Quantum nondemolition measurements: the route from toys to tools

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The history of the theory of quantum nondemolition (QND) measurements from the 1920s until today is reviewed. The definition and main principles of QND measurements are outlined. Achievements in the experimental realization of QND measurements and several new promising schemes of QND measurements are described. A list of the most important problems (from the authors' point of view) in the area of QND measurements is presented. The problem of measurement of a quantum oscillator phase is considered. A new method of phase measurement is proposed. Examples of possible solutions of fundamental physical problems using QND methods are given.

CONTENTS

I. HISTORICAL INTRODUCTION

Fragmentary notes concerning the problem of quantum nondemolition (QND) measurements may be found dating back to as early as the 1930s in the publications by the founders of the quantum theory. For example, a footnote in the paper by Landau and Peierls (1931) contains the following statement (translated from the original German): if there existed an interaction Hamiltonian depending on velocity only, one would be able to measure the velocity of a free particle with the arbitrary high precision. Further, in the same footnote, an incorrect conclusion was given: such a Hamiltonian does not exist, and therefore such a measurement is not possible.

In the outstanding monograph by von Neumann (1932) published one year later, one can find an analysis of the measurement of free mass velocity using the Doppler effect. Such a meter must have a resolution different from the coordinate meter in Heisenberg's microscope. Von Neumann did not complete this analysis. It was made several decades later.

In the fundamental monograph by Bohm (1952), published 20 years after von Neumann's book, one can find the following condition of a QND measurement: diagonality of the meter-object interaction Hamiltonian in the representation of the observable to be measured. Many years later, special analysis showed this condition to be excessive.

The lack of interest in quantum measurements before the 1960s could probably be explained by the fact that in the overwhelming majority of experimental methods of that time, physicists dealt only with serial tests (ensemble measurements). In this type of measurement, arbitrary desirable precision can be achieved by an increase in the number of tests. The interest in quantum measurements with single objects grew again with the emergence of quantum electronics and nonlinear optics. Simultaneously, the interest of physicists was drawn to nonclassical states of electromagnetic (e.m.) fields—the squeezed states [first described by Schrödinger (1927); the term itself appeared later]. At the same time, substantial progress was being made in the development of the mathematical apparatus of the measurement theory (Stratonovich, 1973; Helstrom, 1976; Kholevo, 1982).

The main impetus for a detailed analysis of ultimate sensitivity in quantum measurement with a single object was given by the problem of detection of gravitational waves. A burst of gravitational radiation caused by astrophysical catastrophe (see, e.g., Thorne, 1987) produces very weak a.c. tidal forces (acceleration gradients) which may be detected either by the relative displacement of two separated masses or by the occurrence of a.c. strain in one spatially extended mass. Simple analysis, made in 1967 (Braginsky, 1967), has shown that by improving the isolation of such macroscopic test masses from the heat bath (by reducing friction), one can bring them into the domain of substantially quantum behavior, even if the temperature T is high. In this case, the sensitivity in measurement of displacement or acceleration is limited by the (quantum) back action of the meter. The corresponding characteristic limits of sensitivity in coordinate measurements, as suggested by Thorne, were named the standard quantum limits (SQL). In 1974 (Braginsky and Vorontsov, 1974), it was proposed that a sensitivity better than SQL can be achieved if the meter "extracts" information only on one specially chosen observable. This article contained, however, an incorrect example; the first correct example—in essence, a scheme of gedanken QND experiment—was published three years later (Braginsky, Vorontsov, and Khalili, 1977; Unruh, 1978). In this example, it was proposed that the energy in e.m. resonators be measured by registering the ponderomotive pressure which acts on the resonator's walls. In this procedure the energy is not absorbed, and it may be repeated many times in the absence of dissipation. One year later, Thorne, with colleagues (1978)

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showed that there exists another type of QND meter for an oscillator which registers one of the two quadrature amplitudes.

Since then, more than 200 papers have been published on the subject of QND measurement, describing different aspects of this problem and suggesting and analyzing different measurement schemes, for both mechanical and e.m. systems. It has recently become evident that the area of probable applications of QND meters is much larger than the solution of the problem of sensitivity in gravitational-wave antennas. In the 1980s, the problem of QND measurements attracted quantum opticians, and the first practical success was achieved; QND measurements were realized experimentally, however, in the form of feasibility demonstrations.

From a theoretical point of view, as shown by special analysis after the first publications, QND measurements are the most fundamental type of quantum measurement, free of any nonfundamental uncertainties. On the other hand, successful development of QND methods on the engineering level undoubtedly promises a qualitative improvement of sensitivity in many experiments. As a result, intense studies in this field are now carried out in relatively large numbers of laboratories in several countries. The goals of this review are (1) to familiarize the reader with the principles of QND measurements; (2) to describe the present state of experimental programs and their prospects; (3) to outline unsolved problems in the area of QND measurements; and (4) to give a few examples of possible solutions of fundamental physical problems on the basis of QND methods.

II. DEFINITION AND MAIN PROPERTIES OF QUANTUM NONDEMOLITION METHODS

This section is intended to familiarize readers with QND measurement schemes without their needing to refer to the original papers. Let us first consider a simple example demonstrating the origin of the standard quantum limit. Suppose that the coordinate $x(t)$ of the mass m of an oscillator with eigenfrequency ω is continuously monitored. The value $x(t)$ may be expressed by two quadrature amplitudes X_1, X_2,

$$x(t)=X_1\cos(\omega t)+X_2\sin(\omega t), \tag{1}$$

which satisfy the uncertainty relation

$$\Delta X_1\Delta X_2\geqslant\frac{\hbar}{2m\omega}. \tag{2}$$

The continuous monitoring of the coordinate with the time-independent accuracy is evidently equivalent to the simultaneous symmetrical measurement of quadrature amplitudes:

$$\Delta X_1=\Delta X_2, \tag{3}$$

where ΔX_1 and ΔX_2 are the measurement errors. Substituting this condition in the uncertainty relation (2), we obtain the standard quantum limit for the coordinate of the oscillator:

$$\Delta X_{\mathrm{SQL}}=\Delta X_1=\Delta X_2=\sqrt{\frac{\hbar}{2m\omega}}. \tag{4}$$

If the coordinate is continuously monitored with the accuracy ΔX_{SQL}, the amplitude of oscillations A will be known with the same accuracy:

$$\Delta A=\Delta X_{\mathrm{SQL}}\geqslant\sqrt{\frac{\hbar}{2m\omega}}. \tag{5}$$

It follows that, for the energy of oscillations

$$\mathscr{E}=\frac{m\omega^2A^2}{2}+\hbar\omega/2=\hbar\omega(N+1/2) \tag{6}$$

with N quanta in the mode, the standard quantum limit is equal to

$$\Delta\mathscr{E}_{\mathrm{SQL}}=m\omega^2A\Delta A=\hbar\omega\sqrt{N}. \tag{7}$$

For an electromagnetic oscillator, the analog of formula (4) is the standard quantum limit for the electrical field stress

$$E_{\mathrm{SQL}}=\sqrt{\frac{\hbar\omega}{4\pi V}}, \tag{8}$$

where V is the effective volume occupied by the field. Analogous simple analysis gives the following value for the standard quantum limit for the coordinate of a free mass:

$$\Delta X_{\mathrm{SQL}}=\xi\sqrt{\frac{\hbar\tau}{2m}}, \tag{9}$$

where τ is the duration of measurement and ξ is the factor of the order of unity depending on the form of the signal. For example, for sinusoidal 2π pulse,

$$\Delta X_{\mathrm{SQL}}=\frac{1}{2\pi}\sqrt{\frac{\hbar\tau}{m}}. \tag{10}$$

Apparently, to overcome the standard quantum limit, the meter must extract information *only* on the single specified observable. The meter, designed in accordance with this principle, does not disturb the value to be measured, and the others (noncommuting with it) are disturbed precisely to the extent that provides satisfaction of the uncertainty principle. This type of measurement is called the QND measurement.

Main properties of the ideal QND measurement precisely reproduce the properties of the abstract quantum measurement determined by von Neumann's postulate of reduction (1932).

If the object is initially in an arbitrary state with the density operator $\hat{\rho}$ and if the value q is measured, then the QND measurement will yield one of the eigenvalues q of the operator $\hat{q}$ with probability $\langle q|\hat{\rho}|q\rangle$ (where $|q\rangle$ is the corresponding eigenstate). After the QND measurement, the object will be in the state $|q\rangle$. If a quantum object is in the state with a certain defined value of the measured observable, then the same value will be obtained as the result of measurement. After the measurement, the object will remain in the same state. The measurement may be repeated many times, each time giving the same result. (It is assumed here that the evolution of the measured value can be neglected: either it is small or the measured value is an integral of motion; see below.) That is why the ideal QND measurement is an exact one: the meter does not add any perturbation, and possible variance is the consequence of the *a priori* uncertainty of the value to be measured.

Some quantum observables may have their own additional uncertainties which will limit the accuracy of measurement, e.g., $\hbar/\tau$ for the energy or $1/N$ for the phase of oscillator. The problem of existence of such limitations is not yet solved.

The origin of the term "quantum nondemolition" translates from the intention to emphasize the following basic property: if, before a measurement, an object is not in one of the eigenstates of the measured value, the QND measurement destroys this state but does not demolish it. For example, if an oscillator is initially in the coherent quantum state, the QND measurement of energy will destroy this state and create N-state, although this measurement does not include demolition, as in classical photodetectors.

On the basis of the above consideration, one can formulate a general necessary and sufficient condition that the QND meter must satisfy (see Braginsky and Khalili, 1992):

$$[\hat{q},\hat{U}]|\psi\rangle=0. \tag{11}$$

Here, $\hat{q}$ is the operator of the value to be measured; $|\psi\rangle$ is the initial state of the quantum meter; and $\hat{U}$ is the operator of the joint evolution of the quantum meter and the object under study. Condition (11) is usually replaced by the simpler sufficient (not necessary) condition

$$[\hat{q},\hat{U}]=0 \tag{12}$$

(see details in Sec. IV).

To test implementation of conditions (11) and (12), one has to know the operator $\hat{U}$, i.e., to solve the problem of evolution of the coupled system "quantum meter + object under study." In most cases this is a rather complicated problem, and therefore usually another sufficient (not necessary) condition is used: the measured value should be an integral of motion for the coupled system. From the general form of a dynamic equation in a Heisenberg evolution approach, it can be shown that the latter condition is equivalent to the following equation:

$$i\hbar\frac{\partial\hat{q}}{\partial t}+[\hat{q},\hat{H}]=0, \tag{13}$$

where $\hat{H}$ is the *Hamiltonian* of the coupled system. If the operator of the measured value does not depend directly on time,

$$\frac{\partial\hat{q}}{\partial t}=0, \tag{14}$$

then condition (13) is reduced to the requirement that $\hat{q}$ commute with the Hamiltonian:

$$[\hat{q},\hat{H}]=0. \tag{15}$$

Apparently, this condition is more rigid than condition (12). If it is secured, the measurement is nonperturbing independently of the interaction time of the meter with the object; i.e., equality (12) turns to identity independent of the measurement time.

At about the same time that the term "QND measurement" was introduced, another term—"QND observable"—began to be used. The values of the operator of QND observable $\hat{q}$ commute with those taken at a different time:

$$[\hat{q}(t),\hat{q}(t')]=0, \tag{16}$$

in the Heisenberg picture of evolution. This feature permits continuous monitoring of such an observable with the error less than SQL, and, as a result of this procedure, it is possible to detect very weak external action on the probe quantum object.

For simplest objects, (1) free mass and (2) oscillator, QND observables are, correspondingly, (1) momentum and energy, and (2) quadrature amplitude, energy, and phase. The latter is an example of a QND observable which is not an integral of motion

$$\hat{\varphi}(t)=\hat{\varphi}+\omega t. \tag{17}$$

For the observables-integrals of motion, condition (13) can be simplified. The Hamiltonian $\hat{H}$ in most cases can be presented in the form of a sum,

$$\hat{H}=\hat{H}_0+\hat{H}_M+\hat{H}_I, \tag{18}$$

where $\hat{H}_0$ is the Hamiltonian of the object under study; $\hat{H}_M$ is the Hamiltonian of the meter; and $\hat{H}_I$ is the interaction Hamiltonian. If the measured variable q is an integral of motion for the object under study, then the following equality is valid:

$$i\hbar\frac{\partial\hat{q}}{\partial t}+[\hat{q},\hat{H}_0]=0. \tag{19}$$

From formula (19) and from the evident fact that

$$[\hat{q},\hat{H}_M]=0, \tag{20}$$

we further derive that, for integrals of motion, condition (13) is reduced to the commutation of the measured value with the interaction Hamiltonian:

$$[\hat{q},\hat{H}_I]=0. \tag{21}$$

As an example of QND meter, one can consider a slightly modified scheme of the ponderomotive meter of e.m. energy, proposed in Braginsky, Vorontsov, and Khalili (1977).

Let one wall in an e.m. resonator be flexible or movable (as a piston in a cylinder). By measuring the pressure of an e.m. field imposed on the movable wall, one can evidently calculate the energy. If the inertia of the wall is large enough, then the phase of e.m. oscillations will not influence its motion. The motion of the heavy wall is slow compared to the frequency of electromagnetic oscillations (i.e., it is adiabatic). It is known, on the other hand, that the number of quanta in a resonator does not change during its adiabatic deformation. The ponderomotive force acting on the resonator wall is equal to

$$F=\frac{\mathscr{E}}{d}, \tag{22}$$

where $\mathscr{E}$ is the energy in the resonator, and d is the value of the order of resonator dimensions, depending on the chosen e.m. mode. Under the action of this force, the momentum of the wall will change by the value

$$\delta p=\frac{\mathscr{E}\tau}{d}, \tag{23}$$

where τ is the duration of measurement. It is evident, therefore, that the better the initial momentum of the wall is defined, the higher will be the precision of measurement of the energy:

$$\Delta\mathscr{E}_{\text{meas}}=\frac{d}{\tau}\Delta p, \tag{24}$$

where Δp is the initial uncertainty of the momentum.

On the other hand, in accordance with the uncertainty principle, the less Δp is, the greater will be the initial uncertainty of the wall coordinate Δx. The presence of the uncertainty of coordinate translates into the uncertainty of the resonator frequency $\Delta\omega$ during measurement time τ and therefore to the random-phase shift

$$\Delta\varphi_{\text{pert}}=\Delta\omega\tau=\omega\tau\frac{\Delta x}{d}. \tag{25}$$

From relations (24) and (25), taking into account the inequality

$$\Delta x\Delta p\geqslant\hbar/2, \tag{26}$$

we obtain that

$$\Delta\mathscr{E}_{\text{meas}}\Delta\varphi_{\text{pert}}\geqslant\frac{\hbar\omega}{2}. \tag{27}$$

It is interesting to note that the interaction Hamiltonian in the considered scheme is proportional to the square of the generalized coordinate in the resonator and not to the energy; i.e., condition (21) is not fulfilled. However, nondiagonal matrix elements of the Hamiltonian (in presentation of the measured value) oscillate with the frequency 2ω. If the measurement time is large enough,

$$\omega\tau\gg 1, \tag{28}$$

then their contribution to the operator of evolution is small, and the more general condition (12) is secured. Thus the considered measurement is actually a nonperturbing one. A more detailed and rigorous analysis shows that the ultimate precision of measurement in the considered scheme depends substantially on the *a priori* information on the value of energy in the resonator. In particular, if $\Delta\mathscr{E}_{a\ priori}\simeq\mathscr{E}$, the minimal measurement error is equal to

$$\Delta\mathscr{E}_1=\hbar\omega\sqrt{N}\frac{1}{\sqrt{\omega\tau}}\ll\Delta\mathscr{E}_{\text{SQL}}, \tag{29}$$

where N is the mean number of quanta. In the meantime, by repeating the measurement several times with the increasing precision of *a priori* information, one can obtain in the ultimate limit the following precision of measurement:

$$\Delta\mathscr{E}_{\text{QND}}=\frac{\hbar}{\tau}\ll\Delta\mathscr{E}_1. \tag{30}$$

Derivation of formulas (29) and (30), as well as a more detailed analysis of different aspects of QND measurement theory, can be found in the monograph by Braginsky and Khalili (1992).

III. STATE OF THE ART IN QUANTUM NONDEMOLITION MEASUREMENTS

Before reviewing the realized QND measurement schemes, it is worth touching briefly on one rather important condition of their feasibility: the degree of isolation of the specified quantum object from the heat bath. The criterion of sufficient isolation can be easily obtained by comparing the random change of the chosen variable under the action of the heat bath (during chosen averaging time $\tau\ll\tau^*$, τ^* is the relaxation time) with the *a priori* defined accuracy of measurement. For example, if one has to achieve a resolution slightly better than SQL for a free mass or oscillator in a heat bath with high temperature T, then the following inequalities must be fulfilled:

$$\frac{2kT\tau^2}{\tau^*}\leqslant\hbar, \tag{31}$$

or

$$\frac{2kT\tau}{Q}\leqslant\hbar, \tag{32}$$

where k is the Boltzmann's constant, and Q is the quality factor of the oscillator (Braginsky, 1967).

It is obvious that inequalities (31) and (32) are, in fact, the conditions of quantum behavior of the chosen objects. If, further, one has to measure, for example, the energy of the oscillator with the accuracy of one quantum, then the condition will be more rigid:

$$\frac{kT\tau(2N+1)}{Q}\leqslant\hbar, \tag{33}$$

where N is the initial (premeasurement) number of quanta in the oscillator.

It is worth noting that the above conditions have long been familiar to physicists involved in the development of gravitational-wave antennas. [Condition (32) was obtained as early as 1967.] Relatively recently, they were derived anew on the basis of a rigorous analysis of the decoherence process of a quantum object in a heat bath (see Caldeira and Leggett, 1983; Zurek, Habib, and Paz, 1993; Zurek, 1993, and references therein).

When the value of T is small enough so that $kT\ll\hbar\omega$ for an oscillator or $kT\ll\hbar/\tau$ for a free mass,