SHOT NOISE LIMITED SENSITIVITY OF THE LIGO 40 METER INTERFEROMETER*

R. E. SPERO
LIGO Project, California Institute of Technology, Pasadena, CA 91125

ABSTRACT

The LIGO 40 m interferometer is limited in sensitivity by shot noise (photon counting statistics) at fluctuation frequencies above 2 kHz. Calculations based on the interferometer parameters—optical power, modulation level and waveform, mirror transmissions and reflectivities, and fringe visibility—agree with the measured interferometer strain sensitivity over a factor of ten range of optical powers, within the calibration uncertainty of 10%.

1. Calculation of Shot Noise Limited Sensitivity

The 40 m interferometer at Caltech operates as two optically independent $L =$ 40 m long resonant Fabry-Perot cavities.[1] Sinusoidal phase modulation is used to extract signals from the light reflected from the two input mirrors that are proportional to the lengths of the respective cavities, and these signals are subtracted electronically to form the interferometer output, ΔL. The shot noise equivalent displacement sensitivity is defined as the following amplitude spectral density of ΔL associated with the statistics of photon counting:[2]

$$\Delta\tilde{L}(f) = L f_k \sqrt{3\mathcal{B}} \left[\frac{\lambda h}{cP} \left(1 + [f/f_k]^2 \right) \right]^{1/2} \tag{1}$$

Here $f_k = 1/(4\pi\tau_E)$ is the "knee frequency," at which $\Delta\tilde{L}(f)$ is increased over $\Delta\tilde{L}(0)$ by $\sqrt{2}$, where τ_E, the energy storage time of each cavity (assumed identical), depends on cavity length and mirror reflectivities. λ is the wavelength of the resonant light, c is the speed of light, and P is the quantum efficiency-corrected total power incident on the two cavities. The factor $\mathcal{B}$ contains terms that depend on the modulation:

$$\mathcal{B} = \frac{1}{3} \left[\frac{M^{-1} + A^2 J_0^2 - 2AJ_0^2 + 2AJ_0J_2}{MA^2 J_0^2 J_1^2} \right] \tag{2}$$

Here M is the intensity mode matching fraction (maximum = 1), J_0, J_1, and J_2 are Bessel functions evaluated at argument equal to the depth of modulation Γ, and A depends on the intensity transmission of the input mirror T and on the other losses in the cavity, $\mathcal{L}$: $A = 2T/(\mathcal{L} + T)$. When $M = 1$, $A = 1$, and Γ is vanishingly small, $\mathcal{B}$ takes on its minimum value of unity.

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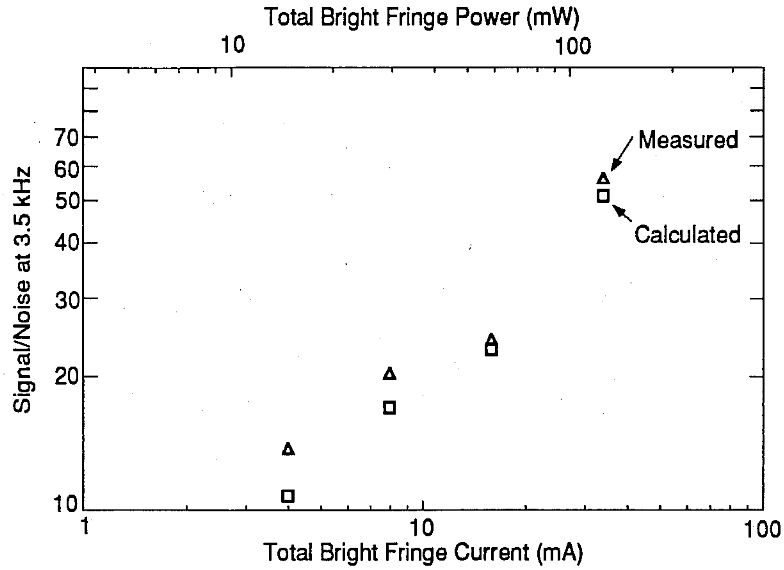

Figure 1: Power dependence of shot noise. The ordinate is the ratio in the interferometer output of (1) an imposed sinusoidal calibration of frequency 3.5 kHz and amplitude 2.7×10^{-16} m, to that of (2) the noise level at the same frequency in the absence of the calibration, measured with a bandwidth of 7 Hz (*triangles*). The abscissa is the sum of photocurrents in the two photodiodes that detect the light reflected from the cavities, when the cavities are out of resonance, (*lower scale*), or the total optical power incident on these photodiodes (*upper scale*); the latter is P of Equation (1) divided by the quantum efficiency of the photodetection. The calculated points (*squares*) depart slightly from a line with slope proportional to $P^{1/2}$ owing to a slight power dependence of parameters such as M.

2. Measurement

Figure 1 shows the result of measurements conducted in 1992, when the interferometer was in its Mk I configuration*. Typical operating parameters are $\lambda = 514\,\text{nm}$, $P = 50\,\text{mW}$, $f_k = 350\,\text{Hz}$, $\mathcal{B} = 3$. The interferometer approaches the sensitivity limit of Equation (1) at $f \gtrsim 2\,\text{kHz}$; at lower frequencies there are additional sources of noise, such as thermal noise in the test masses.

The noise level was calibrated by comparison with the signal resulting from applying a known sinusoidal displacement to one of the test masses. The agreement between the measurement and calculation is within uncertainties in the calibration of displacement signal and the parameters in Equations (1) and (2); that is to within 10% at the highest power level (120 mW total incident power).

References

1. R. L. Savage, "Status of the LIGO 40-Meter Interferometer," these Proceedings.
2. J. Hough et. al., in *The Detection of Gravitational Waves,* ed. David G. Blair, (Cambridge University Press, Cambridge, 1991), p. 329 ff.

*The current (Mk II) configuration uses mirrors of lower loss and lower input transmission, but otherwise the parameters that determine the shot noise are similar to those used in the Mk I measurements.