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A low-frequency vibration isolation table using multiple crossed-wire suspensions

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A practical low-frequency one-dimensional vibration isolation system has been developed exploiting a previously developed crossed wire suspension technique. A table is supported by two such suspensions so as to be stiff in 5° of freedom but with a very long period in one horizontal direction. The rotation of the suspended mass in the original system is eliminated and the transfer function at high frequencies is greatly improved. Periods of 10 s were achieved in experimental tests, giving isolation of 40 dB at 1 Hz and up to 60 dB at 10 Hz. The system is suitable for one-dimensional optics experiments or as a base for further stages of vibration isolation. © *1996 American Institute of Physics.* [S0034-6748(96)02611-1]

I. INTRODUCTION

The X pendulum,[1,2] a method of suspending a mass with a very low restoring force in one horizontal direction, has recently been proposed with a view to possible use in the seismic vibration isolation system of a laser interferometric gravity wave detector, such as GEO,[3] LIGO,[4] TAMA,[5] and VIRGO.[6] Seismic noise is expected to be the dominant noise source at low frequencies, and will require very effective filtering. The basis of such filtering is a good low frequency mechanical oscillator, which has a transfer function that falls as the square of the frequency above the resonance. It is particularly desirable to have a low resonant frequency because lowering this by one order of magnitude causes the fall off to begin earlier and reduces the residual vibration at any particular frequency above that by two orders. Accordingly, several groups are investigating systems with low resonant frequencies.[7] The X pendulum is a promising competitor among these for its stability and reliability.

A X pendulum uses typically four (but at least three) suspension wires to support an intermediate "X plate" which in turn supports the main test mass via a rigid bar. It can be shown[1] that the locus of the center of the X plate is approximately an inverted parabola, so the X plate alone is unstable. However, at a critical position on the bar which depends on the geometry of the X plate, the locus is flat. If the test mass is mounted so that the center of mass of the system is just below the critical point, the system will be stable with a very long period.

The basic technique works as described, with an accurately predictable period, but is susceptible of improvement, particularly in two respects. First, there is the effect of finite mass and moment of inertia of the X plate and bar. An ideal X pendulum, with an infinitely strong and zero-mass superstructure, supporting a suspended mass concentrated at a point, would have a transfer function dropping indefinitely with increasing frequency. However, a more typical response, for any sort of massive pendulum, is for the response to saturate at some nonzero value above a certain frequency. In the original design,[2] the moment of inertia of the superstructure is dominant and has the effect of displacing the "sweet spot," where the vibration filtering is most effective, from the center of mass by a distance proportional to the fraction of mass in the superstructure.

Second, it is desirable to eliminate the rotation of the suspended mass. The system is equivalent to a very tall simple pendulum as far as horizontal motion of the center of mass is concerned, but for tilt there is no improvement on the physical length because the top of the bar (actually, a point just off the top, where the X wires appear to cross) is approximately stationary, with no horizontal motion and only a slight (second order) vertical motion. The motion of a general point will be contaminated by an amount proportional both to the tilt angle and the distance from the sweet spot, and this will be much greater than in the long simple pendulum.

In a laser interferometric gravity wave detector, the tilt could possibly be tolerated but only if the beam could be centered accurately on the sweet spot so as to limit the effect on the optical path length to second order in displacement on average. It would also create a heavy load on the beam alignment servosystems and is best eliminated.

To solve these problems, we evolved a new design in which the bar is replaced by wires. (We call these the intermediate wires to distinguish them from the X wires.) The wires are attached at well separated points on the X plate but are brought to a common clamp at the bottom, outlining the shape of an inverted pyramid. Since the pendulum never departs by more than a few degrees from the vertical, the wires are always in tension and are effectively rigid. Moreover, flexure of the wires at the lower clamp constitutes a hinge which can decouple the load from the rotation of the X plate. Rather than allow the load to dangle we decided to use two such modified X pendulums in parallel.

Finally, we made a detailed analysis of the dynamics, as given in the next section, and discovered that with such a light system, the mass and moment of inertia of the X plate could be contrived to cancel.

II. ANALYSIS OF DYNAMICS

We consider a load table as seen in Figs. 1 and 2 which is suspended via two identical X pendulums from a support

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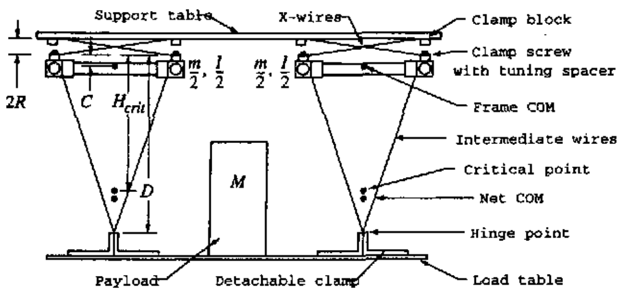

FIG. 1. Side view of the system: two X pendulums support a table so that it nearly moves inertially in one direction but is stiff against all other motions. The rigid bar of a standard X pendulum is replaced by wires in the form of an inverted pyramid. The wires are free to bend at the vertex of the pyramid and form a hinge there.

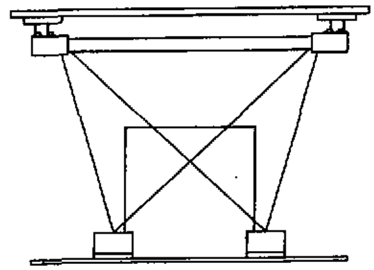

FIG. 2. End view of the system.

table which moves horizontally due to seismic disturbances. Although X pendulums can be constructed with different wire separations at the top and bottom of the "X," for simplicity we consider only the case where these are equal, with common value L. The level on the X plates where the X wires attach is the nominal top and the distance from this reference down to the respective clamps is D. (All other vertical positions on the X pendulums are measured similarly.) Since the X pendulums move identically, for the purposes of analysis we consider only the total mass m and moment of inertia I. The moment of inertia is measured about the center of mass of the plate/wire assembly, which is at a vertical position C. The mass and moment of inertia of the wires are negligible, so C depends only on the shape of the X plate. (It could conceivably be negative if the X plate had protrusions of substantial mass above the nominal top.) The angle of the wires from the horizontal when the pendulum is central is α_0, so giving a gap size of $L \tan \alpha_0$. We are then in a position to use a number of results from Barton *et al.*,[1] putting $L=L_1=L_2=l \cos \alpha_0$ and simplifying. It is convenient to introduce a parameter $R=(L \tan \alpha_0)/2$, i.e., half the gap size. The critical vertical position at which there is a purely horizontal locus is then

$$H_{crit}=L(\cot \alpha_0-\tan \alpha_0)/4=L^2/8R-R/2. \tag{2.1}$$

The locus of a point at an arbitrary vertical position H is described parametrically in terms of the tilt angle θ by

$$x_H(\theta)=(R+H)\theta, \tag{2.2}$$

where

$$y_H(\theta)=\tfrac{1}{2}(H-H_{crit})\theta^2, \tag{2.3}$$

where positive y_H is up and the constant term has been dropped. Using these relations, we can then immediately write the potential and kinetic energies in terms of θ and the displacement of the support, x. We have

$$V=\tfrac{1}{2}[mg(C-H_{crit})+Mg(D-H_{crit})+K_W]\theta^2 \tag{2.4}$$

and

$$T=\tfrac{1}{2}\{m[\dot{x}+(C+R)\dot{\theta}]^2+I\dot{\theta}^2+M[\dot{x}+(D+R)\dot{\theta}]^2\}. \tag{2.5}$$

Since we want the response of the system to an arbitrary displacement $x(t)$, we assume that the support can slide freely in the horizontal direction. The term K_W in the potential represents the angular restoring force due to the elasticity of the wires and is calculated in detail below.

Using Lagrange's formalism[8] then gives the equations of motion for the system which can easily be solved in the frequency domain for $x(\omega)$ and $\theta(\omega)$ as functions of the force $F(\omega)$ required to cause the motion of the support. The position of the load table is $x_D(\omega)=x(\omega)+(D+R)\theta$, so the transfer function is $1+(D+R)\theta/x(\omega)$. The force $F(\omega)$ then cancels giving

$$x_D(\omega)/x(\omega)=1+\frac{(D+R)[m(C+R)+M(D+R)]\omega^2}{K_W+[m(C-H_{crit})+M(D-H_{crit})]g-[I+m(C+R)^2+M(D+R)^2]\omega^2}. \tag{2.6}$$

The form of this function at high frequencies depends critically on the values of mass, moment of inertia, and center of mass position. In general, it will saturate at a nonzero value, but this may be positive or negative, as shown in Fig. 3. This high-frequency limit is

$$\frac{I-m(C+R)(D-C)}{I+m(C+R)^2+M(D+R)^2} \tag{2.7}$$

and for optimum performance we want it to be zero. The design constraint that ensures this is conveniently expressed as a preferred value for the moment of inertia:

$$I=m(D-C)(C+R). \tag{2.8}$$

Note that gravity and the elasticity of the wire do not appear; the balance is due purely to geometrical factors. This moment of inertia should be compared with the value $mL^2/12$ which would result from a completely uniform distribution of mass in the X plate. In the dominant term DR, D is typically $1.5L$ to $3L$, and R is around $L/10$ to $L/20$, so that

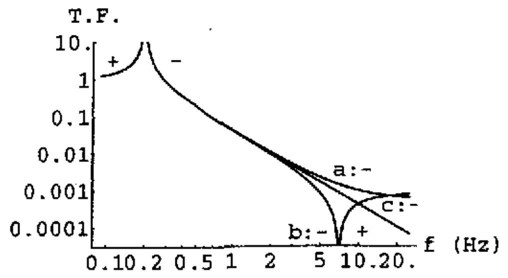

FIG. 3. The form of the transfer function for (a) a system dominated by the plate moment of inertia, (b) one dominated by the plate mass, and (c) one optimized according to Eq. (2.8). The sign of the transfer functions at key points is indicated.

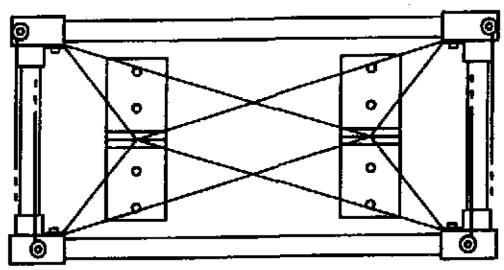

FIG. 4. One of the X frames, viewed from above, with the intermediate wires and lower clamps. A portion of each X wire is also shown.

the preferred moment of inertia is around twice the reference value. If C is large compared with R it may be larger still. This is easily arranged by designing an X plate which is heavier by a modest amount near the edges than near the center.

Alternatively, we can give a preferred value for the vertical position of the center of mass of the X plate

$$C=\{D-R-[(D-R)^2-4(I/m-DR)]^{1/2}\}/2. \tag{2.9}$$

The elastic restoring force K_W is due mainly to the X wires, which are under considerable tension. According to elementary elasticity theory,[9] the restoring torque G_W due to a long thin wire under tension T_W is

$$G_W=(T_W E_W I_W)^{1/2}\Delta\alpha, \tag{2.10}$$

where $\Delta\alpha$ is the wire bending angle, E_W is the Young's modulus, and I_W is the moment of area. Allowing for the fact that there are $N=8$ X wires inclined at an angle α_0 supporting a total mass $m+M$ the tension is

$$T_W=(m+M)g/(N \sin \alpha_0). \tag{2.11}$$

Further allowing for the fact that there are $2N=16$ wire endpoints contributing torque and for the mechanical advantage $a=0.5$ between the wire bending angle and the pendulum angle ($\Delta\alpha=a\theta$),

$$K_W=2a^2[(m+M)gNE_W I_W/(\sin \alpha_0)]^{1/2}$$
$$=[2(m+M)gE_W I_W/(\sin \alpha_0)]^{1/2}. \tag{2.12}$$

In our system, using 0.5 mm diam piano wire, the X wire elasticity would contribute all the restoring force for periods around 15 s. The elasticity of the intermediate wires can be calculated similarly but is negligible due to their small diameter and low tension.

III. EXPERIMENTAL DESIGN

A. Design of pendulum

In designing the pendulum, we aimed to produce a system with generally good performance so that we could get a rough idea of the limitations of the technique, but one which could readily be "detuned" to give a test of the optimization condition. After some experimentation, calculating the masses, and moments of inertia of various X-plate designs, we concluded that by far the most convenient parameter to vary while maintaining a low mass is the center of mass height C. This can be done by placing spacers on the X-wire clamp screws between the X wires and the X plates. This lowers the X plates and increases C.

The final "plate" is shown in Fig. 4 and is actually a frame consisting of four corner blocks milled from 14 mm aluminium plate, held apart by thin-walled brass tubing of 10 mm diam and average wall thickness (determined by weighing) 0.187 mm. The tubes fitted snugly into holes in the blocks and were secured with epoxy adhesive. Each corner block was fitted with two clamp screws: one, on the top surface, was a termination for one of the X-wires, and the second, on the inside surface, held two of the intermediate wires. This construction technique gave ample strength with a net weight of only 94 g per frame with screws. The distance L between the X-wire clamp screws on each side was 100 mm.

The load table consisted of a sheet of 3 mm aluminium with four detachable clamps for the intermediate wires. Three clamps (two below the first X pendulum and one below the second) is the minimum required to constrain against all rotations; however this wastes the transverse rigidity of the second X pendulum, so we chose four clamps at the slight risk of some overconstraint. A two clamp design could be considered if rigidity against rolling motions was not important.

Each of the two load table clamps below a frame was connected to each of the four corner blocks. This can be seen in the end view of Fig. 2. Thus, there were eight wires per frame and sixteen in total. Three wires on each load table clamp would have sufficed, but four was more natural given the symmetry of the frame. To guarantee even tensions in the wires, we built a special jig which held the load table clamp at the correct separation from the frame and applied uniform tension to the intermediate wires while the clamp screws were tightened. A lead block and small additional masses were stacked on the center of the load table to bring it up to a target weight of 5.0 kg. With these loads and using 0.2 mm piano wire, there was no significant unevenness of tension among the sixteen wires.

A Mathematica program was used to model the mass distribution in the X frames as functions of spacer thickness, based on the nominal dimensions. (Although the aim is to change C, the other parameters m, I, and D also change slightly.) The total frame mass, 188 g, which could be checked directly, was within 1% of the calculated value. The center of mass position C without spacers was half the thickness, or 7 mm. The moment of inertia I was 4.86×10^{-4} kg m^2, which is 3.04 times the reference moment of inertia (for a uniform X plate of the same mass and thickness). The model was then used to derive the optimum spacer thickness for a 10 s pendulum, 4.76 mm. (There is a very slight dependence on the frequency via the gap size R; for a 5 s period the optimum is 4.56 mm.)

B. Tuning procedure

Because of the large number of wires involved in supporting the load, it was not convenient to tune the system by varying the position of the center of mass (as in Ref. 1) and we implemented a new strategy as follows. The support table for the system was a 5 mm sheet of aluminium reinforced by two steel ribs bolted lengthwise along the top. Each X wire was run from the clamp screw on the X frame to a clamp on the support table. However it then continued, turning through 90° around a cylindrical post set into the underside of the support table and was joined via a turnbuckle to the symmetrically opposite wire on the far side. There were two turnbuckles for each X mechanism and four in total. The turnbuckles were also free to slide along their lengths in slots milled into the support table. By sliding the turnbuckles bodily, the two X's could be made symmetrical, and by rotating them the wire length and hence the gap size and critical height could be set. We chose a relatively short pendulum ($D=150$ mm, $H_{\mathrm{crit}}\approx120$ mm) because in this regime the critical height varies slowly with wire length and the pendulum is easy to adjust.

A preliminary adjustment was performed for each X pendulum separately. Because slight stretching of the wires causes the gap size and hence critical height to vary with load, it was necessary to use a test mass with approximately half the weight of the load table and payload. The gap size was checked with vernier calipers and was made equal at each side by sliding the turnbuckles. Secondly, the turnbuckles were shortened or lengthened individually to make the pendulum hang vertically. Finally the turnbuckles were tightened in parallel to set the period. A modest 5 s period was considered adequate for the preliminary adjustment.

After the preliminary adjustment on each side the respective X-wire clamps on the support table were tightened and the test mass was removed. To avoid unnecessary stretching and relaxing of the wires as the load was changed, spacer screws were fitted to the support table. When extended they pressed down on the frame from above and held it at the loaded separation from the support table even after the load was removed.

For the final adjustment, the load table and lead block were fitted and the spacer screws were retracted. The period was set to the target value (e.g., 10 s) using adjustments of one pair of X wires only. The gap size at each corner was measured and the average value was calculated. Three corners were then set directly as close as possible (±0.1 mm) to the average, using symmetrical motions of the turnbuckles. The final corner was adjusted taking notice only of centering and period, but naturally it also converged on the average gap.

Provided the pendulum is kept well centered, tuning for periods around 10 s is straightforward. As the period increases, any slight imperfection in the initial centering is magnified, so it is typically necessary to return to the centering step several times. If the tightening is overdone the system becomes unstable and will not hang centrally. (Note that "unstable" does not imply a catastrophic result because the fourth order term in the potential is always positive and of a convenient magnitude to act as a safety net.) Very close to the critical point it is not always clear whether a slight tilt indicates the onset of instability or not. A sensitive test is that the response to tightening one turnbuckle described above is reversed: if, when the wires leading to say the left side are tightened in an attempt to raise it, it instead goes further down, then the system has already gone unstable. In such a case, for our system, which had a considerable degree of frictional hysteresis, it was typically quickest to loosen both turnbuckles substantially and repeat the tuning from the beginning, using only adjustments in the tightening direction.

C. Measurement of transfer function

For measuring the transfer function of the system, the support table was mounted on a tripod of three linear bearings on a specially constructed stand. The stand was anchored by metal ties to a large stack of lead blocks on the floor of the laboratory. A magnetic coil driver with a dc-capable amplifier was used to apply horizontal forces to the support. The displacement of the support and load tables were measured by high frequency inductive sensors. Because the input displacement was large (up to 0.5 mm) the sensor for the support table was mounted directly on the main stand, but the lower sensor was mounted on a separate stack of lead blocks to minimize any spurious vibrations.

A HP-35665A signal analyzer was used to measure the transfer function of the system. The analyzer was operated in "swept sine" mode, wherein it injected pure sine waves into the actuator and used digital Fourier transform techniques to monitor signals at the same frequency from the two sensors. To take a spectrum over a frequency band extending down to the pendulum resonance would have created problems of both measurement time and saturation of the sensors, so runs were started at 1 Hz and the pendulum period was determined separately from the display of a digital oscilloscope. A preliminary calibration run was done with both sensors mounted on the support table.

IV. RESULTS

Runs were made without spacers, and with 4.7 and 10.0 mm spacers. For each run, data was taken with the system tuned to nominal periods of 5 and 10 s. All the 5 s transfer functions and the one for 4.7 mm spacers at 10 s are plotted in Figs. 5–8. The theoretical prediction based on the measured resonant frequencies are superimposed. For comparison, the calculated transfer function of an ideal simple pen-

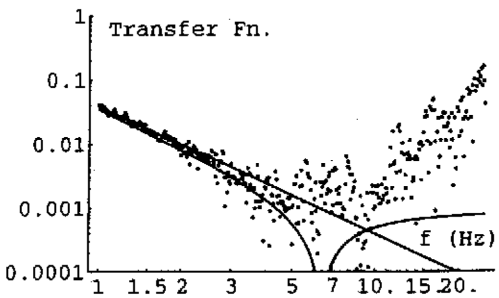

FIG. 5. Transfer function for the pendulum with no spacers, tuned to 5.4 s.

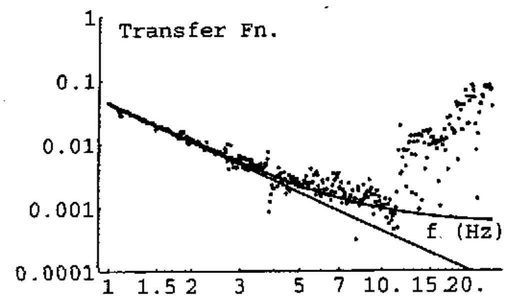

FIG. 7. Transfer function for the pendulum with over-long (10 mm) spacers, tuned to 4.9 s.

dulum with the same frequency is also plotted. The 10 s data (Fig. 8) shows horizontal isolation of 40 dB of at 1 Hz and 60 dB or more at 10 Hz.

Due to the fact that both the transfer function of the X pendulum and the output of the shaking mechanism fall rapidly with frequency, the noise floor of the position sensor is encountered at about 10–15 Hz. Unfortunately, due to the lightweight design, this is quite close to the frequency at which saturation is expected to set in. Bearing this in mind the agreement with theory is quite reasonable, and the characteristic curve of the transfer function relative to the ideal case can be clearly seen, especially for the 5 s data.

Three elastic mode resonance peaks were clearly visible in data from early runs not reproduced here: a oscillation sideways at 4 Hz, a yawing mode at 9 Hz, and a pitching mode at 11 Hz. Because the payload was a single lead block in the very center of the load table, the moment of inertia was low and the frequencies of tipping modes were relatively high compared to a design where most of the mass is in the load table itself. We succeeded in suppressing the excitations almost totally by paying more attention to the symmetrical hanging of the X plates. However for the data shown here, we also moved the sensor from the edge of the load table to the lead block itself, which is at a node of all the tipping modes.

V. DISCUSSION

The observed behavior of the system was in accordance with the theoretical analysis within the limitations of the experiment. For best results therefore, such a system should be as light as possible and should be optimized according to Eq. (2.8). However, the position sensor used in these experiments was close to its noise limit above 10 Hz, and the transfer function in this region needs to be explored with an accelerometer. The raw output of the lower sensor was a minimum at around 10 Hz in all data sets and tended to rebound slightly from 10 to 25 Hz. This is probably not noise but a real degradation due to leakage through elastic modes such as those identified above. It is critically dependent on the degree of symmetry between the X plates and as we refined our technique, the performance above 10 Hz greatly improved. However, many of the adjustments were done by eye and there is undoubtedly scope for further improvement.

The system is reasonably easy to tune, but a few modifications could improve the usability substantially. The sliding turnbuckle system worked tolerably well to make the X wires symmetrical automatically, but we discovered that it was a simple matter to diagnose any asymmetries, and that we could do a better job of adjustment by working with one corner at a time. Thus, we would recommend a system with an independent adjustment for each X wire. The X-wire clamps on the support table tended to disturb the centering when tightened and we doubt that periods longer than 10 s could be obtained with the current system for this reason. A better scheme would be to carry out the tuning by moving the tightened clamps bodily.

A double X pendulum supporting a table is therefore a

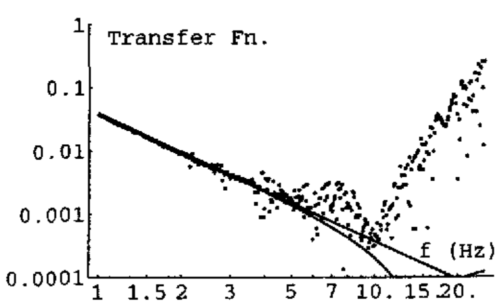

FIG. 6. Transfer function for the pendulum with optimal (4.7 mm) spacers, tuned to 5.1 s.

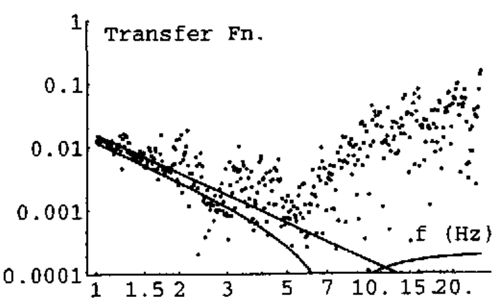

FIG. 8. Transfer function for the pendulum with optimal (4.7 mm) spacers, tuned to 9.2 s.