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Thermal motion associated with monolithic fused silica cradle suspensions for gravitational wave detectors

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Abstract

A finite-element analysis of a monolithic fused silica cradle suspension is presented. In particular the level of thermal motion of this design is evaluated with regard to employing such a novel suspension in an interferometric gravitational wave detector.

1. Introduction

There is currently much work being carried out around the world towards the development of interferometric gravitational wave detectors. Such instruments will use laser interferometry between test masses suspended freely as pendulums to detect the strain in space induced by the passage of a gravitational wave, see e.g. Ref. [1]. Most detectors will incorporate at least four pendulum suspensions. A serious limit to the sensitivity of these detectors will be set by thermal motion associated with the internal modes of the test masses, with the transverse (violin) modes of the suspension fibres and with the pendulum modes of the suspended masses. In order to minimise the effect of these motions, it is important that the test masses and suspension fibres should be formed from material that has intrinsically low internal friction in the bandwidth of interest for the detection of gravitational waves (~ a few Hz to a few kHz). Furthermore the overall suspension design must minimise the introduction of any other sources for energy dissipation in this bandwidth. This Letter describes work towards designing such a suspension and in particular considers in detail a less conventional design than that currently employed in the prototype detectors around the world. This novel design uses a cradle to hold each test mass as will be described below.

2. A brief reminder of the thermal noise power spectrum of a harmonic oscillator

The spectral density of thermal motion of a harmonic oscillator is given by e.g. [2]

$$\langle \tilde{x}^2(\omega) \rangle = \frac{4k_B T \omega_0^2 \phi(\omega)}{\omega m_0[(\omega_0^2-\omega^2)^2+\omega_0^4\phi^2(\omega)]}, \qquad (1)$$

where k_B is Boltzmann's constant, T is the temperature, ϕ is the phase angle associated with the internal

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friction of the oscillator, ω is the angular frequency, m_o is the equivalent mass of the oscillator and ω_o is the resonant angular frequency of the oscillator. The introduction of a reduced or equivalent mass allows a more complicated oscillator (e.g. the fundamental longitudinal mode of a solid cylinder) to be parameterized as a simple harmonic oscillator formed with a point mass of value m_o, vibrating at angular frequency ω_o.

$\phi(\omega)$ is the phase angle associated with internal friction of the oscillator. At ω_o it may be shown that the quality factor, Q, of the oscillator is given by [3]

$$Q = \frac{1}{\phi(\omega_o)}, \quad \text{for } \tan\phi(\omega_o) \ll 1, \tag{2}$$

and thus it is evident that low internal friction implies high Q. The form of the power spectrum given by Eq. (1) is dictated by the frequency dependence of ϕ and thus it is important that this is known, in order to correctly predict the level of thermal motion for a suspended test mass in a laser interferometric gravitational wave detector. In order to attain low thermal motion, it is clear from Eq. (1) that the materials of interest for fabricating such suspensions must have intrinsically low ϕ. It is, however, experimentally very difficult to measure ϕ as a function of frequency in materials with very low losses ($\phi \sim 10^{-6}$–10^{-7}). An indication of the behaviour may be gained by measuring the Q of samples of the material under study resonant at different frequencies, together with an understanding of the dynamics of the system. There is widespread evidence that for many materials of interest ϕ is in fact independent of frequency over large bandwidths (see e.g. Ref. [3] and references therein). This behaviour, resulting from what is known as structural damping, will be assumed for the rest of this Letter.

2.1. The thermal motion associated with a simple pendulum suspension

Consider a simple pendulum placed under vacuum so that viscous damping of the motion is negligible. Almost all of the energy associated with the pendulum mode is stored in the lossless gravitational field. A fraction of the energy will however be stored in the bending of the suspension wire and since there will inevitably be some loss associated with this, a small fraction of this energy will be dissipated. Thus the pendulum mode of the suspension is expected to have a much lower loss, ϕ_{pend}, than the loss associated with the suspension material, ϕ_{mat}. Indeed it may be shown that [2]

$$\phi_{\text{pend}}(\omega) \approx \phi_{\text{mat}}(\omega)\frac{N\sqrt{TEI}}{2mg\ell}, \tag{3}$$

where N is the number of suspension wires (one loop is equivalent to two wires), T is the tension in the wire, E is Young's modulus, I is the moment of inertia of the wire cross section ($I = \pi r^4/4$ for a wire of circular cross section and radius r), m is the mass of the pendulum bob, g is the acceleration due to gravity and ℓ is the length of the wire. The factor multiplying ϕ_{mat} is typically of the order of a few times 10^{-3} for the suspensions of interest here.

The GEO 600 project, a German-British project to build a 600 m interferometric gravitational wave detector [4], has as its design goal a strain sensitivity of $5 \times 10^{-23}/\sqrt{\text{Hz}}$ at 100 Hz. To achieve this, with a factor of 10 safety margin, would require pendulums with $Q_{\text{pend}} = 4 \times 10^7$ (assuming a 1 Hz pendulum frequency and a 16 kg test mass).

González and Saulson [5] showed that for a single loop suspension, where the mass is free to rotate, if ϕ_{mat} is frequency independent,

$$\phi_{\text{violin}} = 4\phi_{\text{pend}}. \tag{4}$$

Note that (4) is only true for lower frequency harmonics since for higher harmonics the effects of stiffness of the wire increasingly dominate over those of tension. For typical parameters appropriate to suspensions to be employed in a gravitational wave detector Eq. (4) holds to a good approximation for the first few harmonics.

Thus the total thermal motion due to the suspension of a test mass, at angular frequencies much higher than the pendulum angular frequency ω_{pend}, may be expressed as [2,6]

$$\langle \tilde{x}^2(\omega)\rangle = \frac{4k_B T\omega_{\text{pend}}^2\phi_{\text{pend}}}{\omega^5 m_{\text{pend}}} + \frac{4k_B T\omega_n^2\phi_{\text{violin}}}{\omega m_n[(\omega_n^2-\omega^2)^2+\omega_n^4\phi_{\text{violin}}^2]}, \tag{5}$$

where ω_n is the angular frequency and $m_n = \frac{1}{2}m_{\text{pend}}(\omega_n^2/\omega_{\text{pend}}^2)$ is the equivalent mass of the nth violin mode, where it is assumed that the suspension wires are kept at a fixed fraction of their breaking stress irrespective of how many wires are employed.

In order to evaluate the total thermal noise from a pendulum suspension, the thermal motion associated with the internal modes of the test mass must also be estimated. This is more complicated than for the pendulum or violin modes since m_0 must be known for each mode. Furthermore a coupling factor between the laser beam of the interferometer and the test mass is required since only a fraction of the face of the test mass will be sampled by the laser beam. Gillespie and Raab recently carried out such a calculation for a laser beam of waist 22 mm incident centrally on a cylindrical test mass of diameter 0.25 m and height 0.1 m and also for the same laser beam incident on a cylindrical mass 0.1 m in diameter and 0.875 m in height [7]. This work brought to light the unexpectedly large contribution of some of the higher order axisymmetric modes to the total thermal motion of the test mass as sensed by the laser beam. Similar calculations have been carried out by Bondu and Vinet [8]. The GEO 600 project will employ fused silica test masses of order 0.25 m diameter by 0.15 m long. Extrapolating from Gillespie and Raab's work it is estimated that for the same size of laser beam, the spectral density of thermal motion of the internal modes for such a test mass will be

$$\langle \tilde{x}^2(\omega) \rangle = \frac{2.5 \times 10^{-30} \phi_{\rm tm}}{\omega}, \qquad (6)$$

where all modes are assumed to have the same loss $\phi_{\rm tm}$. To achieve the GEO 600 strain sensitivity goal at 100 Hz ($5 \times 10^{-23}/\sqrt{\rm Hz}$) with no safety margin requires $\phi_{\rm tm} \sim 2 \times 10^{-7}$ ($Q \sim 5 \times 10^6$). Thus a material of very low loss, such as fused silica, must be used for the test masses.

3. Current suspension design

In the above discussion, it was implicitly assumed that the suspension design was such as to allow simultaneously low losses in all modes, or equivalently high Q. How practical is this to realise? We first consider the basic suspension design used in all the prototypes worldwide, as shown in Fig. 1. The stand-offs provide a well-defined break-away point for the suspension wires and in general have a groove to constrain the position of the wire. Thus in effect the stand-offs behave like a clamp. The wires are also clamped at the top. Recently, however, Quinn et al. [9] observed stick–slip damping at flexure clamps which gave rise to damping of 2 orders of magnitude higher than the intrinsic damping of the flexure material. They found that if instead they clamped a much wider piece of material which then tapered to form the required flexure, the losses at the clamp became negligible. Although stick–slip damping is amplitude dependent and should thus become negligible for small movements of the test mass, it would be safer, when designing the suspensions for interferometric gravitational wave detectors, either to avoid wire clamps or to clamp to much a wider piece of material which then narrows to form the required fibre.

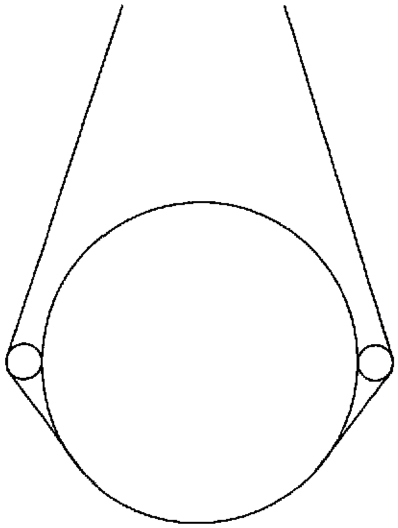

Fig. 1. The conventional design of pendulum suspension. The relative scaling of the stand-offs and mass has been exaggerated for the sake of clarity.

The presence of stand-offs, which are usually glued in place, can also have an uncertain effect on the Q of the internal modes of the test mass. Gillespie compared the Q values of the five lowest axisymmetric modes of a fused silica cylinder when the sample was simply suspended on a single wire loop to when it was suspended with stand-offs attached to the mass [10]. He found that when stand-offs were employed, the Q of three of the modes were significantly degraded, but the other two modes showed enhanced Q values due to the suspension wire having a better defined position and thus not being as free to rub against the side of the mass. It has also been observed by one of the authors that very high Q values may be measured with attachments glued to a test mass, but on removing and then re-gluing the attachments it is quite possible to greatly lower these values [11]. It is thought that this behaviour is due to the character of the glue bonds. Thus a reliable and reproducible method of connecting attachments is required.

Consideration of all the above effects suggests that perhaps a better suspension design, from the point of view of thermal noise, could be achieved if quasi-monolithic suspensions of a low loss material such as fused silica are constructed.

4. Monolithic fused silica suspensions

There are three possible approaches currently under consideration to using fused silica in the construction of quasi-monolithic suspensions.

(a) Fusing of fused silica fibres directly onto a silica test mass. This is being investigated by Braginsky et al. [12]. With a light test mass (30 g) suspended by a fibre a factor of 10 away from its breaking stress, a very high pendulum Q (1.3×10^8) has been demonstrated. It remains to be determined if this technique can readily be extended to large test masses suspended by fibres ~ 3 away from their breaking stress; the effects of thermally induced stresses may be a problem [13].

(b) Optically contacting expanded ends of fused silica fibres onto polished flats on the test mass. This approach is currently under investigation at Glasgow.

(c) The use of a silica cradle suspension as shown schematically in Fig. 2a. This is of great interest since it does not require any direct work to be done to the test mass. However there is the question of whether the thermal noise of such a suspension is increased in the frequency bandwidth of interest for gravitational wave detection. It is this issue that we seek to address here.

5. Finite-element modelling of a cradle suspension

Finite-element modelling of such a cradle suspension was undertaken in order to determine where in frequency space the cradle resonances fell and what effect they had on the thermally displaced motion of the test mass. This analysis was carried out using a computer package "Abaqus (version 5.3)" [14].

5.1. *The finite-element model*

As noted above, the GEO 600 project will employ fused silica test masses of order 25 cm in diameter by 15 cm long, a mass of 16 kg. In order to minimise losses in the pendulum mode and thus reduce the off-resonance thermal motion a small moment of inertia of the fibre cross section is required (see Eq. (3)). For circular fibres this may be achieved by using as small a radius of fibre as possible. This also has the additional advantage of pushing the violin resonances to higher frequencies and thus fewer of these resonance peaks occur in the frequency band of interest. (Note that at Glasgow we are also actively investigating the possibility of using fibres of a ribbon geometry which for a given cross-sectional area have a smaller value of I than a circular fibre. We have successfully pulled such ribbons from fused silica [15].) Consider then such a mass suspended by two fused silica circular fibres subject to a third of their breaking stress, which was measured by one of the authors to be 800 MPa, thus implying a fibre radius of 0.3 mm.

The model of the cradle suspension, schematically shown in Fig. 2b, was formed using beam elements which have a single node at each end. It was assumed that the change in length of the elements was negligible and that their cross section remained unchanged. The fibres were modelled with a radius of 0.3 mm, a length of 0.27 m and were each formed by a chain of 46 elements. The cradle was modelled as having an inner diameter of 0.127 m with a 5 mm square cross section. This was formed from a chain of 40 elements. Together the fibre and cradle formed a monolithic structure which was subject to the boundary condition that the top node of each fibre had both zero displacement and rotation. The fused silica mass was represented by a point mass of 16 kg with the appropriate moments of inertia for the cylindrical mass described above. This was positioned 1 mm below the level at which the fibres broke free from the cradle, and was initially attached to the centre node of the cradle using a beam constraint. This may be thought of as a massless, infinitely rigid rod which constrains the displacement

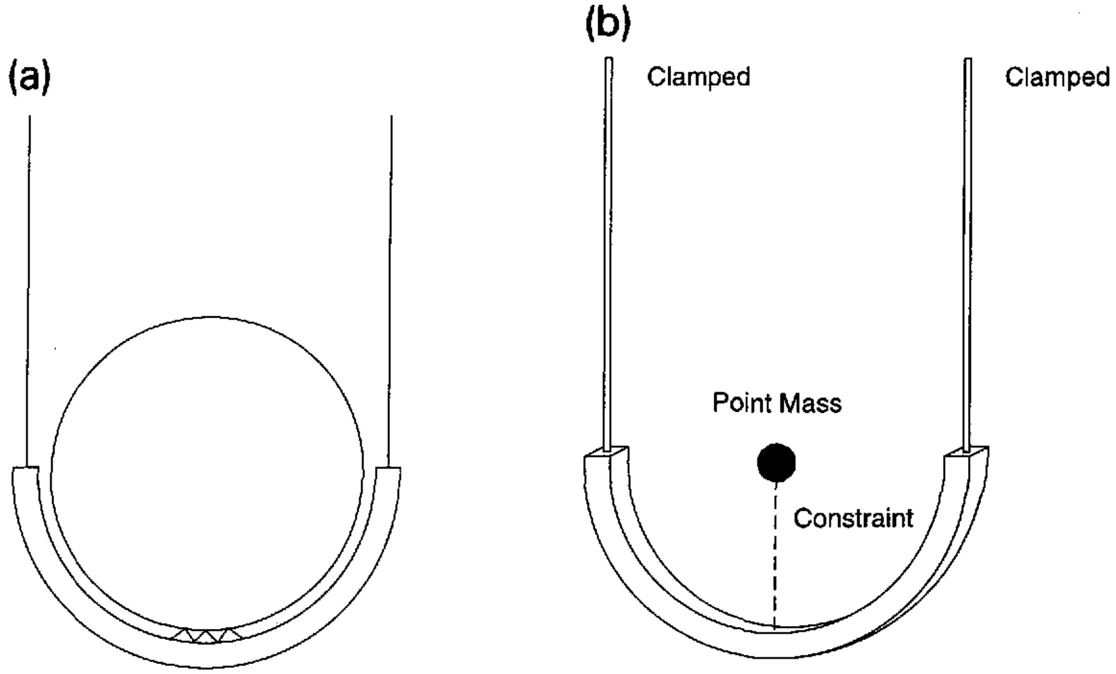

Fig. 2. (a) Schematic of the proposed cradle suspension where the mass rests on three sharp points, the size and positioning of which have been exaggerated for the sake of clarity. (b) Representation of the first finite-element model of the cradle suspension.

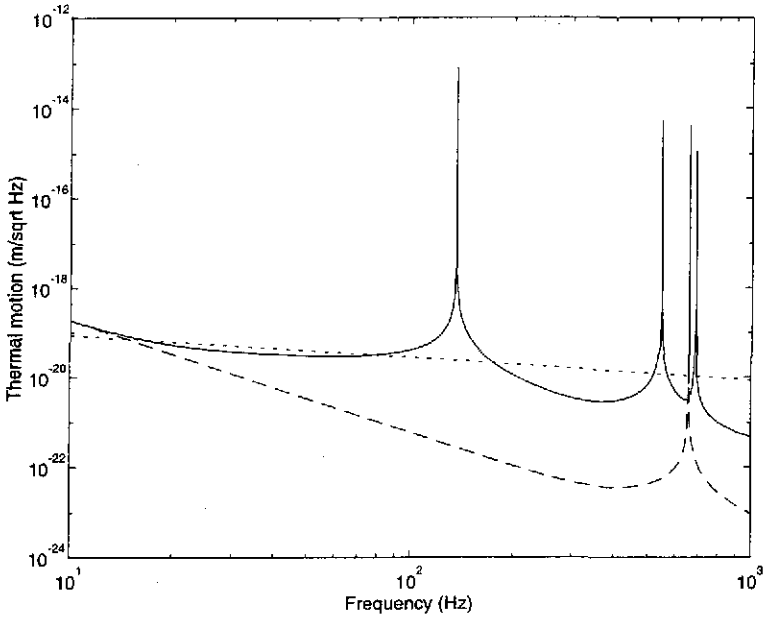

Fig. 3. (—) Predicted form for the thermal motion of the test mass due to thermal motion of its cradle suspension alone. (···) Predicted thermal motion from internal modes of the test mass. (- -) Predicted thermal motion of a simple pendulum suspension as described in the text.

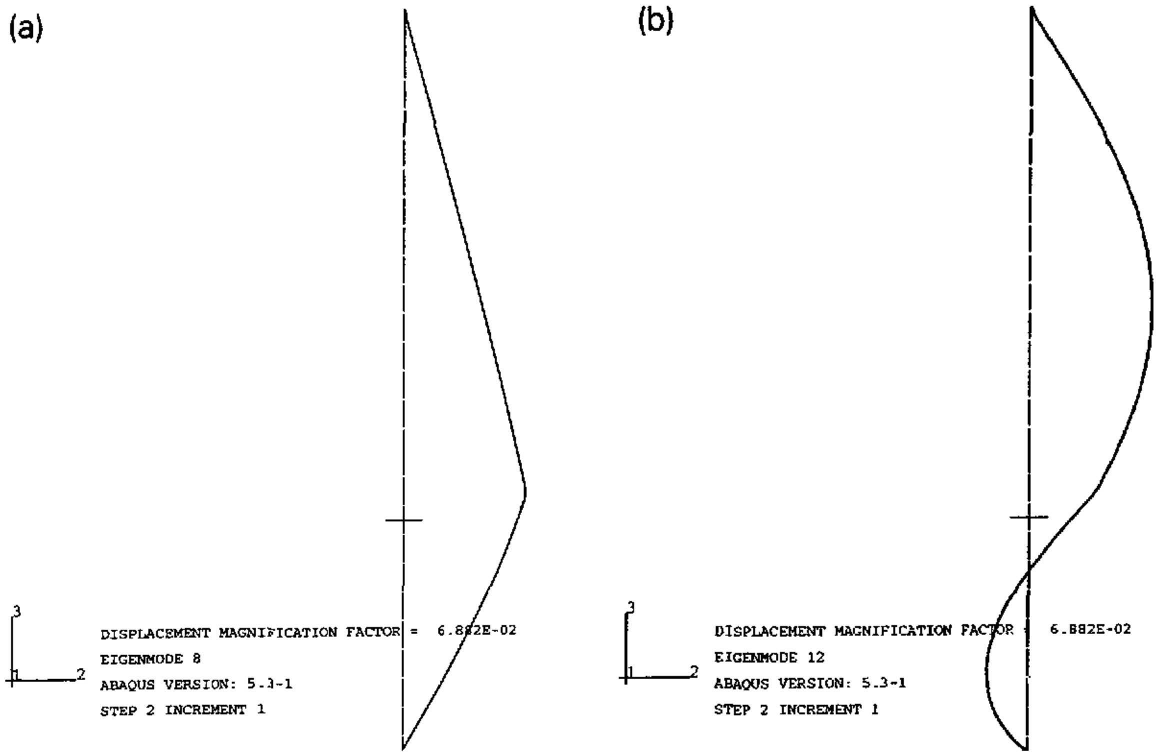

Fig. 4. The cradle modes that appear in the frequency band of Fig. 3. The figure shows a side view of the cradle structure where the dashed line represented the undistorted structure and the solid line shows the mode shape. The horizontal bar represents the position of the point mass which is placed 1 mm below the fibre and cradle leg join, as described in the text. The orientation of the bar indicates the direction of the optic axis for the incident laser beam. The modes of (a), (b) (first and second resonances respectively in Fig. 3) both involve torquing of the cradle legs. (c) First violin mode of the suspension wires.

and rotation of the point mass to that of the central cradle node.

5.2. The analysis

The complete model was first subjected to the force of gravity. The normal modes of the structure were then found and excited in order to determine the displacements and rotations of the nodes and the total energy in the mode for each mode of the system. This then allowed the peak motion of the point mass along the direction of the optic axis to be calculated when the mode was considered as being driven purely by thermal energy.

From the law of equipartition we have

$$\tfrac{1}{2}k_\mathrm{B}T = \tfrac{1}{4}m_\mathrm{o}\omega_\mathrm{o}^2 x_\mathrm{o}^2, \tag{7}$$

where x_o is the peak amplitude of oscillation. If this is combined with Eq. (1), the form of the spectral density of the thermal noise power associated with a mode of the oscillator may be re-written as

$$\langle \tilde{x}^2(\omega)\rangle = \frac{2\omega_\mathrm{o}^4 x_\mathrm{o}^2\phi}{\omega[(\omega_\mathrm{o}^2-\omega^2)^2+\phi^2\omega_\mathrm{o}^4]}, \tag{8}$$

where ω_o and x_o for each mode have already been determined from the finite-element analysis. Thus to generate the form of the spectrum requires only the choice of ϕ. For fused silica a reasonable choice of $\phi_\mathrm{mat} = 2\times10^{-7}$, see e.g. Ref. [10], which is equivalent to a Q of 5×10^6. Thus from the parameters of the system and Eq. (3), $Q_\mathrm{pend} = 10^9$, and from Eq. (4) $Q_\mathrm{violin} = 2.5\times10^8$. Other cradle modes were assumed to have a Q of 5×10^6. The spectral density of thermal noise power for displacement of the point mass along the optic axis for the first 35 modes of the structure was then summed. The result of this is shown in Fig. 3.