency in Fig. 2 for four different sets of weight plates.

To test Eq. (8), the equation $\log Q_p = m + e \log f$ was fitted to the data by the least squares method, and the resulting values of the exponent e are shown in Table 2. The values are quite close to (but a little lower than) 2. This is quite good support for Eq. (8), and very similar to the results of Quinn et al. [12].

Because Q_p is very nearly proportional to f^2 , we conclude from Eq. (8) that to the same extent ϕ_e is independent of frequency. This result shows that the complex spring constant model with a frequency independent loss factor is much more accurate for estimating internal friction than a velocity proportional model in this range of frequency and tensile stress. We made a second fit of the data to the equation $\log Q_p =$

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Results of fitting a power law in frequency to Q_p data. e is the power law exponent from a two parameter fit and Q_0 is the coefficient from a one parameter fit assuming $e = 2$

	A	B	C	D
e	1.91 ± 0.054	1.85 ± 0.045	1.73 ± 0.035	1.96 ± 0.053
$10^{-3} \times Q_0$	23.8 ± 2.8	18.4 ± 3.3	17.4 ± 4.4	12.7 ± 2.3
$10^3 \times \phi_e$	0.98 ± 0.12	1.34 ± 0.24	1.08 ± 0.27	2.17 ± 0.39

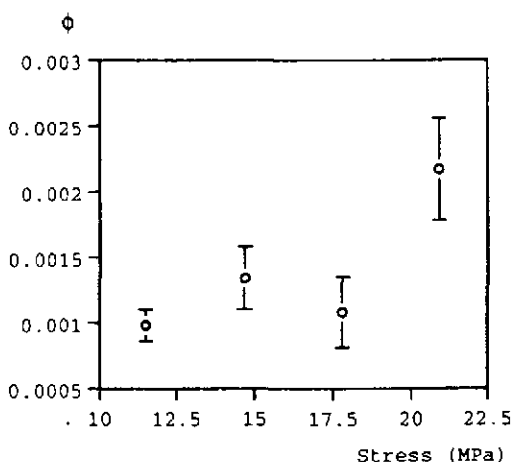


Fig. 3. Dependence on tensile stress.

$\log Q_0 + 2 \log f$ and calculated ϕ_e using Eq. (8). These values are shown in Table 2 and are plotted as a function of the stress in the Cu-Be in Fig. 3.

Since there are only four data points, we cannot make a firm conclusion about the dependence of ϕ_e on tensile stress, but a slight increasing trend can be seen.

There may be several possible damping sources other than the intrinsic damping in the flexure hinges. We considered two sources of friction, clamping friction and air damping. The friction caused by clamping Cu-Be foil is critical in this kind of pendulum experiment. To look for such an effect, we analyzed the dependence of Q_p to the oscillation amplitude. If the clamp slips, the friction is likely to be worse when the pendulum swings with a large amplitude. However, Fig. 4 shows that the damping is almost independent of the amplitude of pendulum oscillation.

Air damping is caused by air viscosity and turbulent flow. In our experiment, the velocity of the pendulum swing is extremely low and a simple calculation (using an approximation based on Stokes law, assuming a

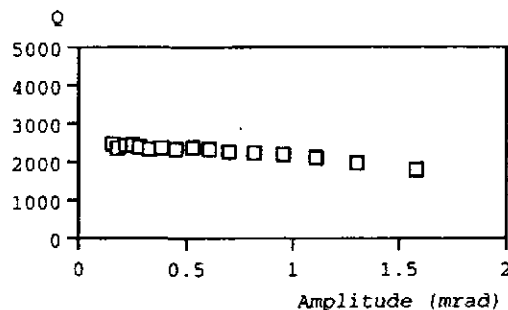


Fig. 4. Dependence of Q_p on amplitude of swing for pendulum B with a resonant period of 4.6 s. The data for other pendulums were almost the same as this plot.

sphere with the same cross section) shows the effect on Q_p is less than 0.1%.

The experiment of Quinn et al. [12,13] adopted a pendulum in the form of a vertical bar with masses at the ends and suspended in the center by a Cu-Be flexure hinge machined from a single block. Their data was similarly consistent with Eq. (8) and gave a pendulum damping factor (equivalent to ϕ_p) of $3.6 \times 10^{-5} \omega^{-2}$. Upon conversion to material damping factors using their quoted values $I = 0.6 \text{ kg m}^2$ and $k_e = 0.0044 \text{ N m/rad}$, this becomes $\phi = 4.9 \times 10^{-3}$. The stress in the strip was approximately 39 MPa or about double that for our pendulum D so a slight extrapolation is required. As noted above, Fig. 3 suggests a positive dependence of damping on stress. This is similar to the findings of Quinn et al., who noticed a slight increase in damping after a process of electropolishing which they attribute to thinning of the hinge and a resulting increase in tensile stress. If we then suppose that damping is exactly proportional to stress, then we would expect $\phi_e \approx 4 \times 10^{-3}$, which is quite good agreement. Therefore we believe that we have been at least as successful in eliminating extraneous damping as Quinn et al. and that the clamping friction was small enough to allow the intrinsic damp-

ing of Cu–Be film to be seen.

Even if some of this damping is not intrinsic but due to the interface, the results are still suggestive, as there will necessarily be such an interface in any practical application and the existing technique to suspend mirrors is likely to have a similar stress-dependence of ϕ_e . In an IGWD, the tensile stress of wires suspending mirrors would be set near to the yield strength in order to reduce the moment of area of the wire. If this suggestion of stress dependence is correct, even if the increase occurs in the interface region of the support, it will increase thermal noise of the mirror suspension system, resulting in much worse sensitivity than anticipated. Needless to say, further experimentation on this point is needed.

In conclusion, the frequency independent damping model is confirmed for Cu–Be film in the frequency range 0.03 Hz to 0.5 Hz under tensile stresses of 11 to 21 MPa. This augments the results of other exper-

iments. This study also shows the suitability of the X-pendulum for this sort of experiment.

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