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Optical flat surfaces: direct interferometric measurements of small-scale irregularities

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Abstract. Some applications of optical surfaces place very stringent limits on small-scale surface irregularities, such as ripples. A method that can be used to make direct interferometric measurements of such small-scale irregularities is described.

Subject terms: interferometry; optical testing; flat surfaces; surface ripples.

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Several methods are now available for evaluating the absolute deviations from flatness of optical surfaces.[1–3] Some applications, such as mirrors for gravitational wave detectors, however, place much more stringent limits on small-scale surface irregularities, such as ripples. This paper describes a method that can be used to make direct measurements of such small-scale irregularities.

Consider the interference pattern formed in a Fizeau interferometer between a reference surface R and the test surface T, as shown in Fig. 1. At a point (x,y) on the test surface, the optical path difference between the beams reflected from the two surfaces is given by the relation

$$p(x,y)=2[f_T(x,y)-f_R(x,y)+d], \qquad (1)$$

where $f_T(x,y)$ and $f_R(x,y)$ are, respectively, the deviations of the test surface and the reference surface from two parallel planes separated by a distance d. Obviously, if the small-scale irregularities on the reference surface are comparable to those on the test surface, no useful information can be obtained on the latter.

To make direct measurements of the small-scale irregularities on the test surface, the reference surface is translated laterally in a series of steps of magnitude Δx and Δy in the x and y directions, and an interferogram is recorded at each position. Recordings are made at an $n\times n$ matrix of positions distributed at random over a square with dimensions $na\times na$ (where a is the average displacement of the reference surface along each axis between successive measurements) centered on the initial position of the reference surface, to obtain a data set consisting of $n\times n$ interferograms.

We assume that when the reference surface is translated laterally by an amount $\Delta x_i, \Delta y_j$, to record one of these interferograms, it also undergoes a displacement $\Delta z_{i,j}$ along the z axis and is tilted through small angles $\Delta\theta_{i,j}$ and $\Delta\phi_{i,j}$ about the y and x axes, respectively. As a result, the optical path difference, at a given point (x,y) on the test surface, that is recorded in this interferogram will be

$$p_{i,j}(x,y)=2[f_R(x-\Delta x_i, y-\Delta y_j)-f_T(x)+d+\Delta z_{i,j} + x\Delta\theta_{i,j}+y\Delta\phi_{i,j}]. \qquad (2)$$

Accordingly, if we average the data from the entire set of interferograms, the value obtained for the optical path difference at this point will be

$$\langle p_{i,j}(x,y)\rangle=2[\langle f_R(x-\Delta x_i, y-\Delta y_j)\rangle-f_T(x,y) +\langle d+\Delta z_{i,j}\rangle+\langle x\Delta\theta_{i,j}+y\Delta\phi_{i,j}\rangle], \qquad (3)$$

where $\langle f_R(x-\Delta x_i, y-\Delta y_j)\rangle$, $\langle d+\Delta z_{i,j}\rangle$, and $\langle x\Delta\theta_{i,j}+y\Delta\phi_{i,j}\rangle$ represent the individual averages for these quantities. The piston term $\langle d+\Delta z_{i,j}\rangle$ can be neglected, while the term $\langle x\Delta\theta_{i,j}+y\Delta\phi_{i,j}\rangle$, which corresponds to a tilt, can be calculated from a polynomial fit to the averaged data and subtracted from it. We can then drop the suffixes and rewrite Eq. (3) as

$$\langle p(x,y)\rangle=2[\langle f_R(x,y)\rangle-f_T(x,y)], \qquad (4)$$

where $\langle f_R(x,y)\rangle$ represents a profile of the reference surface obtained by averaging its actual surface deviations over an $na\times na$ square centred on each measurement point (x,y).

The left-hand side of Eq. (4) therefore represents the interferogram that would be obtained with a smoothed version of the reference surface. The large-scale deviations from flatness of the reference surface remain, but if we assume that its surface irregularities have a stationary, ran-

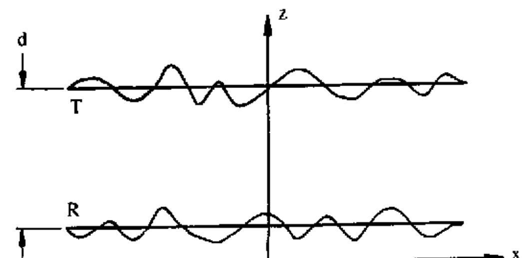

Fig. 1 Comparison of a test surface T with a reference surface R in a Fizeau interferometer.

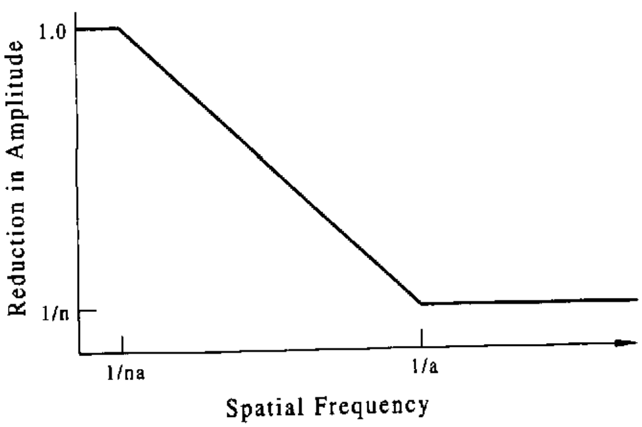

Fig. 2 Reduction in the apparent amplitude of ripples on the reference surface as a function of the spatial frequency.

dom distribution,[4] the apparent amplitude of small-scale irregularities on the reference surface with a spatial period less than a (that is, with a spatial frequency greater than $1/a$), will be reduced, as shown in Fig. 2, by a factor approximately equal to n in this interferogram. If the amplitude of these small-scale irregularities on the reference surface is less than or equal to those on the test surface, and the data are averaged over a sufficiently large number of points (say, $n=10$), the data obtained from Eq. (4) will map small-scale irregularities on the test surface with spatial periods less than a with an error less than 10%.

Further information on the statistics of the small-scale irregularities of the test surface can be obtained by using the same polynomial fit to eliminate the combined large-scale deviations from flatness of the reference and test surfaces from the data given by Eq. (4). The residuals then consist only of the small-scale irregularities of the test surface, which can be processed to obtain such parameters as the peak-to-valley amplitude and the root mean square (rms) deviation.

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