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Effects of misalignments and beam jitters in interferometric gravitational wave detectors

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Abstract

We present a calculation of the phase noise in a recycled interferometer with Fabry–Perot cavities in the arms, induced by the coupling of the geometrical fluctuations of the input laser with geometrical asymmetries of the interferometer. By comparison with the shot-noise limit planned for long-baseline interferometric gravitational wave detectors, upper limits in interferometer misalignments are established.

1. Introduction

In recent years a great interest has been shown in the long-baseline Michelson interferometers as possible detectors of gravitational waves [1–3]. Because of the weakness of the forces exerted by a gravitational wave on the test masses (the mirrors of the interferometers), particular care has been devoted to analyze all possible effects which can decrease the sensitivity of these interferometers with respect to the ultimate limit imposed by the photon counting noise [4]. In 1981, Rüdiger et al. [5] pointed out that geometrical fluctuations of the input laser beam generate a phase noise if the interferometer is not perfectly aligned, through coupling to the geometrical asymmetries of the interferometer. In their case, the interferometer analyzed was a multipass Michelson interferometer without recycling. Nowadays various detectors which are being built in the world, like LIGO [2] and VIRGO [3], utilize Michelson interferometers with Fabry–Perot cavities as optical multiplicators in the arms and a recycling mirror [6] to increase the power circulating in the interferometer. Extending the calculation for this type of interferometers, and for typical experimental values of geometrical fluctuations of the input laser beam, allows one to evaluate the upper limits for the interferometer misalignments. In order to not to limit the analysis to a particular phase-detection scheme (frontal modulation [7], external modulation

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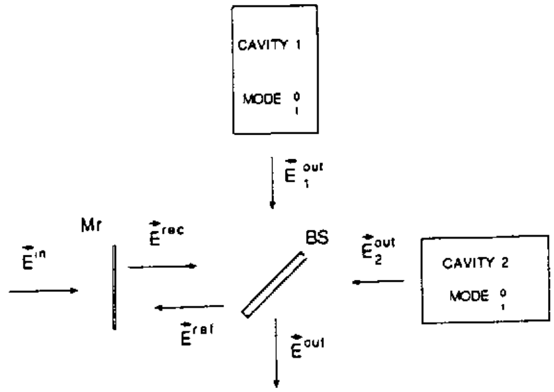

Fig. 1. The fields in the interferometer. "Mr" is the recycling mirror, "BS" is the beam splitter, "cavity 1" and "cavity 2" are the Fabry–Perot cavities.

[8], ...) we have calculated the phase difference of the two interfering beams at the output photodiode, and compared it to the shot-noise limited phase sensitivity of VIRGO/LIGO. The calculation is analytical, at the first order in the misalignments and jitters, and it is frequency dependent.

2. The field in the recycling cavity

A useful way to describe small misalignments of a Fabry–Perot cavity is that of referring its eigenmodes to a set of perfect Gaussian modes, chosen by convenience, in a particular plane of the space, typically at the waist [9,10].

In our case, we can describe the misalignments of the interferometer by fixing the position of the recycling mirror and allowing the two Fabry–Perot cavities of the arms to move. By considering that a rotation of the beam-splitter, which is flat, is equivalent to a rotation of the two Fabry–Perot cavities, we see that a general description is given in Fig. 1, where the modes of the cavities are slightly different from the modes of the interferometer, as referred to the fixed position of the recycling mirror.

The misalignments that are going to be considered are, without loss of generality, lateral translations and rotations in only one dimension, say the one corresponding to the x-direction (because the y-direction will lead to identical expressions) and waist mismatches, in size and position. In the first case *it is convenient* to use a Cartesian coordinate system, and consider the Hermite–Gauss spatial modes, ϕ^m (one spatial mode number m is sufficient, since we are only interested in the x-direction); for the waist mismatches, it is convenient to use the Laguerre–Gauss modes, V^n, with angular mode numbers $l = 0$, because we are interested in the axisymmetric modes (the ones being excited by waist mismatches).

In case of translations $x_{1,2}$ and rotations $\theta_{1,2}$, the two cavities fundamental modes ϕ_1^0 and ϕ_2^0 can be written in first order [10] as

$$\phi_{1,2}^0 \approx \phi^0 + \epsilon_{1,2}\phi^1, \tag{1}$$

where in case of translation $\epsilon_{1,2}^{t} = x_{1,2}/w_0$, and in case of rotations $\epsilon_{1,2}^{r} = i\theta_{1,2}(\pi w_0/\lambda)$; here λ is the light wavelength and w_0 is the waist size of the Gaussian modes of the perfectly aligned cavities, supposedly identical. In a similar way, when we consider waist size mismatches $\delta w_{1,2}$ and waist positions mismatches $\delta z_{1,2}$, the two cavities' fundamental modes ϕ_1^0 and ϕ_2^0 can be written as [10]

$$V_{1,2}^0 \approx V^0 + k_{1,2}V^1, \tag{2}$$

where in case of size mismatches $k^{\rm s}_{1,2} = \delta w_{1,2}/w_0$, and in case of position mismatches $k^{\rm p}_{1,2} = {\rm i}(\lambda\delta z_{1,2}/2\pi w_0^2)$.

Analogously, the incident beam (supposedly perfectly matched to the perfect interferometer in the absence of jitter) can be written as

$$E^{\rm in} \approx A(\phi^0 + \beta\phi^1)\exp({\rm i}\omega_0 t), \tag{3}$$

where $\beta_{\rm l}(t) = l(t)/w_0$ if lateral beam jitter $l(t)$ is considered and $\beta_{\rm a}(t) = {\rm i}\theta(t)\,(\pi w_0/\lambda)$ if angular jitter $\theta(t)$ is considered. If the size or position fluctuations are considered, similar expressions hold, except for modes V^n instead of ϕ^m and corresponding expansions must be considered.

As a case of particular interest, let us suppose that the two cavities are translated (by an amount x_1, x_2) and that the input beam is oscillating at an angle $\theta(t)$ in the x-direction.

Because the response of a Fabry–Perot cavity is frequency dependent, it is useful to consider a given frequency of the jitter of the beam and write $\beta_{\rm a}(t) = \frac{1}{2}{\rm i}\beta_0({\rm e}^{{\rm i}\omega t} + {\rm e}^{-{\rm i}\omega t})$, where $\beta_0 = \theta_0(\pi w_0/\lambda)$.

The incident field is now the superposition of three electric fields with different frequencies: one, E^0, having components only on the fundamental mode ϕ^0, with frequency ω_0,

$$E^0 = A\,\phi^0\exp({\rm i}\omega_0 t), \tag{4}$$

and the two others, E^1, having components on the transverse mode ϕ^1 with the frequency shifted from ω_0 by $\pm\omega$,

$$E^1 = A\phi^1{\rm i}(\beta_0/2)\exp[{\rm i}(\omega_0 \pm \omega)t]. \tag{5}$$

The misaligned interferometer mixes up these incident modes, so that the interferometer field $E^{\rm rec}$ at a particular sideband frequency, say $\omega_0 + \omega$, can be conveniently written as

$$E^{\rm rec}(\omega_0 + \omega) = M(\omega_0 + \omega)E_{\rm in}(\omega_0 + \omega), \tag{6}$$

where $E(\omega)$ is a two-element vector having as components the projections of the electric field onto the ϕ^0 and ϕ^1 modes. The matrix M, taking into account only the first-order terms in the misalignments, can be calculated as a sum of all the reflections of the beam in going forth and back in the interferometer, as shown in Appendix A.

In this approximation, the matrix M is given by

$$M(\omega_0 + \omega) = t_{\rm R}\begin{pmatrix} \dfrac{1}{1 - r_{\rm R}r_0 f(\omega)\,{\rm e}^{{\rm i}\varphi}} & \dfrac{\epsilon}{2}\left(\dfrac{1}{1 - r_{\rm R}r_0 f(\omega)\,{\rm e}^{{\rm i}\varphi}} - \dfrac{1}{1 + r_1 r_{\rm R}}\right) \\ \dfrac{\epsilon}{2}\left(\dfrac{1}{1 - r_{\rm R}r_0 f(\omega)\,{\rm e}^{{\rm i}\varphi}} - \dfrac{1}{1 + r_1 r_{\rm R}}\right) & -\dfrac{1}{1 + r_1 r_{\rm R}} \end{pmatrix}. \tag{7}$$

Here $r_0 f(\omega)\,{\rm e}^{{\rm i}\varphi(\omega)}$ is the reflectivity of the Fabry–Perot cavities, with $f(\omega) \approx [1 + (c\omega/4\pi LF)^2]^{-1/2}$, $\varphi = 4LF\omega/\pi c$, c is the velocity of the light, $\epsilon = \epsilon_1 + \epsilon_2 = (x_1 + x_2)/w_0$, L is the length of the cavities in the arms, F their finesse, and $t_{\rm R}$ and $r_{\rm R}$ are the transmittivity and the reflectivity of the recycling mirror. The reflectivity of the cavities for the first transverse mode, r_1, is assumed to be real because the cavities are not degenerate (see Appendix A).

If there is no jitter ($\boldsymbol{E}^{\rm in} = (1;0)A{\rm e}^{{\rm i}\omega_0 t}$) and if the interferometer is perfectly aligned ($\epsilon = 0$), the field circulating in the interferometer remains on the ϕ^0 (TEM00) mode, and it is equal to the incident field amplified by the recycling gain factor $G = t_{\rm R}/(1 - r_{\rm R}r_0)$, as expected: $\boldsymbol{E}^{\rm rec} = (t_{\rm R}/(1 - r_{\rm R}r_0);0)A{\rm e}^{{\rm i}\omega_0 t}$.

In the more general case the electric field circulating in the recycling cavity can be obtained by superposing the (recirculated) incident fields: the field incident at the fundamental mode E^0, with the frequency ω_0 perfectly resonating in the cavities, and the E^1 mode, with a frequency shifted by $\pm\omega$.

Multiplying for the matrix M, with proper frequency, and superposing them, we can express the field in the recycling cavity as

$$E^{\mathrm{rec}} = [\,M(\omega_0)(1;0) + M(\omega_0 \pm \omega)(0;\tfrac{1}{2}\mathrm{i}\beta_0)\,\mathrm{e}^{\pm \mathrm{i}\omega t}\,]\, A\mathrm{e}^{\mathrm{i}\omega_0 t} \tag{8}$$

$$= \left\{ \frac{t_\mathrm{R}}{1 - r_\mathrm{R} r_0} + t_\mathrm{R}\frac{\mathrm{i}\beta_0\epsilon}{4}\left[\left(\frac{1}{1 - r_\mathrm{R} r_0 f(\omega)\,\mathrm{e}^{\mathrm{i}\varphi}} - \frac{1}{1 + r_1 r_\mathrm{R}}\right)\mathrm{e}^{\mathrm{i}\omega t} + \left(\frac{1}{1 - r_\mathrm{R} r_0 f(-\omega)\,\mathrm{e}^{-\mathrm{i}\varphi}} - \frac{1}{1 + r_1 r_\mathrm{R}}\right)\mathrm{e}^{-\mathrm{i}\omega t}\right];\; t_\mathrm{R}\frac{\epsilon}{2}\left(\frac{1}{1 - r_\mathrm{R} r_0} - \frac{1}{1 + r_\mathrm{R} r_1}\right) + \frac{\beta_0 t_\mathrm{R}}{1 + r_\mathrm{R}}\right\} A\mathrm{e}^{\mathrm{i}\omega_0 t}. \tag{9}$$

We may already note the coupling of the jitter (β_0) and lateral misalignments (ϵ) in the ϕ^0 component of E^{rec}, at the sideband frequencies $\omega_0 \pm \omega$.

3. Effects of misalignments

Starting from the expression of E^{rec} we can calculate the two electric fields interfering at the output of the interferometer. With the notation of Fig. 1 we obtain

$$E_1^{\mathrm{out}} = \tfrac{1}{2}(\langle E^{\mathrm{rec}}|\phi_1^0\rangle \hat{r}_1^0 \phi_1^0 + \langle E^{\mathrm{rec}}|\phi_1^1\rangle \hat{r}_1^1 \phi_1^1), \tag{10}$$

$$E_2^{\mathrm{out}} = \tfrac{1}{2}(\langle E^{\mathrm{rec}}|\phi_2^0\rangle \hat{r}_2^0 \phi_2^0 + \langle E^{\mathrm{rec}}|\phi_2^1\rangle \hat{r}_2^1 \phi_2^1). \tag{11}$$

The phase of the interfering beams is dominated by the relative phase of the electric fields on the ϕ^0 mode, and the components of the fields on the ϕ^0 mode are given by

$$\langle E_1^{\mathrm{out}}|\phi^0\rangle = \frac{1}{2}t_\mathrm{R}\left\{\frac{r_0}{1 - r_0 r_\mathrm{R}} + \mathrm{i}\frac{\beta_0\epsilon}{4} r_0 f(\omega)\left[\left(\frac{1}{1 - r_\mathrm{R} r_0 f(\omega)\,\mathrm{e}^{\mathrm{i}\varphi}} - \frac{1}{1 + r_1 r_\mathrm{R}}\right)\mathrm{e}^{\mathrm{i}(\omega t + \varphi)} + \left(\frac{1}{1 - r_\mathrm{R} r_0 \mathrm{e}^{-\mathrm{i}\varphi}} - \frac{1}{1 + r_1 r_\mathrm{R}}\right)\mathrm{e}^{-\mathrm{i}(\omega t + \varphi)}\right] + \frac{\mathrm{i}\beta_0\epsilon_1}{1 + r_1 r_\mathrm{R}}[\,r_1\cos(\omega t) + r_0 f(\omega)\cos(\omega t + \varphi)\,]\right\}, \tag{12}$$

$$\langle E_2^{\mathrm{out}}|\phi^0\rangle = \frac{1}{2}t_\mathrm{R}\left\{\frac{r_0}{1 - r_0 r_\mathrm{R}} + \mathrm{i}\frac{\beta_0\epsilon}{4} r_0 f(\omega)\left[\left(\frac{1}{1 - r_\mathrm{R} r_0 f(\omega)\,\mathrm{e}^{\mathrm{i}\varphi}} - \frac{1}{1 + r_1 r_\mathrm{R}}\right)\mathrm{e}^{\mathrm{i}(\omega t + \varphi)} + \left(\frac{1}{1 - r_\mathrm{R} r_0 \mathrm{e}^{-\mathrm{i}\varphi}} - \frac{1}{1 + r_1 r_\mathrm{R}}\right)\mathrm{e}^{-\mathrm{i}(\omega t + \varphi)}\right] + \frac{\mathrm{i}\beta_0\epsilon_2}{1 + r_1 r_\mathrm{R}}[\,r_1\cos(\omega t) + r_0 f(\omega)\cos(\omega t + \varphi)\,]\right\}. \tag{13}$$

The phase difference at the frequency ω is finally given by

$$\delta\psi = \mathrm{Re}\left((\epsilon_1 - \epsilon_2)\beta_0 \frac{r_1 + r_0 f(\omega)\,\mathrm{e}^{\mathrm{i}\varphi}}{r_0}\,\frac{1 - r_0 r_\mathrm{R}}{1 + r_1 r_\mathrm{R}}\right). \tag{14}$$

In this formula it is worth noting that the phase noise is proportional to the product of the amplitude of the component of the asymmetry of the interferometer on the ϕ^1 mode, and the amplitude of the quadrature-component of the incident field on the same mode. It is easy to verify that this result is also valid, if we consider the coupling between the lateral jitters of a laser with angular misalignments of the interferometer, while coupling between components with the same geometrical phase (like translations of the cavities – fluctuations in positions of the beam) produces only a change in the light circulating in the interferometer (a power decrease), but no phase noise, within our approximations.

In the same way, considering the waist fluctuations of the input laser, we obtain the same expression, except that the coupling is now between the diameter fluctuations of the laser and the difference in position of the waist in the two cavities, and vice-versa. In the previous formula (14) ϵ and β must now be interpreted as the components of the analogous expansions on the first Laguerre–Gaussian modes.

4. Results and conclusion

Looking at Eq. (14), let us first note that in the absence of the recycling cavity and in the zero-frequency limit, which implies $\varphi = 0$, we find the same results as Rüdiger et al. [5]. Also we notice (non-surprisingly) the filtering effects of the recycling cavity, which reduces the phase noise, in the zero-frequency limit, by the factor $(1 - r_R r_0)/(1 + r_R r_1)$, as is expected and as we have checked also by beam-propagation computer code [11].

If we consider that the cavities are not degenerate, so that $\hat{r}_1$ is essentially not frequency dependent, the frequency behaviour is described by the factor $r_1 + r_0 f(\omega)\, e^{i\varphi}$, which shows a reduction of the noise effect for frequencies that are antiresonant in the recycling cavity ($\varphi = \pi$).

Nevertheless, when we consider the extreme sensitivity of the interferometers designed for gravitational wave detection, the limits that formula (14) imposes for the misalignments of the interferometers, are quite stringent. In fact, a spectral density of the angular jitter of the laser, cleaned by an input mode-cleaner at the level of $\theta \approx 10^{-11}$ rad/$\sqrt{\text{Hz}}$ [12], and a recycling-mirror reflectivity $1 - r_R \approx 2 \times 10^{-2}$, would lead, for example in the case of VIRGO ($\lambda = 1.064\ \mu$m, $w_0 = 2$ cm, recycling cavity power gain $F = 100$, laser power = 10 W), to the phase noise

$$\delta\psi \approx 6 \times 10^{-7}\, \delta_x\, \frac{\text{rad}}{\sqrt{\text{Hz}}}. \tag{15}$$

In order to have a noise (three times) less than the shot-noise limited phase sensitivity of VIRGO, $\psi_{sn} = 1.4 \times 10^{-11}$ rad/$\sqrt{\text{Hz}}$, the transverse position of the optical axis of the cavities has to be maintained as well as

$$\delta_x < 10^{-5}\ \text{m}.$$

Taking into account that the position of the axis is controlled by orienting the curved mirrors (and that these mirrors have a curvature radius typically of the order of the length of the cavities), we see that the mirrors have to be aligned with an accuracy of the order of

$$\Delta\theta \approx 3 \times 10^{-9}\ \text{rad}. \tag{16}$$

In the case of the initial LIGO, the limits are of the same order of magnitude, even if a little less stringent, because the planned shot-noise limit is a bit worse than for VIRGO, and imposes angular residual misalignments of the order of 10^{-8} rad, while the other possible kinds of fluctuations (waist mismatches) do not impose particularly stringent limits, in both cases. In fact, the laser fluctuations [12] are of the order of $\delta W/W_0 \approx 10^{-9}/\sqrt{\text{Hz}}$ and $\delta z(\lambda/\pi W_0^2) \approx 10^{-9}/\sqrt{\text{Hz}}$, after mode cleaning. By inserting this values in the above formula (14) and considering asymmetries of the cavities of the order of 10^{-3}, we obtain a phase noise $\delta\psi \approx 10^{-13}$ rad/$\sqrt{\text{Hz}}$, which is negligible.

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Appendix A. The matrix M

Referring to Fig. 1, let us first suppose that the incident field has a component only on the ϕ^0 mode: $E^{in} = A\phi^0 \exp[i(\omega_0 + \omega)t]$. After the first reflection on the cavities, the fields E_1^{out} and E_2^{out} are given by

$$E_1^{\rm out} = \frac{A}{\sqrt{2}}\left(\langle\phi^0|\phi_1^0\rangle\hat{r}_1^0\phi_1^0 + \langle\phi^0|\phi_1^1\rangle\hat{r}_1^1\phi_1^1\right), \quad E_2^{\rm out} = \frac{A}{\sqrt{2}}\left(\langle\phi^0|\phi_2^0\rangle\hat{r}_2^0\phi_2^0 + \langle\phi^0|\phi_2^1\rangle\hat{r}_2^1\phi_2^1\right).$$

The modes of the interferometer and the modes of the cavities are related by the expressions

$$\phi_0^1 = \phi^0 + \epsilon_1\phi^1, \quad \phi_1^1 = \phi^1 - \epsilon_1\phi^0, \quad \phi_0^2 = \phi^0 + \epsilon_2\phi^1, \quad \phi_1^2 = \phi^1 - \epsilon_2\phi^0.$$

For the sake of simplicity, assuming that the frequency of the laser, even if shifted by the amount ω due to the jitter, is always resonant in the cavities, even if not perfectly, we can write the complex reflection $\hat{r}_1^0 = \hat{r}_2^0 = -r_0 f(\omega)\exp(i\varphi)$, where $f(\omega) = [1 + (c\omega/a\pi LF)^2]^{-1/2}$ and $\varphi(\omega) = 4LF\omega/\pi c$.

Here $\hat{r}_1$ is the reflectivity of the cavity for the first transverse mode: because in the typical case of LIGO/VIRGO the frequency distances of transverse modes are of the order of some kHz, in the frequency range of interest these modes are never excited, and $\hat{r}_1 = r_1$ can be assumed real.

The fields $E_1^{\rm out}$ and $E_2^{\rm out}$ then take the expression

$$E_1^{\rm out} = \frac{A}{\sqrt{2}}\{-r_0 f(\omega)\exp(i\varphi)\,\phi^0 - \epsilon_1[r_0 f(\omega)\exp(i\varphi) + r_1]\phi^1\},$$

$$E_2^{\rm out} = \frac{A}{\sqrt{2}}\{-r_0 f(\omega)\exp(i\varphi)\,\phi^0 - \epsilon_2[r_0 f(\omega)\exp(i\varphi) + r_1]\phi^1\}.$$

In this way, after the recombination at the beam splitter, and the reflection on the recycling mirror, the contribution of this first round trip on the interferometer to the field of the recycling cavity can be written as

$$E_1^{\rm rec} = At_{\rm R}\{r_0 f(\omega)\exp(i\varphi)\,\phi^0 + \tfrac{1}{2}(\epsilon_1 + \epsilon_2)[r_0 f(\omega)\exp(i\varphi) + r_1]\phi^1\}.$$

In the same way, considering an incident field with components only on the ϕ^1 mode, the contribution after a round trip is

$$E_1^{\rm rec} = At_{\rm R}\{\tfrac{1}{2}(\epsilon_1 + \epsilon_2)[r_0 f(\omega)\exp(i\varphi) + r_1]\phi^0 - r_1\phi^1\},$$

so that we can write $E_1^{\rm rec} = E^{\rm in}t_{\rm R}B$, where the matrix B is

$$B = \begin{pmatrix} r_0 f(\omega)\,e^{i\varphi} & \frac{1}{2}(\epsilon_1 + \epsilon_2)[r_0 f(\omega)\,e^{i\varphi} + r_1] \\ \frac{1}{2}(\epsilon_1 + \epsilon_2)[r_0 f(\omega)\,e^{i\varphi} + r_1] & -r_1 \end{pmatrix}.$$

The total field in the recycling cavity is then the sum of all the contributions, and it is given by

$$E^{\rm rec} = E^{\rm in}t_{\rm R}\sum_{n=0}^{\infty} B^n.$$

The sum can easily be performed taking into account that the matrix B is symmetric and than the off-diagonal elements are much smaller that the diagonal ones.

In this case it is easy to see that the off-diagonal elements do not give a contribution, at the first order, to the diagonal elements of the resulting matrix M. Furthermore, in the same approximation, the off-diagonal elements of M can be expressed as

$$M_{12} = \frac{B_{12}}{B_{11} - B_{22}}\left(\frac{1}{1 - B_{11}} - \frac{1}{1 - B_{22}}\right),$$

so that the matrix M assumes the form

$$M = t_R \begin{pmatrix} \dfrac{1}{1 - r_R r_0 f(\omega)\, e^{i\varphi}} & \dfrac{\epsilon}{2}\left(\dfrac{1}{1 - r_R r_0 f(\omega)\, e^{i\varphi}} - \dfrac{1}{1 + r_1 r_R}\right) \\ \dfrac{\epsilon}{2}\left(\dfrac{1}{1 - r_R r_0 f(\omega)\, e^{i\varphi}} - \dfrac{1}{1 + r_1 r_R}\right) & -\dfrac{1}{1 + r_1 r_R} \end{pmatrix},$$

where $\epsilon = \epsilon_1 + \epsilon_2$.

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