INTEROFFICE MEMORANDUM

CALIFORNIA INSTITUTE OF TECHNOLOGY

TO Albert Lazzarini DATE 28 May 2006

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SUBJECT Light Scattering Noise for Advanced LIGO [Document LIGO-P070050-00-Z]

Using the inputs from your document LIGO-T060013-02-E, I have computed the two dominant forms of light scattering noise for Advanced LIGO: that due to backscatter off the baffles, and that due to diffraction off the baffles. I will provide the details of my calculations as part of my long Physical Review D manuscript with Eanna, but that won't happen any sooner than mid summer. I suggest you incorporate these results into a revised version of your document LIGO-T060013-02-E.

1. Mirror Scattering Distribution

In my analysis I have assumed a power-law probability distribution function for photons scattered off the mirror into a unit "mirror-scattering" solid angle (what you call the BRDF—a terminology I'll adopt):

$$P(\theta) \equiv \frac{dP}{d\Omega_{\rm ms}} = \frac{\alpha}{\theta^{2q}} \ . \tag{1}$$

Equation (1) with

$$q = 1.84 \ , \quad \alpha = 1.4 \times 10^{-13} \tag{2}$$

is a good approximation to the *forward scattering* distribution you deduced for initial LIGO's mirrors

$$P(\theta) = \frac{P_o}{(1 + A\theta^2)^q} \quad P_o = 503 \ , \quad A = 2.9 \times 10^8 \ , \quad q = 1.84 \tag{3}.$$

throughout the range of scattering angles relevant for the beam-tube walls and baffles: $\theta_{\rm min} \equiv \mathcal{R}/L \simeq 1.3 \times 10^{-4} < \theta < \pi/2$ (with $\mathcal{R} \simeq 51.5$ cm the radius of the mean baffle top and $L = 4$ km the beam tube length). Specifically, at $\theta = \theta_{\rm min}$, Eq. (1) is larger by 40 per cent than your BRDF (3); at the beam-tube midpoint ($\theta = 2\theta_{\rm min}$), it is larger by 10 per cent; and it becomes more accurate in the near part of the tube. Throughout my analysis, as here, I have attempted to be accurate to within a factor 2 but not better than that.

My power-law distribution (1) in the *isotropic* case, $q = 0$, is a reasonable approximation, I presume, to the light scattered off point defects and particles with size $\lesssim 1\mu$m on the mirror surface. I have explored the scattering noise for values of the power parameter q ranging from $q = 0$ (isotropic scattering) to $q = 2$ [moderately more forward peaked than your distribution (2)].

I have chosen to parametrize my results by the power parameter q and by the *total probability* $\mathcal{P}$ for a main-beam photon, hitting a mirror, to scatter into an angle between $\theta_{\rm min}$ and ~ 90 degrees. I use the approximate formula

$$\mathcal{P} \simeq \int_{\theta_{\rm min}}^{1} \frac{\alpha}{\theta^{2q}} 2\pi\theta d\theta = \frac{\pi\alpha}{(q-1)} \left[(L/\mathcal{R})^{2(q-1)} - 1 \right] , \tag{4}$$

to relate the scattering amplitude α to the total scattering probability $\mathcal{P}$. In particular, for isotropic scattering I will assume a fiducial value of $\mathcal{P} = 10^{-5} = 10$ ppm because that is the value at which we must begin to worry (see below). For the forward scattering from LIGO-I mirrors, the values (2) give $\mathcal{P} \simeq 1.7$ ppm. Summarizing: My fiducial parameters will be

$$\text{For isotropic scattering } q = 0 , \quad \alpha = 3 \times 10^{-6} , \quad \mathcal{P} = 10\text{ppm} ; \tag{5a}$$

$$\text{For LIGO-I forward scattering } q = 1.84 , \quad \alpha = 1.4 \times 10^{-13} , \quad \mathcal{P} = 1.7\text{ppm} . \tag{5b}$$

2. Brief Summary of Conclusions

2.1 Light scattering noise for light that backscatters or diffracts off beam-tube baffles need not be a significant driver for the Advanced-LIGO mirrors and coatings, IF the forward scattering is kept near or below the level (5b) for the initial-LIGO mirrors and the isotropic scattering is kept below 100ppm.

2.2 Isotropic scattering is a very serious issue for light that scatters at large angles, $15^{\rm o} \lesssim \theta \lesssim 90^{\rm o}$, toward the near beam tube wall (or whatever else the mirror can see there), then backscatters to the mirror with a backscatter probability (BRDF) β. The tube wall's BRDF at these angles is $\beta \simeq 0.04$ (to within a factor 2; see Fig. 1 of your LIGO-T060013-02-E). The backscatter noise is

$$\tilde{h}_{\rm bs} = \left[\frac{4\pi\lambda^2}{L^2\mathcal{R}^2} \beta \int_0^{\pi/2} \theta^2 P^2 \sin\theta d\theta \right]^{1/2} \tilde{\xi}_L \simeq 6 \times 10^{-18} \left(\frac{\mathcal{P}}{10\text{ppm}} \right) \left(\frac{\beta}{0.04} \right)^{1/2} \tilde{\xi}_L(f) , \tag{6}$$

where λ is the light wavelength, $L = 4$ km is the distance between mirrors, $\mathcal{R} \simeq 51.5$ cm is the beam tube radius from the tube's centerline to the mean baffle top, and $\tilde{\xi}_L(f)$ is the spectrum (square root of spectral density) of the scattering-surface displacement toward the mirror ("longitudinal" displacement). This gravitational-wave noise is plotted in Fig. 1 for $\tilde{\xi}(f)$ equal to the tube's radial displacement spectrum as measured by Weiss far from the mirrors (Table 4 and Fig. 4 of your LIGO-T060013-02-E). Actually, that displacement spectrum is not at all likely to be representative of the motions of the near-mirror tube wall and objects off which the light backscatters (for $15^{\rm o} \lesssim \theta \lesssim 90^{\rm o}$), since they are inside the end or corner building, with a different environment than the far tube wall. The displacement spectrum might be substantially larger than in Weiss's measurements due to acoustic excitation, different structural resonance frequencies and Q's, and absence of thermal insulation to damp the resonances; and correspondingly, the noise might be substantially larger than shown in Fig. 1. This danger and Fig. 1 show that:

a. For advanced LIGO careful baffling near the mirrors may be required to keep backscatter noise at all frequencies below 1/10 the advanced-LIGO noise specification, even if the total mirror scattering is as small as $\mathcal{P} = 10$ ppm.

b. For initial LIGO (where the isotropic scattering is $\mathcal{P} \sim 100$ ppm), this backscatter is a possibly significant contributor to the noise bump seen at a few tens of Hz.

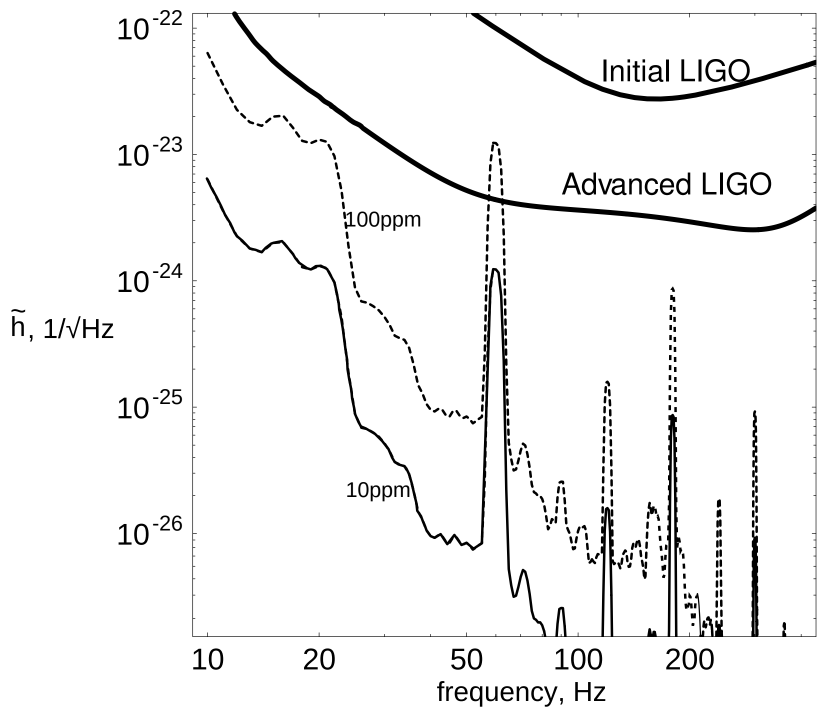

Figure 1: Backscatter noise due to light that scatters isotropically off the mirrors at large angles, $\theta \simeq 15^{\rm o}$ to $90^{\rm o}$, backscatters off the near tube wall or other objects with BRDF $\beta = 0.04$, and then scatters back into the main beam. The noise level is shown for 100ppm isotropic scattering from each mirror (dashed curve; about the value in initial LIGO) and for 10ppm (solid curve). The spectrum of tube-wall vibrations toward the mirror is the "radial spectrum" measured by Weiss for segments of tube outside the corner and end stations, as reported in Table 4 and Figure 4 of LIGO-T060013-02-E.

3. More Quantitative Conclusions: BackScatter Noise

A general formula for the square root of the spectral density of the interferometer's strain noise, produced by backscatter, is:

$$\tilde{h}_{\rm bs}(f) = \left[\frac{2\lambda^2}{L^2 \mathcal{R}^2} \int \beta P^2 \theta^2 d\Omega_{\rm ms} \right]^{1/2} \tilde{\xi}_L(f) \; . \qquad (7)$$

Here λ is the light wavelength, $\mathcal{R} \simeq 51.5$ cm is the beam tube radius from the tube's centerline to the mean baffle top, $L = 4$ km is the distance between mirrors, β is the backscatter distribution (BRDF) $dP/d\Omega$ for the baffles or whatever else the light may hit, θ is the scattering angle off the mirror, the integral is over the mirror scattering solid angle $d\Omega_{\rm ms} = \sin\theta d\theta d\phi$, and $\tilde{\xi}_L(f)$ is the square root of the spectral density of the scattering surface's displacement toward the mirror ("longitudinal" displacement). This formula is valid for any mirror scattering distribution $P \equiv dP/d\Omega_{\rm ms}$, axially symmetric or non-axially symmetric.

3.1 For isotropic scattering and mirrors centered in the beam tube, Eq. (7) is easily seen to give the results quoted and discussed in Sec. 2.2 above. When the mirrors are off center, it is not hard to show (cf. Sec. 3.2 and Fig. 4 below) that the noise, in this isotropic case, is essentially the same as for centered mirrors. Section 2.2 actually only deals with backscatter from the near tube wall (before the gate valves). The baffled tube beyond the gate valves subtends an angle $\theta_{\rm gate} \simeq 60{\rm cm}/1200{\rm cm} \simeq 0.05$ radians for the end mirrors (ETM's), and $\theta_{\rm gate} \simeq 60{\rm cm}/4000{\rm cm} \simeq 0.015$ radians for the corner mirrors (ITM's). From Eq. (6) we see that the backscatter noise due to this baffled tube is smaller, by a factor $\theta_{\rm gate}^2 \lesssim 0.003$ (at fixed β) than the backscatter noise from the near tube wall. For a total isotropic scattering as large as 100ppm and for the baffle BRDF $\beta = 0.0048$, this is factor ~ 30 below the advanced-LIGO noise curve at all frequencies. Thus, isotropic scattering into the baffled beam tubes is negligible.

3.2 For the axisymmetric power-law scattering distribution (2) and mirrors centered in the beam tube, and position-independent backscatter BRDF β, Eq. (7) gives the following simple formula (where I have approximated $\sin\theta = \theta$ and integrated θ from $\theta_{\rm min} = \mathcal{R}/L$ to 1 rather than to $\pi/2$):

$$\tilde{h}(f)_{\rm bs} \simeq \tilde{h}_{\rm bso} H(q) \; . \tag{8a}$$

The q-dependence is all in

$$H(q) = \left[(1-q) \frac{1 + (\mathcal{R}/L)^{2(1-q)}}{1 - (\mathcal{R}/L)^{2(1-q)}} \right]^{1/2} \; , \tag{8b}$$

which, as shown in Fig. 2, is 1 for $q = 0$ and about 1 for $q = 2$, and falls to a minimum of about 0.4 at $q = 1$.

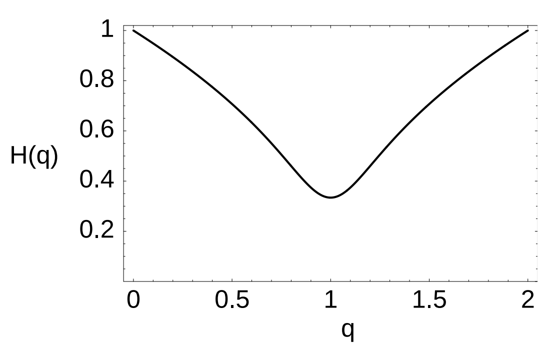

Figure 2: The function H(q).

The quantity $\tilde{h}_{\rm bso}$ (the actual diffraction noise for strongly forward scattering $q \simeq 2$ including your value 1.84, and also for isotropic scattering $q = 0$ if the tube were baffled all the way up to each mirror) is

$$\tilde{h}_{\rm bso} = \frac{\lambda \mathcal{P}}{L\mathcal{R}}\sqrt{\frac{\beta}{\pi}}\tilde{\xi}_L \simeq 3.4 \times 10^{-19}{\rm cm}^{-1}\left(\frac{\mathcal{P}}{1.7\ {\rm ppm}}\right)\left(\frac{\beta}{0.0048}\right)^{1/2}\tilde{\xi}_L(f)\ . \qquad (8c)$$

Here the fiducial $\beta = 0.0048$ is the value for the baffles in your LIGO-T060013-02-E. For the beam-tube axial displacement spectrum in your LIGO-T060013-02-E ($\tilde{\xi}_L = \tilde{\xi}_{\rm axial}$), and for the LIGO-I forward-scattering probability $\mathcal{P} \simeq 1.7$ ppm [Eq. (5b)], this is about 1/60 the Advanced-LIGO noise goal near 60 Hz, $\sim 1/300$ near 15 Hz, and much smaller than at other frequencies; see Fig. 3. Thus, backscatter noise due to long wavelength ripples in the mirrors (sharply forward scattering from the mirrors) is not at all a danger for advanced LIGO, when the mirrors are centered — except, possibly, for backscatter from objects in the far chamber, not hidden by the baffles, which I have not considered.

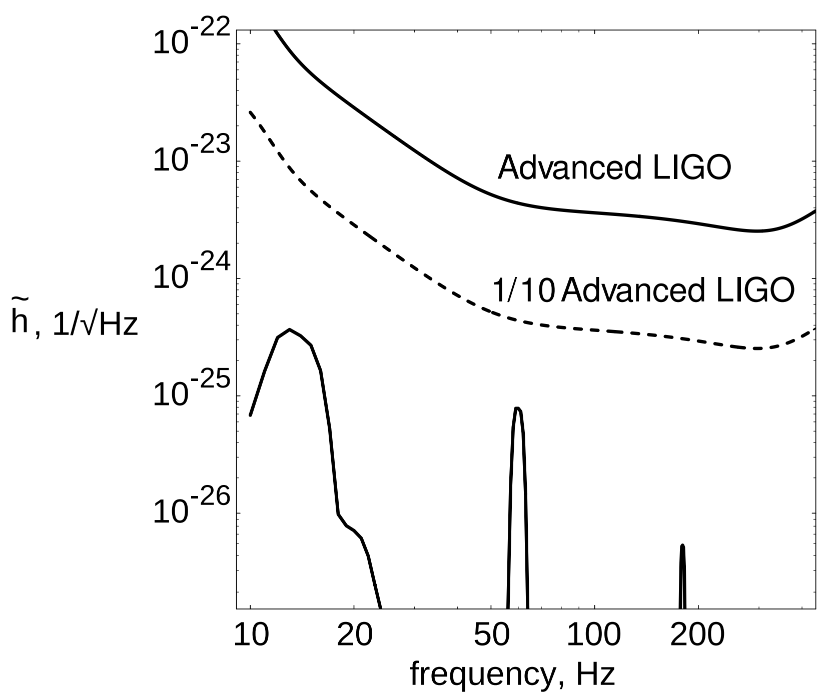

Figure 3: Backscatter noise due to light that scatters sharply forward [LIGO-I mirror scattering law (5b)], backscatters off baffles with $\beta = 0.0048$, and then scatters back into the main beam. The spectrum of the baffle displacement toward the mirror is the "axial spectrum" measured by Weiss for segments of tube outside the corner and end stations, as reported in Table 5 and Figure 4 of LIGO-T060013-02-E; $\tilde{\xi}_L = \tilde{\xi}_{\rm axial}$.

3.2 For off-center mirrors and the axisymmetric power-law scattering distribution (2), the backscatter noise is given by Eq. (8) multiplied by a function $\sqrt{J_q(\rho)}$, where $\rho \equiv \Delta r/\mathcal{R}$ is the mirrors' fractional offset (the mirror centers' radial offset distance Δr divided by the radius of the mean baffle top $\mathcal{R}$):

$$\tilde{h}_{\rm bs}(f) = \tilde{h}_{\rm bso}(f) H(q) \sqrt{J_q(\rho)}\ . \tag{9a}$$

The offset function is given by the integral

$$J_q(\rho) = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\phi}{\left[\sqrt{1-\rho^2 \sin^2\phi} + \rho\cos\phi\right]^{4q-2}}\ ; \tag{9b}$$

its square root is plotted in Fig. 4. From the figure we see that, *for LIGO-I mirrors [Eq. (5b)] and for mirror offsets less than $\rho \simeq 0.6$ ($\Delta r < 30$ cm, $\sqrt{J_q}$ is less than 6, and correspondingly (Fig. 3), the backscatter noise is less than 1/10 the advanced LIGO noise specification at all frequencies.*

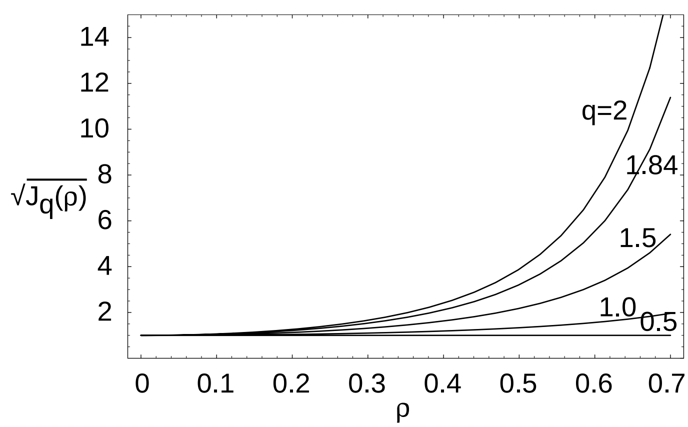

Figure 4: The mirror offset function $J_q(\rho)$. For $q \lesssim 0.5$, $J_q(\rho) \simeq 1$.

4. Noise from Scattered Light that Diffracts off Baffles

4.1 Baffle parameters. LIGO's baffles have saw-tooth serrations with a Gaussian probability distribution for the heights of their peaks and valleys; see your LIGO technical report LIGO-E950083-B-E, 07/09/96. The mean depth of serrations (peak to valley) projected into the beam tube's transverse plane is $\Delta H = 0.8$ cm, and the pitch angle of the serration teeth projected into the transverse plane is $\mu = \arctan(8/3) = 69^{\rm o}$. We restrict attention to 4 km interferometers, under the assumption that in Advanced LIGO the 2 km interferometer at Hanford will be converted to 4km. The 4 km interferometers see only the full serration (FS) baffles and not the non-serrated (NS) baffles, so we ignore the NS baffles. The total number of FS baffles in each beam tube is $N_b = 229$ at Hanford and 213 at Livingston. We denote by l_n the distance from the corner mirror (ITM) to baffle n, and by $l'_n = L - l_n$ the distance from the end mirror (ETM) to baffle n; and we define

$$\beta_n \equiv \frac{l_n l'_n}{(L/2)^2} \; . \tag{10}$$

This β_n is ~ 0.01 near the mirrors and rises to a value of 1 at the beam-tube midpoint, $l_n = l'_n = L/2$.

4.2 Centered Mirrors and Power-Law Scattering Distribution. When the mirrors are centered in the beam tube and their BRDF has the power-law form (1), (4), then the noise due to light that scatters off a mirror, diffracts off a serrated, vibrating baffle edge, and scatters back into the main beam at the distant mirror has the following form:

$$\tilde{h}_{\rm diff}(f) = \frac{\lambda^2 \mathcal{P}}{32\pi^2 \mathcal{R}^3} \sqrt{\frac{\mathcal{R} N_b}{4\pi \Delta H \tan\mu}}\, Q(q) \tilde{\xi}_{\rm radial}(f) \simeq 1 \times 10^{-20} \left(\frac{\mathcal{P}}{1.7 \text{ ppm}} \right) Q(q) \tilde{\xi}_{\rm radial}(f) \; . \tag{11a}$$

Here $\xi_{\rm radial}(f)$ is the baffles' radial displacement noise spectrum, and

$$Q(q) = \frac{8(q-1)(L/2\mathcal{R})^{2(q-1)}}{(L/\mathcal{R})^{2(q-1)} - 1} \left[\frac{1}{N_b} \sum_{n=1}^{N_b} \beta_n^{2q} \right]^{1/2} \; . \tag{11b}$$

This formula is based on the prediction (which Eanna and I have verified by numerical simulations) that the Gaussian distribution of the baffle heights breaks any relevant coherence that the scattered light might have, so the noise contributions from the baffle teeth add incoherently as do the contributions from the individual baffles. The function $Q(q)$, which differs negligibly between Hanford and Livingston due to the different baffle distributions, is plotted in Fig. 5. This figure shows that for isotropic scattering by point defects and submicron particles in the mirror surface ($q = 0$), diffraction noise is negligible — as should be obvious from the fact that isotropic scattering sends very little light into the small forward angles subtended by the baffled beam tube. For $q \gtrsim 1.3$, including the LIGO-I mirrors' forward-scatting $q = 1.84$, $Q(q)$ is essentially one, so $\tilde{h}_{\rm diff} \simeq 1 \times 10^{-20} (\mathcal{P}/1.7 \text{ ppm}) \xi_{\rm radial}(f)$. This is about 30 times smaller than the noise due to backscatter off baffles [Eqs. (8) and Fig. 3 with $\tilde{\xi}_L \simeq \tilde{\xi}_{\rm axial} \sim \tilde{\xi}_{\rm radial}$] and thus is $\sim 30 \times 60 \simeq 2000$ times smaller than the advanced LIGO noise specification at all frequencies; i.e., utterly negligible.

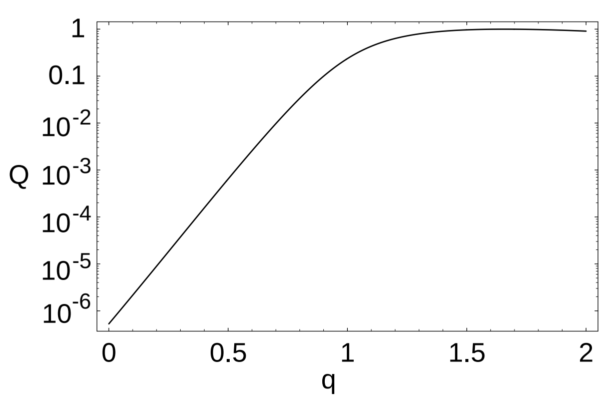

Figure 5: The function $Q(q)$.

4.3 Off-Center Mirrors and Power-Law Scattering Distribution. Eanna and I have been unable to derive analytic formulae for the dependence of the scattering/diffraction noise on mirror offset. However, using formulae that he and I derived in the 1990s, I have computed that dependence by a simulation that sums over contributions from all teeth of all baffles, and I have then fit the results analytically to obtain the following result:

$$\tilde{h}_{\rm diff}(f,\rho) = \tilde{h}_{\rm diff}(f,\rho=0) D_q(\rho)\ . \tag{12a}$$

Here $\tilde{h}_{\rm diff}(f,\rho=0)$ is given by Eqs. (11) which we have just explored, and

$$D_q(\rho) \simeq \exp(2.5\rho^4) \text{ for } q=0\ , \quad D_q(\rho) \simeq \exp(5\rho^4) \text{ for } q=1.84\ . \tag{12b}$$

These functions rise slowly enough with increasing fractional offset $\rho = \Delta r/\mathcal{R}$ to keep scattering/diffraction noise always negligible compared to backscatter noise.

4.4 Combined Isotropic and Sharply Forward Scattering. The LIGO-I mirrors have both isotropic scattering ($q=0$, $\mathcal{P} \sim 70$ ppm) and sharply forward scattering ($q=1.84$, $\mathcal{P} \simeq 1.7$ ppm). Assuming the Advanced-LIGO mirrors are similar, then the diffraction noises due to the sharply forward component and isotropic component add in quadrature, and there is also a third contribution to be added in quadrature. This third contribution arises when light scatters off one mirror via its long-wavelength deformations (sharply forward scattering), diffracts off a baffle, and scatters back into the mean beam via the other mirror's point defects and sub-micron surface particles (isotropic scattering) — or scatters isotropically, diffracts, and scatters back in via sharply forward scattering. I have evaluated this third contribution and found (not surprisingly) that the relative magnitudes of the three contributions are:

$$\text{(Sharply forward, } q=1.84) \gg \text{(Combined, } q=1.84 \text{ and } q=0) \gg \text{(Isotropic, } q=0). \tag{13}$$

Since the sharply forward scattering/diffraction noise is negligible compared to backscatter noise (Secs. 4.2 and 4.3), the other components are also negligible.