

Frequency and surface dependence of the mechanical loss in fused silica

Steven D. Penn,¹ Alexander Ageev,² Dan Busby,³ Gregory M. Harry,⁴

Andri M. Gretarsson,⁵ Kenji Numata,⁶ and Phil Willems³

¹*Department of Physics, Hobart and William Smith Colleges, Geneva, NY 14456, USA.**

²*Department of Physics, Syracuse University, Syracuse, NY 13244, USA.*

³*LIGO Laboratory, California Institute of Technology Pasadena, CA 91125, USA.*

⁴*LIGO Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.*

⁵*LIGO Livingston Observatory, Livingston, LA 70754, USA.*

⁶*Exploration of the Universe Division, Code 663,
NASA/Goddard Space Flight Center, Greenbelt, MD 20771, USA*

(Dated: October 30, 2005)

Abstract

We have compiled measurements of the mechanical loss in fused silica from samples spanning a wide range of geometries and resonant frequency in order to model the known variation of the loss with frequency and surface-to-volume ratio. This improved understanding of the mechanical loss has contributed significantly to the design of advanced interferometric gravitational wave detectors, which require ultra-low loss materials for their test mass mirrors.

PACS numbers: 95.55.Ym, 04.80.Nn, 62.40.+i

INTRODUCTION

As part of the research and development for the LIGO [1] and TAMA [2] gravitational wave detectors, we have conducted investigations into the internal friction of fused silica. Displacement of the interferometer's mirror faces arising from thermal motion of the fused silica test mass mirrors sets a fundamental limit to the detector sensitivity. The frequency distribution of this noise is directly related to the internal friction of the mirror material.

An Advanced LIGO detector has recently been proposed [3] with better sensitivity than initial LIGO. The Advanced LIGO mirror thermal noise must be as low as possible. Two materials have been under consideration for the mirror substrate: fused silica and single crystal sapphire. To its advantage, sapphire has the higher Young's modulus and a low bulk mechanical loss ($\phi \leq 3 \times 10^{-9}$) [4]. However, sapphire also has high thermoelastic noise [5].

In the advanced detectors, thermal noise in the mirror coatings makes a significant contribution to the total noise budget in the central frequency region of 30-500 Hz. Discussion on the mechanical loss in the mirror coatings can be found elsewhere [6–8].

Recent measurements on the mechanical loss in fused silica have revealed a dependence on frequency [9, 10] and on surface-to-volume ratio [11–13]. This paper combines data from several of these research groups in order to model both of these effects. The frequency dependence of the loss agrees well with results from Weidersich *et al.* [14]. In that work, loss data spanning six decades in frequency is modeled by an asymmetric double-well potential in the bond angle. Together these results provide a more complete picture of the loss in ultra-pure glasses and a more physically motivated prediction for the thermal noise in advanced interferometric detectors. It was previously predicted that fused silica's loss dependence would make it suitable for low frequency detectors (10 – 100 Hz) [15]. Indeed, this model's prediction of a very low mechanical loss in the LIGO frequency regime has motivated the recent selection of fused silica as the Advanced LIGO test mass substrate [16].

THEORY OF LOSS IN FUSED SILICA

The thermal noise motion of the mirror surface is related to the internal friction of the substrate by the fluctuation-dissipation theorem [17]. The internal friction of very pure fused silica is associated with strained Si-O-Si bonds, where the energy of the bond has minima

at two different bond angles, forming an asymmetric double-well potential. Redistribution of the bond angles in response to an applied strain leads to mechanical dissipation, which at audio frequencies has a peak in the cryogenic range 20-60K. Because fused silica is an amorphous material, there is a distribution of potentials which must be inferred from measurements of the dissipation. It can be shown [14] that the frequency dependence of the loss should exhibit a power law spectrum with exponent $k_B T/V_0$ at low temperatures. Both this power law, with $V_0/k_B = 319K$, and the distribution of potentials have been measured [14]. The power law exponent of a relaxation process cannot exceed 1, and is expected to saturate near 300 K. At room temperature the exponent is 0.76.

At elevated temperatures there is another loss peak arising from a double-well potential associated with the Si-O-Si bond angles. For this peak the bond angle shift and potential barrier are much larger; the double-well of the cryogenic loss peak is a small feature at the minima of this larger potential well. At room temperature, thermal fluctuations allow the bonds to span the cryogenic double-well but not to cross the larger potential barrier, where $V_0/k_B = 3.54 \times 10^4 K$ [18]. The calculated internal friction for this loss peak at audio frequencies and room temperature is utterly negligible compared to other loss mechanisms cited herein.

A separate loss mechanism exists in the surface of the glass. The contribution from the surface loss depends on the mode of the sample. The total energy lost per oscillation in an isotropic sample undergoing slowly decaying vibration, can be described by the integral of the local loss angle, $\phi(\vec{r})$ with the energy density $\rho_E(\vec{r})$

$$\Delta E = 2\pi \int_{\mathcal{V}} \rho_E(\vec{r}) \phi(\vec{r}) d^3r \quad (1)$$

where \mathcal{V} is the sample volume. Assuming that the local loss angle is constant and equal to ϕ_{bulk} everywhere except within a distance h of the surface, and that the energy density in that surface layer is approximately the energy density at the surface, then the loss can be expressed as [10]

$$\phi = \phi_{\text{bulk}} + \mu \alpha_s \frac{S}{V} \quad (2)$$

where S is the surface area of the sample and μ is a factor of order unity that depends on the mode shape. The surface loss parameter, α_s , is typically several picometers for flame polished or flame drawn fused silica but much higher for abrasively polished surfaces.

EXPERIMENTAL METHOD

Three laboratories performed resonant Q experiments to measure the mechanical loss, $\phi = 1/Q$ at resonance. The measurements at Syracuse University (SU) [10–13] were performed on fiber/rod samples with diameters ranging from 0.1 – 8 mm over resonant frequencies less than 5 kHz. The samples were drawn from and left attached to a massive bob of Suprasil [19], thus forming a cantilever beam. This bob was welded to a vibration isolating suspension formed by similar silica bobs connected by thin silica fibers. In a vacuum of $\approx 10^{-6}$ torr, the samples were made resonant by an electrostatic comb exciter, and their position was measured using a shadow sensor.

The measurements at University of Tokyo [9] were performed on cylindrical samples with optically polished surfaces. The diameters and heights were 70 mm and 60 mm, respectively. The samples were annealed in a vacuum furnace. To exclude the support loss, the samples were supported at nodal points of their vibrational modes during the Q measurements.

The Caltech measurements were performed on a spare input test mass for the initial LIGO interferometers, a superpolished right cylinder made from Suprasil 312 with a diameter of 25.4 cm and a thickness of 10 cm. It was suspended in a $\approx 10^{-6}$ Torr vacuum by a loop of polished stainless steel wire greased with lard. The elastic modes of the mass were excited with an electrostatic actuator and the mode amplitude was monitored using a birefringence sensor. Since friction at the wire could reduce Q , only modes with small motion at the point of wire contact were used in the fit.

MODELING METHOD

Resonant Q measurements from each of the labs were submitted for generating this model of the loss. The measurements spanned several types of fused silica, V/S ratios from 0.03 – 28 mm, and frequency up to 10^5 Hz. The data was first separated by silica type since the loss is known to vary significantly between varieties of fused silica [9, 11, 20]. Only Suprasil 2 and 312 had sufficient data to warrant a fit over both frequency and V/S ratio. Characteristics of these samples are listed in Table I.

We chose a model for the mechanical loss that included terms describing the frequency dependence, the surface loss, and the thermoelastic loss. The loss function took the form:

$$\phi(f, \frac{V}{S}) = \phi_{\text{surf}} + \phi_{\text{bulk}} + \phi_{\text{th}} \quad (3)$$

$$= C_1(\frac{V}{S})^{-1} + C_2(f/1 \text{ Hz})^{C_3} + C_4\phi_{\text{th}} \quad (4)$$

where $C_1 = \mu\alpha_s$ from Eqn. 2. Given that the surface loss term only contributes significantly to the rod (fiber) samples, we have assumed for all samples that $\mu \approx 2$ which is appropriate for cylindrical rods. We have also not distinguished the loss angle arising from the Young's modulus from that due to the shear modulus.

The thermoelastic loss, ϕ_{th} , which is negligible in all but the thinnest fiber samples, is described for fibers by:

$$\phi_{\text{th}} = \frac{Y\alpha^2 T}{\rho C_m} \frac{2\pi f\tau}{1+(2\pi f\tau)^2} \quad (5)$$

$$\tau = (d^2 \rho C_m)/(13.55 \kappa)$$

where Y is the Young's modulus, α is the coefficient of thermal expansion, T is temperature, ρ is the density, C_m is the mass specific heat capacity, d is the diameter, and κ is the thermal conductivity. We fit the amplitude of ϕ_{th} to account for small changes in the coefficient of thermal expansion among samples. Variations in the fiber diameter can also slightly alter the shape of the thermoelastic peak. Neither of these effects significantly affect the frequency or surface loss terms.

Measurements of large resonant Q 's are subject to numerous mechanisms that can greatly reduce the Q and few processes that can increase it. These effects produce a distribution in the systematic error that is asymmetric, heavily skewed toward lower Q , and unique for each experiment. Standard data analysis techniques based on normally distributed error, such as linear least squares (LLS) fitting, are therefore inappropriate for analyzing our full data set. We circumvent this problem by first limiting our data to the best measurement at each $(f, V/S)$ point for each sample. A LLS fitting routine is applied with the sample variance approximating the actual variance of the data. This method is commonly used in analyzing mechanical loss measurements where the lowest loss measurement closely approximates actual mechanical loss for a sufficiently large set of measurements [12, 13]. The results of the method are displayed in Figure 1 for Suprasil 2 and in Figure 2 for Suprasil 312. The fit coefficients are listed in Table II. The frequency dependence, C_3 , agrees well

with results from Weidersich *et al.* [14]. The thermoelastic amplitude, C_4 , is similar to earlier measurements [10]. Assuming no unforeseen loss mechanisms, the Advanced LIGO test masses ($V/S \approx 40$ mm) have a predicted loss ($\phi(100 \text{ Hz}) \approx 4 \times 10^{-10}$) that is a several-fold improvement over previous estimates.

IMPLICATIONS FOR ADVANCED DETECTORS

The low mechanical loss of silica in the 10 – 1000 Hz bandwidth, coupled with its optical and thermal properties, makes it an attractive material for the optics of next generation interferometric gravitational wave detectors. Fused silica has recently been chosen as the test mass substrate for Advanced LIGO [3], which has been approved and recommended for funding by the US National Science Foundation.

If the bulk and surface loss predicted herein can be achieved, the mirror thermal noise in Advanced LIGO with fused silica mirrors will likely be dominated by the coating [6, 7, 21]. The mirror thermal noise contributions to the total Advanced LIGO noise budget are shown in Figure 3. Table III shows the predicted sensitivity of Advanced LIGO with silica optics to two possible sources of gravitational waves: binary neutron star inspirals (BNSI) and binary $10 M_{\odot}$ black hole inspirals. Two different scenarios of coating thermal noise are shown: the best measurements to date [23] and the research goal. The sensitivity goal for a single Advanced LIGO interferometer is to observe BNSI, averaged over sky position and polarization, to a distance of ≈ 200 Mpc. (See Harry [22] for a description of a LIGO range calculation.)

CONCLUSIONS

We have shown that the mechanical loss of fused silica can be described by a model that includes surface loss and a frequency dependent bulk loss. The frequency dependent loss, thought to arise from an asymmetric double-well potential of the bond angle, agrees well with earlier measurements [14] that spanned six decades in frequency. This improved understanding of the loss indicates that at large geometries and low frequency, fused silica is an excellent material for test masses in advanced interferometric gravitational wave detectors.

ACKNOWLEDGMENTS

The authors would like to thank the LIGO laboratory and LIGO Science Collaboration for their support and review of this work. This research was supported by the National Science Foundation under cooperative agreements PHY-9210038 & PHY-0107417 (LIGO laboratory) and awards PHY-9801158 & PHY-0098715 (Caltech), PHY-0245118, PHY-0321144, & PHY-0355118 (HWS), and PHY-0140335 (SU). This paper has been assigned LIGO Document Number LIGO-P050049-00-R.

* Electronic address: penn@hws.edu

- [1] A. Abramovici *et al.*, *Science*, **256** (1992) 325.
- [2] K. Kawabe, *Class. Quantum Grav.* 14 (1997) 1477.
- [3] LIGO internal document, LIGO-M030023-00 available at <http://www.ligo.caltech.edu/docs/M/M030023-00/M030023-00.pdf>
- [4] V. B. Braginsky, V. P. Mitrofanov, and V. I. Panov, *Systems with Small Dissipation*, University of Chicago Press, 1985.
- [5] V. B. Braginsky, M. I. Gorodetsky, and S. P. Vyatchanin, *Phys. Lett. A* 264 (1999) 10.
- [6] G. M. Harry *et al.*, *Class. Quantum Grav.* 19 (2002) 897.
- [7] S. D. Penn, *et al.*, *Class. Quantum Grav.* 20 (2003) 2917-2928.
- [8] D. R. M. Crooks, *et al.*, *Class. Quantum Grav.* 21 No 5 (7 March 2004) S1059-S106.
- [9] K. Numata *et al.*, *Phys. Lett. A* 327 (2004) 263.
- [10] Andri Gretarsson, Ph.D. thesis, Syracuse University, 2004.
- [11] A. Ageev, *et al.*, *Class. Quantum Grav.* 21 (2004) 3887.
- [12] S. Penn, *et al.*, *Rev. Sci. Instrum.* 72 (2001) 7630.
- [13] A. M. Gretarsson and G. M. Harry, *Rev. Sci. Instrum.* 70 (1999), 4081.
- [14] J. Weidersich, S. V. Adichtchev, and E. Rössler, *Phys. Rev. Lett.* 84 (2000) 2718.
- [15] G. Conforto and R. DeSalvo, *Nucl. Instr. Meth. A* 518 (2004), 228-232.
- [16] “Advanced LIGO Substrate Selection Recommendation,” LIGO internal document LIGO-M040405-00-R
- [17] Callen, H. B. and Greene, R. F., *Phys. Rev.* 86 (1952), 702-710.

- [18] G. M. Bartenev, V. A. Lomovskoi, and G. M. Sinitsyna, *Inorg. Mater.* 32 (1996), 671.
- [19] Heraeus Quarzglas GmbH & Co. KG, Quarzstrasse 8, D-63450 Hanau, Germany.
- [20] W. J. Startin, M. A. Beilby, and P. R. Saulson, *Rev. Sci. Instrum.* 69 (1998), 3681.
- [21] Y. Levin, *Phys. Rev. D* 57 (1998), 659.
- [22] G. M. Harry, J. L. Houser, and K. A. Strain, *Phys. Rev. D* 65 (2002) 082001.
- [23] G. M. Harry, *et al.*, *Photonic Applications in Astronomy, Biomedicine, Imaging, Materials Processing, and Education*, Proceedings of SPIE: 5578 (SPIE, Bellingham, WA, 2004).

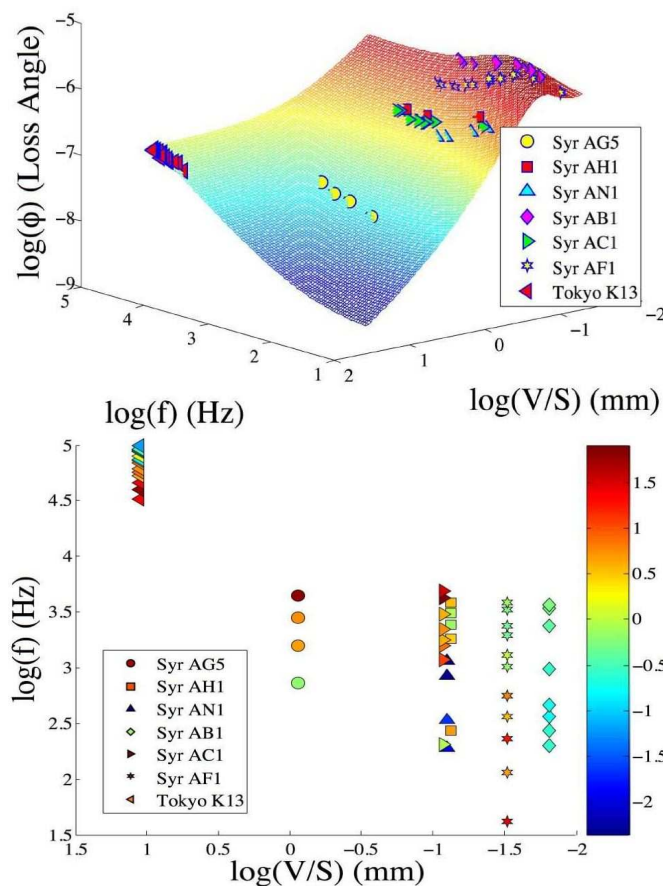


FIG. 1: Suprasil 2 mechanical loss data: Best fit surface (upper) and Deviation in units of sample variance (lower).

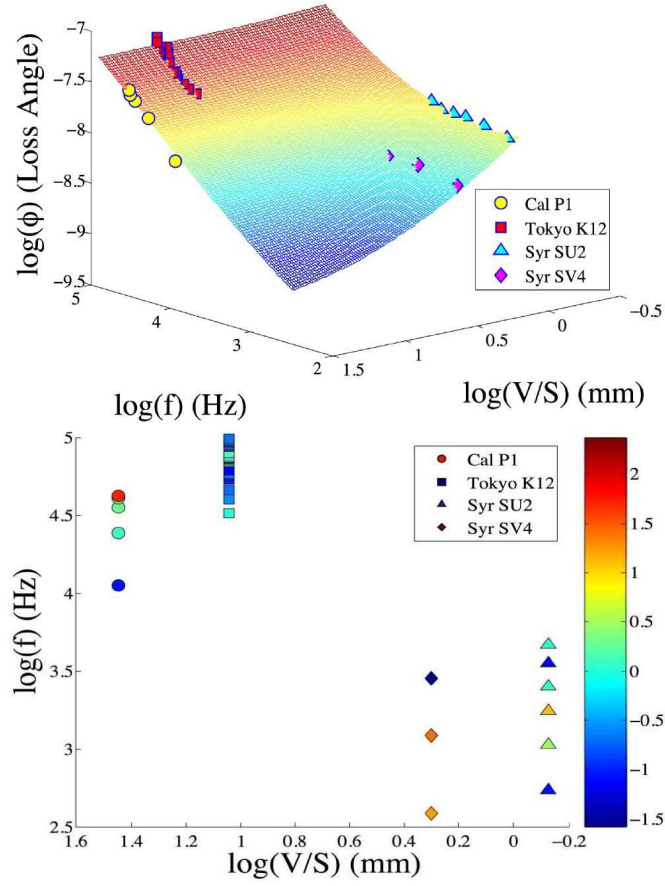


FIG. 2: Suprasil 312 mechanical loss data: Best fit surface (upper) and Deviation in units of sample variance (lower).

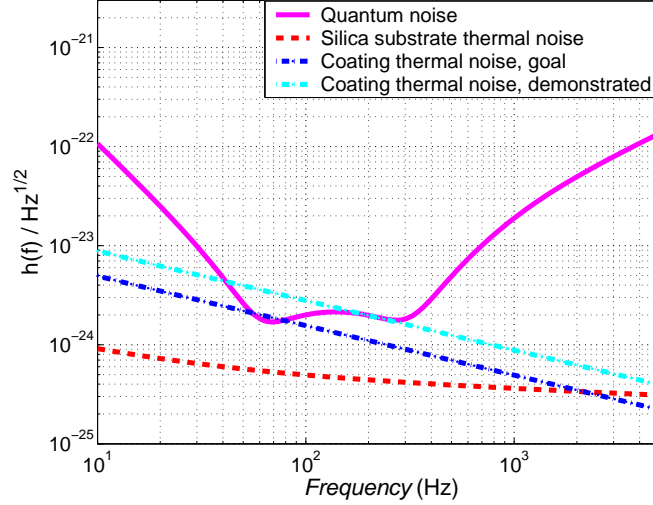


FIG. 3: Estimated Advanced LIGO thermal noise for a Suprasil 312 test mass substrate and for two mirror coatings: the best measured and the research goal. Laser quantum noise provided for comparison.

| Label | Type | D | h | V/S | Surface | Anneal | Lab |
|-------|-----------|-------|-----|-------|---------|--------------|---------|
| P1 | 312, cyl. | 254 | 100 | 28 | SP | None | Caltech |
| K12 | 312, cyl. | 70 | 60 | 11 | SP | 980°C, vac. | Tokyo |
| SU2 | 312, rod | 3 | – | 0.75 | FP | 1025°C in Ar | SU |
| SV4 | 312, rod | 8 | – | 2 | FP | 950°C in Ar | SU |
| AG5 | 2, rod | 3.5 | 188 | 0.88 | FP | None | SU |
| AH1 | 2, rod | 0.300 | 108 | 0.075 | FP | None | SU |
| AN1 | 2, rod | 0.318 | 160 | 0.080 | FP | None | SU |
| AB1 | 2, rod | 0.062 | 175 | 0.016 | FP | None | SU |
| AC1 | 2, rod | 0.340 | 310 | 0.085 | FP | None | SU |
| AF1 | 2, rod | 0.120 | 130 | 0.030 | FP | None | SU |
| K13 | 2, cyl. | 70 | 60 | 11 | SP | 900°C, vac. | Tokyo |

TABLE I: Sample characteristics: *Type* lists Heraeus Suprasil variety and shape. Samples are cylinders with diameter (D), height (h), and volume-to-surface ratio (V/S) given in mm. Surface types are superpolished (SP) and flame polished (FP).

| Type | C_1 (pm) | C_2 ($\times 10^{-11}$) | C_3 | C_4 |
|------|----------------|-----------------------------|-----------------|-----------------|
| 2 | 12.1 ± 0.8 | 1.18 ± 0.04 | 0.77 ± 0.02 | 0.61 ± 0.05 |
| 312 | 6.5 ± 0.2 | 0.76 ± 0.02 | 0.77 ± 0.02 | |

TABLE II: Fit coefficients for Suprasil 2 and 312.

| Coating Loss | Loss Angle $\phi_{ }$ | BNSI Range | BH/BH Range |
|--------------|------------------------|------------|-------------|
| Measured | 1.6×10^{-4} | 190 MPc | 840 MPc |
| Goal | 5.0×10^{-5} | 230 MPc | 1060 MPc |

TABLE III: The distance a single Advanced LIGO interferometer could detect a neutron star or $10 M_{\odot}$ black hole binary inspiral, assuming a Suprasil 312 test mass and two possible mirror coatings: the best measured and the research goal.