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Nonlinear meter for the gravitational wave antenna

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Abstract

A principle of a new readout meter for the large - scale gravitational wave antennae is proposed. This principle is based on registration of the spatial shift of standing optical wave caused by the gravitation wave inside the Fabri-Perot resonator without absorption of optical quanta.

1 Introduction

Several laboratories on our planet at present are involved in the creation of so-called laser gravitational wave antennae (the projects LIGO, VIRGO, GEO - 600). The goal of these projects is to detect small perturbations of the metric h which are produced by astrophysical catastrophes (the merging of binary neutron stars, the merging of neutron stars with black holes etc) many megaparsecs away from our planet. The first step in these projects is to reach the sensitivity at the level $h \simeq 1 \cdot 10^{-21} \div 1 \cdot 10^{-22}$ (see e.g. [1, 2, 3]). This means that the antennae have to register the small displacements ΔL between two masses (the mirrors in optical interferometer) separated at the distance L with the resolution $\Delta L \simeq hL/2 \simeq 2 \cdot 10^{-16} \div 2 \cdot 10^{-17} cm$ if $L \simeq 4 \cdot 10^5 cm$. In the following steps of these projects the sensitivity must be substantionally higher and it will probably reach the level $h \simeq 10^{-23}$ or even a better one. At this level of sensitivity probably it will be possible to detect such features of the gravitational wave burst which will allow to learn the correct equation of the state of neutron stars and even to test General Relativity in the ultrarelativistic case.

"Only" two tasks have to be fulfilled in these projects: i) sufficient isolation of the test masses from all sources of noises, and ii) sufficient sensitivity of the readout meter. There are reasonably optimistic prospects for the isolation of the test masses from the heat bath [4, 5]. On the contrary at present there are no simple realistic schemes for the readout meter which will permit to reach the sensitivity substantionally better than $h \simeq 10^{-22}$. There are two main obstacles if the Fabri - Perot resonator with laser pump is used as a meter. The first one (above the Standard Quantum limit of sensitivity) is of technical origin. The response of the meter in the output phase shift $\Delta\phi_{out}$ has to be larger than the phase uncertainty of the phase pump:

$$\Delta\phi_{out} \simeq \frac{1}{2} h\omega_{opt}\tau > \frac{1}{\sqrt{N}} \tag{1}$$

where ω_{opt} is the optical frequency, τ is the averaging time, N is the number of "used" photons. If $h \simeq 10^{-22}$, $\omega_{opt} \simeq 2\cdot 10^{15} s^{-1}$, $\tau \simeq 1\cdot 10^{-3} s$, then $\Delta\phi_{out} \simeq 1\cdot 10^{-10}$, $N \simeq 1\cdot 10^{20}$ photons and the power of the laser

$$W_{opt} \simeq \frac{\hbar\omega_{opt} N}{\tau} \simeq 2\cdot 10^{11} erg/s$$

A small leak of the laser power (for example into the heating of the mirror) may be a serious obstacle [6] in this scheme.

The second obstacle is the SQL: the increase of N in the meter with continuous monitoring of the coordinate will inevitably encounter the Standard Quantum Limit of sensitivity:

$$h_{SQL} = \frac{2}{L}\sqrt{\frac{\hbar\tau}{m}} \simeq 5\cdot 10^{-23} \times \left(\frac{4\cdot 10^5 cm}{L}\right) \times \left(\frac{1\cdot 10^{-3} s}{\tau}\right)^{-1/2} \times \left(\frac{1\cdot 10^4 gram}{m}\right)^{1/2} \tag{2}$$

In this numerical example the necessary power of the laser (in coherent state!) has to be $\simeq 4\cdot 10^{12} erg/s$.

Several schemes of the meter which in principle permit to overcome the SQL were proposed recently [7, 8, 9, 10, 11]. Unfortunately in these schemes the power has to be even larger than in the above example or the possibility of technical implementation of the scheme is not elaborated.

The goal of this article is the description of the principle of the new readout meter for gravitational wave antennae.

2 The readout meter based on nonlinear optical element

The first feature of this meter is the use of long relaxation time τ^*_{opt} of the optical resonator which may be much longer than the averaging time τ. In the 40 - meter prototype of LIGO at present $\tau^*_{opt} \simeq 10^{-3}s$. With the best finesse existing today [12] in the full scale LIGO antenna the value of τ^*_{opt} may be $\simeq 10s$ and thus the ratio τ/τ^*_{opt} may be as small as 10^{-4}.

The second feature of the meter is the idea not to measure the phase shift of the output beam outside the interferometer but to measure the spatial shift of the standing optical wave induced by the gravitational wave inside the resonator without absorption of photons.

Let us suppose that the Fabri - Perot resonator is created between mirrors A, B, and C (see Fig.1). The gravitational wave changes the distance between the mirrors and produces the spatial shift of standing optical wave which has to be exited by external source (e.g. by a laser). Amplitude value of this shift in the area near to the mirror B is equal to

$$\delta x = \frac{\delta L_{AB} - \delta L_{BC}}{2} \tag{3}$$

where δL_{AB} and δL_{AB} are the variations of the distances between the mirrors A,B and B,C correspondingly. In the case of optimal direction and polarization of the gravitational wave the value of δx is equal to

$$\delta x = \frac{hL}{2}$$

where L is the unperturbed value of the distances.

The spatial shift of the wave may be measured by the a special device mounted on the mirror B. This device has to consist of two optical lenses separated by double focal length (see Fig.1) and a thin dielectrical plate with cubic nonlinear dielectrical susceptibility $\chi^{(3)}$. This plate has to be situated inside a capacitor part of a microwave resonator. If the plate is located on the slope of the standing optical wave then the shift of the standing wave near B will produce a change of dielectrical susceptibility in the focal zone and therefore a change of capacity and frequency of the microwave resonator:

$$\delta\omega_e = K\frac{h\omega_{opt}}{2} \tag{4}$$

where

$$K = \frac{\pi\chi^{(3)}E_{opt}^2\omega_e L}{\epsilon c}\frac{\sin(\omega_{opt}l\sqrt{\epsilon}/c)}{\omega_{opt}l\sqrt{\epsilon}/c},$$

E_{opt} is the amplitude value of the electric field of the optical wave, c is the speed of light, ϵ is the linear term of the dielectrical susceptibility, l - is thickness of the plate the with nonzero $\chi^{(3)}$.

This change of the frequency may be registered by the measurement of the phase shift in the microwave resonator (see Fig. 2):

$$\delta\phi_e = \delta\omega_e\tau = K\frac{\hbar\omega_{opt}\tau}{2} \tag{5}$$

The device described above evidently has to be a monolithic dielectrical structure with the nonlinear plate created by dopping in the way it is done in superlattices.

It is necessary to note that the fluctuations of the optical energy in the resonator will randomly change the value of ϵ and therefore produce additional uncertainty of ω_e. This effect limits the sensitivity at the level

$$h \simeq \frac{c}{\omega_{opt}L}\sqrt{\frac{\tau}{N_{opt}\tau_{opt}^*}}$$

However it can be avoided by using two dielectrical plates with opposite signs of $\chi^{(3)}$ located symmetrically on the left and right slopes of the standing optical wave.

Comparing the formulas (5) and (1) one has to conclude that the dimensionless parameter K defines the efficiency of this meter. Substituting value $\chi^{(3)} \simeq 1\cdot 10^{-14} CGSE$ (fused silica), $E_{opt}^2 \simeq 2\cdot 10^7 CGSE$ (the optical breakdown of fused silica), $\omega_e \simeq 1\cdot 10^{11}s^{-1}$, $\epsilon \simeq 1$, $L \simeq 4\cdot 10^5 cm$ we obtain $K \simeq 1$.

Thus the task is reduced to the measurement of the phase shift in the microwave resonator and the value of the phase shift is approximately the same as the in traditional optical scheme. To realize this it will be necessary to use same number of quanta $N_e \simeq 10^{20}$ in coherent state (for $h \simeq 10^{-22}$) but the power W_e will be ω_{opt}/ω_e times smaller:

$$W_e = \frac{\hbar\omega_e N_e}{\tau} \simeq 1\cdot 10^7 erg/s \tag{6}$$

To realize a capacitor in microwave band for a resonator of clystron type with $\omega_e \simeq 10^{11}s^{-1}$ it is necessary to have its value in the range of $0.05 \div 0.1 cm$. This means that the cross section of the light in the focal area has to be $S_{opt} \simeq 1\cdot 10^{-4}cm \times 5\cdot 10^{-1}cm \simeq 5\cdot 10^{-5}cm^2$. To create such a distribution of the light beam it is possible to use a combination of two spherical and two cylindrical lenses. The total optical energy $\mathcal{E}_{opt}$ in this case has to be

$$\mathcal{E}_{opt} = \frac{L S_{opt} \sqrt{\epsilon} E_{opt}^2}{16\pi} \simeq 1 \cdot 10^6 erg$$

and the pumping optical power when $\tau_{opt}^* \simeq 10s$ will be

$$W_{opt} = \frac{\mathcal{E}_{opt}}{\tau_{opt}^*} \simeq 1 \cdot 10^5 erg/s$$

Thus the nonlinear meter described above permits to obtain a substantional reduction of the necessary optical power with a relatively modest condition for the microwave power.

3 The uncertainty relation in the nonlinear meter

The meter described above is by essence a coordinate one: the phase shift in a microwave cavity is proportional to the displacement of test masses and thus is proportional to the variation of metric h. If the averaging time τ_{meas} is longer than the relaxation time τ_e^* in the cavity then the resolution in the displacement (3) is equal to

$$\Delta x_{meas} = \frac{L}{2\omega_{opt} K \sqrt{N_e \tau \tau_e^*}} \tag{7}$$

In accordance with the uncertainty relation this measurement has to be accompanied with the perturbation of the canonically conjugated momentum. In the traditional scheme this perturbation is produced by the shot noise of the optical photons which randomly enter and leave the Fabri-Perot resonator. In the discussed meter this effect may be very small (in proportion with the ratio τ_{meas}/τ_e^*), because to get the signal it is not necessary to extract the optical photons from the Fabri-Perot resonator: the measurement is performed inside it without absorption.

A simple calculation (see appendix A) shows that the presence of a dielectrical non-homogenity in the optical resonator (a plate located on the slope of a standing wave) produces a redistribution of the e.m. energy in the two parts of the resonator which are separated by this plate. The ratio of the energy densities in these parts and thus the ratio of the pondermotive forces produced by optical photons which acts on the mirrors A and C and on the plate is equal to

$$\frac{F_A}{F_C} = \left(n - \frac{1}{n}\right) \sin \frac{\omega_{opt} l \sqrt{\epsilon}}{c}$$

where n is the relative value of the nonhomogeneous refraction index.

In the meter discussed above n is created by the cubic dielectrical nonlinearity $\chi^{(3)}$:

$$n = \sqrt{1 + \frac{4\pi\chi^{(3)}E_e^2}{\epsilon}}$$

Hence

$$\frac{F_A}{F_C} \simeq \frac{4\pi\chi^{(3)}E_e^2}{\epsilon}\sin\frac{\omega_{opt}l\sqrt{\epsilon}}{c}$$

where E_e is the strength of the electric field in the capacitor part of the microwave resonator.

The quantum shot noise fluctuations of the energy of the microwave resonator will create a random redistribution of the optical photons in the two parts of the Fabri-Perot resonator (the total number will remain constant). The corresponding fluctuating difference of the forces which acts on the mirrors A and C will be equal to

$$\Delta F = K\frac{\omega_{opt}}{\omega_e}\frac{\Delta\mathcal{E}_e}{L}$$

where $\Delta\mathcal{E}_e$ is the uncertainty of the microwave energy. Due to this force, during the time interval τ the momentum which is canonically conjugated to the measured coordinate will be perturbed. A value of this perturbation will be equal to

$$\Delta p_{pert} = \frac{\hbar\omega_{opt}K\sqrt{N_e\tau\tau_e^*}}{L} \tag{8}$$

As it follows from the formulas (7) and (8) the values of Δx_{meas} and Δp_{pert} exactly satisfy the uncertainty relation.

4 On the possibility to overcome the Standard Quantum Limit

As it was mentioned in the introduction to this article there are several schemes of meters which permit to realize a sensitivity better than the Standard Quantum Limit (SQL) in the gravitational wave antennae on free masses. The key obstacles in the implementation of these schemes are either the necessity to use substantionally nonclassical states of the e.m. field or the enormous pumping power or both of them. In the described above principle of the nonlinear meter practically any of the published schemes may be used to get the a sensitivity better than SQL. The evident advantage is that the implementation has to be done in the microwave band and therefore the power must be much smaller.

Let us consider one concrete procedure of the measurement. In the article [9] it was shown that using a "stroboscopic" sequence of coordinate measurements of a free mass it is possible to obtain the resolution in the measurement of an external classical force better than SQL. The necessary condition for this case is anticorrelation between the additive noise of the coordinate meter and its back action noise. To realize this anticorrelation in the nonlinear meter it is necessary to pump the coherent microwave power not continuously but by shot pulses with duration τ_{pulse} fulfilling the inequalities:

$$\tau_e^* < \tau_{pulse} < \tau_F$$

where τ_F is the characteristic duration of the force which has to be detected. The pulses have to be separated by time intervals $\tau > \tau_F$. The anticorrelation may be obtained by the method proposed in [10]: by the tuning of the phase of reference wave in such a way that the microwave homodyne gives information about the sum of the amplitude and phase quadrature amplitudes in properly weighted ratio. The minimal amplitude of the perturbation of the metric which may be detected by the use of this method is equal to

$$h_{min} \simeq \frac{2}{L}\sqrt{\frac{\hbar\tau_F}{2m}}\sqrt{\frac{\tau_F}{\tau_{meas}}} = h_{SQL}\sqrt{\frac{\tau_F}{\tau_{meas}}} \qquad (9)$$

where τ_{meas} is the time of extraction of the signal from the noises and m is the mass of the mirror (see appendix B). Hence the "price" which has to be "paid" for better sensitivity is the loss of the time resolution. Another price is the rise of the pumping power: the expression (9) can be rewritten in the form:

$$h_{min} \simeq h_{SQL}\sqrt{\frac{W_e^{SQL}}{W_e}} \qquad (10)$$

where W_e^{SQL} is the power sufficient to reach the SQL value and W_e is the power used.

The relatively moderate numerical estimate for the microwave power (see section 2) shows in our view that it will be not too difficult to realize the value of W_e^{SQL}/W_e smaller than unity and thus to obtain a sensitivity better than h_{SQL}.

Concluding this description of a new principle of the readout system for the gravitational antenna we want to emphasize that all estimates presented above were based on the conservative approach: the use of $\chi^{(3)}$ of very weak nonlinearity of the fused silica. The use of a much more nonlinear dielectric will inevitably make the parameter K larger and thus the meter more simple in realization.

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A Back-action mechanism in the nonlinear meter

Let us consider a Fabri-Perot resonator with inserted dielectrical plate with dielectrical susceptibility n^2. Let $\tau_1 c$ is the optical distance between the left mirror and left edge of the plate, $\tau_2 c/n$ is the thickness of the plate, and $\tau_3 c$ is the optical distance between right edge of the plate and the right mirror. Let $E_1^{right}, E_2^{right}, E_3^{right}$ are the amplitudes of the optical waves travelling in right direction. in these three areas, and $E_1^{left}, E_2^{left}, E_3^{left}$ - amplitudes of the waves travelling in left direction. These amplitudes satisfy the following boundary conditions:

$$\begin{aligned}
&E_1^{right} = E_1^{left} \\
&E_1^{right} e^{i\omega\tau_1} + E_1^{left} e^{-i\omega\tau_1} = E_2^{right} = E_2^{left} \\
&E_1^{right} e^{i\omega\tau_1} - E_1^{left} e^{-i\omega\tau_1} = n(E_2^{right} - E_2^{left}) \\
&E_2^{right} e^{i\omega\tau_2} + E_2^{left} e^{-i\omega\tau_2} = E_3^{right} = E_3^{left} \\
&n(E_2^{right} e^{i\omega\tau_2} - E_2^{left} e^{-i\omega\tau_2}) = E_3^{right} - E_3^{left} \\
&E_3^{right} e^{i\omega\tau_3} + E_3^{left} e^{-i\omega\tau_3}
\end{aligned} \tag{11}$$

where ω is the frequency of the light. From these equations follows an equation for eigen frequencies of the resonator:

$$\sin\omega\tau + (n-1)\sin\omega\tau_2 \left(\cos\omega\tau_1 \cos\omega\tau_3 + \frac{1}{n}\cos\omega\tau_1 \cos\omega\tau_3\right) = 0 \tag{12}$$

where $\tau = \tau_1 + \tau_2 + \tau_3$.

Let as assume that

$$|(n-1)\sin\omega\tau_2| \ll 1$$

In this case the eigen frequencies can be presented is the form $\omega_k = \omega_k^{(0)} + \nu_k$, where

$$\omega_k^{(0)} = \frac{\pi k}{\tau}$$

is the zero-order-approximation values and

$$|\nu_k| \ll \omega_k^{(0)}$$

Substituting this presentation into the equation (12) and omitting the terms of second order value one can obtain, that

$$\nu_k \approx -\frac{(n-1)\sin\omega_k^{(0)}\tau_2}{\tau}\left(\cos\omega_k^{(0)}\tau_1 \cos\omega_k^{(0)}\tau_3 + \frac{1}{n}\cos\omega_k^{(0)}\tau_1 \cos\omega_k^{(0)}\tau_3\right)$$

Substitution of this solution into the equation (11) gives the value of a ratio

$$\frac{E_3^{right} e^{i\omega\tau_3}}{E_1^{left}} = (-1)^k + \frac{1}{2}\left(n - \frac{1}{n}\right) \sin\omega_k^{(0)}\tau_2 \sin\omega_k^{(0)}(\tau_1 - \tau_3)$$

Hence the ratio of the pressure forces on the right and left mirrors is equal to

$$\frac{F_{right}}{F_{left}} = \left|\frac{E_3^{right} e^{i\omega\tau_3}}{E_1^{left}}\right|^2 = 1 + (-1)^k \left(n - \frac{1}{n}\right) \sin\omega_k^{(0)}\tau_2 \sin\omega_k^{(0)}(\tau_1 - \tau_3)$$

The difference of this forces is equal to

$$F_{right} - F_{left} \approx \bar{F}\left(\frac{F_{right}}{F_{left}} - 1\right) = \bar{F}\left(n - \frac{1}{n}\right) \sin\omega_k^{(0)}\tau_2 \sin\omega_k^{(0)}(\tau_1 - \tau_3) \qquad (13)$$

where $\bar{F} = (F_{right} + F_{left})/2$.

B Stroboscopic measurement of the coordinate of the free mass

Let a classical force $F(t)$ acts on a free mass m. To detect this force coordinate x of the mass is measured periodically. The result of such a sequence of the measurement has the form of a vector

$$\tilde{x}_j = X_j + \hat{x} + \frac{\hat{p}j\tau}{m} + \hat{x}_j^{fluct} + \sum_{k=-\infty}^{j} \frac{\hat{p}_k^{fluct}(j-k)\tau}{m} \qquad (14)$$

where τ is a time interval between measurements,

$$x_j^{signal} = \frac{1}{m}\int_{-\infty}^{j\tau} F(t')(t-t')dt'$$

is the signal, $\hat{x}$ and $\hat{p}$ are an initial values of coordinate and momentum of the mass, $\hat{x}_j^{fluct}$ and $\hat{p}_j^{fluct}$ describe the error of $j-th$ measurement and perturbation of the momentum during the measurement. We shall assume that measurements are independent:

$$< \hat{x}_j^{fluct} \circ \hat{x}_k^{fluct} >= \Delta_x^2 \delta_{jk}$$
$$< \hat{p}_j^{fluct} \circ \hat{p}_k^{fluct} >= \Delta_p^2 \delta_{jk}$$
$$< \hat{p}_j^{fluct} \circ \hat{p}_k^{fluct} >= \Delta_{xp} \delta_{jk}$$

and

$$\Delta_x^2 \Delta_p^2 - \Delta_{xp}^2 = \frac{\hbar^2}{4} \qquad (15)$$

where Δ_x is the error of measurement, Δ_p is the perturbation of the momentum, $\circ$ denotes symmetrical multiplication.

Simple linear transformation of the output of the meter allows to exclude the initial values $\hat{x}$ and $\hat{p}$:

$$\tilde{p}_j = \frac{m}{\tau}(\tilde{x}_{j+1} - 2\tilde{x}_j + \tilde{x}_{j-1}) = p_j^{signal} + \hat{P}_j^{fluct}$$

where

$$p_j^{signal} = \frac{m}{\tau}(x_{j+1}^{signal} - 2x_j^{signal} + x_{j-1}^{signal}) \tag{16}$$

and

$$\hat{P}_j^{fluct} = \frac{m}{\tau}(\hat{x}_{j+1}^{fluct} - 2\hat{x}_j^{fluct} + \hat{x}_{j-1}^{fluct}) + p_j^{fluct}$$

Signal-to-noise ratio for such a procedure is equal to

$$\frac{s}{n} = \sum_{j=-\infty}^{\infty} v_j p_j^{signal} \tag{17}$$

where filtering vector v_j is defined from the matrix equation

$$\sum_{k=-\infty}^{\infty} B_{jk} v_k = p_j^{signal} \tag{18}$$

and $B_{jk} \equiv < \hat{P}_{jfluct} \circ \hat{P}_{jfluct} >$ is the correlation matrix of the total noise:

$$\begin{aligned} B_{jk} = \; & \tfrac{m^2}{\tau^2}\left(\delta_{j+1k-1} + \delta_{j-1k+1} - 4(\delta_{j+1k} + \delta_{jk+1}) + 6\delta_{jk}\right)\Delta_x^2 + \\ & \tfrac{2m}{\tau}(\delta_{j+1k} + \delta_{jk+1} - 2\delta_{jk})\Delta_{xp} + \delta_{jk}\Delta_p^2 \end{aligned} \tag{19}$$

Let us use a spectral representation: for any vector A_j

$$A(\nu) = \sum_{-\infty}^{\infty} A_j e^{-ij\nu}; A_j = \int_{-\pi}^{\pi} A(\nu) e^{ij\nu} \frac{d\nu}{2\pi}$$

where ν is the dimensionless "frequency". Spectral transformation of correlation matrix of the noise has the form:

$$S(\nu) = \sum_{-\infty}^{\infty} B_{jk} e^{-i(j-k)\nu}$$

where $S(\nu)$ is the "spectral density" of the noise.

Formulas (16 - 19) in the spectral representation have the form:

$$p^{signal}(\nu) = \frac{2m}{\tau}(\cos\nu - 1)x_{signal}(\nu) \tag{20}$$

$$\frac{s}{n} = \int_{-\pi}^{\pi} v^*(\nu) p^{signal}(\nu) \frac{d\nu}{2\pi} \tag{21}$$

$$S(\nu)v(\nu) = p^{signal}(\nu) \tag{22}$$

$$S(\nu) = \frac{4m^2}{\tau^2}(\cos\nu - 1)^2\Delta_x^2 + \frac{4m}{\tau}(\cos\nu - 1)\Delta_{xp} + \Delta_p^2 \tag{23}$$

Hence

$$\frac{s}{n} = \int_{-\pi}^{\pi} \frac{|p_{signal}(\nu)|^2}{S(\nu)}\frac{d\nu}{2\pi} = \int_{-\pi}^{\pi} \frac{|x_{signal}(\nu)|^2(\cos\nu - 1)^2}{\left(\cos\nu - 1 + \frac{\tau}{2m}\frac{\Delta_{xp}}{\Delta_x^2}\right)^2 \Delta_x^2 + \left(\frac{\hbar\tau}{4\Delta_x m}\right)^2}\frac{d\nu}{2\pi} \tag{24}$$

Let

$$\frac{\hbar\tau}{4\Delta_x m} \ll 1$$

and

$$\Delta_{xp} = \frac{4m}{\tau}\Delta_x^2.$$

In this case expression under integration in the formula (24) has sharp narrow maximum near the value of $\nu = \pi$, and the integral is equal to:

$$\frac{s}{n} = \int_{-\infty}^{\infty} \frac{16|x_{signal}(\pi)|^2}{\Delta_x^2\eta^4 + \left(\frac{\hbar\tau}{2\Delta_x m}\right)^2}\frac{d\eta}{2\pi} = \frac{16|x_{signal}(\pi)|^2\Delta_x}{(\hbar\tau/m)^{3/2}}$$

where $\eta = \nu - \pi$ and $|\eta| \ll 1$.

Under the same condition the filtering function $v(\nu)$ is equal to

$$v(\eta) = \frac{-\frac{4\tau}{m}x_{signal}(\pi)}{\Delta_x^2\eta^4 + \left(\frac{\hbar\tau}{2\Delta_x m}\right)^2}$$

and the filtering vector has the form

$$v_j = \frac{4\tau}{m}(-1)^{j+1}x_{signal}(\pi)\int_{-\infty}^{\infty} \frac{e^{ij\eta}}{\Delta_x^2\eta^4 + \left(\frac{\hbar\tau}{2\Delta_x m}\right)^2}\frac{d\eta}{2\pi} =$$

$$\frac{2(-1)^{j+1}Jx_{signal}(\pi)}{\hbar}e^{-|j|/J}\left(\cos\frac{j}{J} + \sin\frac{j}{J}\right)$$

where

$$J = 2\Delta_x\sqrt{\frac{m}{\hbar\tau}}$$

Hence

$$\frac{s}{n} = \frac{4m|x_{signal}(\pi)|^2\tau_{meas}}{\hbar\tau^2}$$

where

$$\tau_{meas} = 2J\tau$$

is the characteristic duration of the filtering.

Let the force $F(t)$ has the form of a pulse with duration $\tau_F \simeq \tau$ and without coordinate memory. In this case

$$|x_{signal}(\pi)| \simeq \frac{F\tau_F^2}{2m}$$

and

$$\frac{s}{n} = \frac{F^2\tau^2\tau_{meas}}{m\hbar} = \left(\frac{s}{n}\right)_{SQL}\frac{\tau_{meas}}{\tau}$$

where

$$\left(\frac{s}{n}\right)_{SQL} = \frac{F^2\tau^3}{m\hbar}$$

is the Standard Quantum Limit value of the signal-to-noise ratio.

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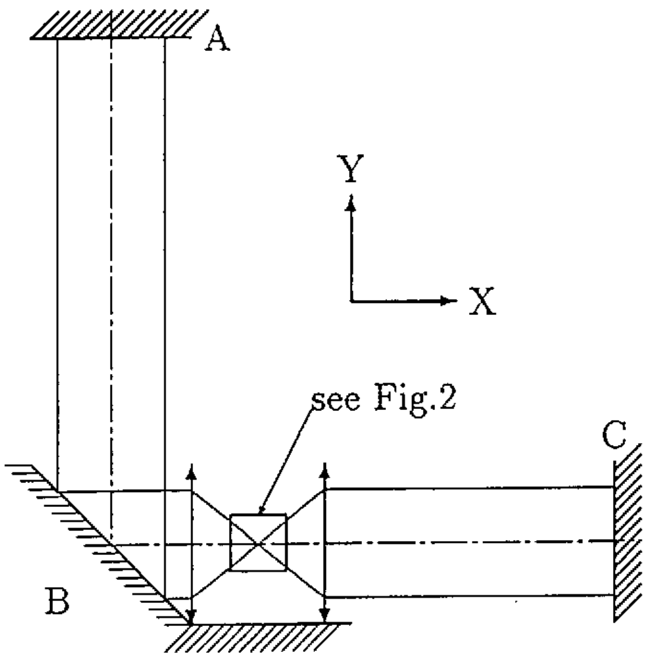

Fig. 1

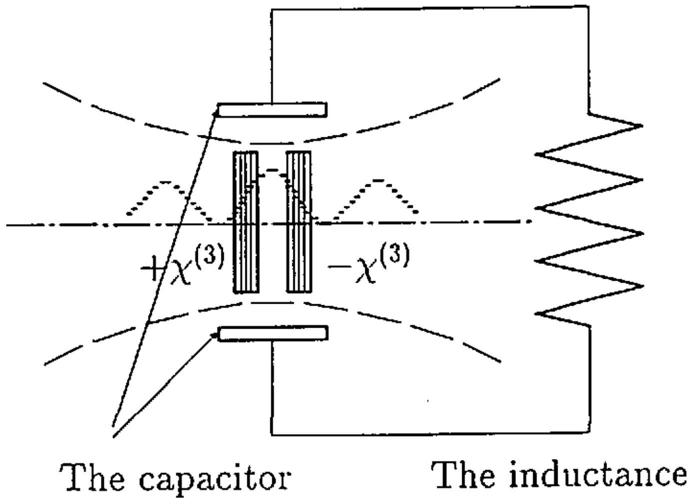

Fig. 2