Position estimation from a network of interferometers

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The use of a network of interferometers for the estimation of the position of a transient source of gravitational radiation is discussed in various contexts. The prospects for the observation of electro-magnetic counterparts to binary coalescences as well as to more generic gravitational wave bursts are analyzed. The requirements for the use of a two-interferometer network to perform a targeted burst search are also discussed.

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I. INTRODUCTION

The scientific output of the gravitational wave (GW) detectors that are now in an advanced stage of their commissioning around the world will be maximized by combining their observations as thoroughly as possible. The incoherent combination of event lists from independent interferometers is one example of an efficient way to use network information to reduce the false alarm rate of a GW search. I comment in this article on the use of the network observation of a GW to infer the position of its source.

In sections II and III, I review the prospects for the observation of electro-magnetic (EM) counterparts to transient GW events. Such observations would certainly carry a wealth of scientific information, as the EM radiation would complement with information about the thermodynamics of the source the information about its dynamics that is contained in its GW signal. I also present in section IV a discussion of the use of a two-interferometer network (such as LIGO) to perform a targeted burst search.

II. EM COUNTERPART SEARCHES FOR BINARY INSPIRALS

The most carefully studied source of gravitational radiation for ground-based interferometers is the inspiral of two neutron stars in a coalescing binary. The "chirp" GW signal can be calculated with enough accuracy using post-newtonian expansion techniques to build a filter that is precisely matched to the signal, thus maximizing the efficiency for detection. Interacting neutron stars present a number of channels for emitting EM radiation, and it is therefore natural to ask what are the prospects for its detection, following a detection of the GW signal. A detailed discussion is presented in [1], and is summarized in this section.

When two neutrons stars or a neutron star and a black hole merge, a significant fraction of the mass of the neutron star(s) might be ejected from the system due to a mass shedding instability. Numerical simulations of the mergers indicate that perhaps as much as 10% of the star material is ejected during the merger [2]. This neutron rich material could power an EM counterpart to the binary merger through its radioactive decay, as it has been argued by [3]. The EM emission should be brief ($\sim$ days), bright ($\sim 10^{44}$ ergs s^{-1} if $10^{-5} M_\odot$ is converted to heat), and it should be primarily in the UV band.

As another possibility, it has been proposed that binary coalescences might be the generator of the blast wave that is popular for explaining the long wavelength counterparts to gamma-ray bursts. While it is possible to look for a direct association between the gamma-ray bursts and gravitational wave events [4], the large distances involved reduces the likelihood of detecting the GW signal. It is possible, however, that most binary coalescences fail to produce a gamma-ray burst, while still producing an afterglow [5]. In that case, it would be plausible that some coalescences occur near enough from the Earth to allow the detection of the GW signal. From what could be expected for these afterglows based on the interpolation of the properties of long duration gamma-ray bursts [6], the EM emission should be bright enough to be easily detected, assuming that the source position can be estimated with enough accuracy using GW data.

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As mentioned above, matched filtering can be used to optimally search for binary inspiral GW signals. This allows to estimate precisely the "coalescence time" of the binary at all interferometers where the signal is detected. If three or more non-colocated interferometers measure the coalescence time, the position of the source can be triangulated. It appears, however, that a more precise way of obtaining the source position is to analyze the data from all interferometers *coherently* [7], by combining the data before they are filtered by a single filter matched to the signal in all interferometers. From the results of [7], the size of the 2σ (95% coverage) error box for the HLV network, which is the network formed by the LIGO Hanford, LIGO Livingston, and Virgo interferometers, is given by

$$\Delta\Omega = \frac{7.4 \times 10^{-4}\ \mathrm{sr}}{|\cos\theta|}\left(\frac{12}{\rho_N}\right)^2, \tag{1}$$

where θ is the angle between the normal to the HLV plane and the direction of the source, and ρ_N is the *network* signal-to-noise ratio (SNR).

To estimate the number of EM counterparts that can be expected to be observed every year, one can fix the area of the sky that can be effectively scanned by a given EM telescope, and invert Eq.(1) to obtain the SNR or, equivalently, the maximal distance for a source to give an error box with size equal to that area. Integrating this distance over the position-dependent response of the HLV network and averaging over polarization angle and inclination angle of the binary gives an observation volume, which is multiplied by the "standard model" rate of binary coalescences of 50 Myr^{-1} per Milky Way equivalent galaxy [8] and by the mean density of galaxies to give the expected number of EM counterpart observations per year for binary coalescences.

This rate is negligible ($\sim 10^{-3}\ \mathrm{yr}^{-1}$) for the HLV network, even is the search area is a few square degrees. It is approximately an order of magnitude larger with Advanced LIGO interferometers and the initial Virgo interferometer, but it reaches $\sim 7\ \mathrm{yr}^{-1}$ for observations with the 3.4 deg^2 field-of-view ROTSE-III robotic telescope, for instance, for Advanced LIGO detectors in America, and a Virgo interferometer with a noise level comparable to that of Advanced LIGO. The presence of an Advanced detector which complements the information provided by the LIGO detectors appears to be critical to achieve good enough position estimations for EM counterpart searches to binary coalescences. It should be kept in mind that the uncertainty on the rate of binary coalescences in probably an order of magnitude either way, so the main conclusion to draw from this analysis is that the observation of EM counterpart to binary coalescences is unlikely for the initial HLV network, possible with Advanced LIGO detectors, and likely for a network of three independent Advanced interferometers.

III. EM COUNTERPART SEARCHES FOR BURSTS

The network of interferometric detectors has a great potential for discovering other types of transient GW signals besides binary inspirals. The waveform of these signals is not expected to be known a priori, and consequently the data analysis will have to be performed without precisely matched filters. A number of robust detection techniques have been proposed in the literature [9–15] for these *burst* sources.

These techniques all rely on detecting segments of the data which are not consistent with stationary noise, using various criteria, such as the amount of excess power [12, 14] or the correlation of the data with generic filters [10]. Consequently, their outputs for independent interferometers have to be compared in a coincidence analysis, in order to reject the spurious events introduced by the non-Gaussian components of the instrumental noise. In particular, the use of cross-correlations in this coincidence analysis is a promising way of reducing the false rate [16] and of providing a precise estimate of the differences in arrival time of the GW signal at various interferometers. Two of these differences can be used to triangulate the position of the source, up to a reflection with respect to the plane defined by the three interferometers. It is not clear at this time whether the precision of the timing performed with cross-correlations will be sufficient to achieve position error boxes that are small enough for EM counterpart searches, especially if the interferometers in the network are not all well aligned with respect to each other.

I have proposed a generalization of the single interferometer power detectors which could alternatively be used to infer the position of a GW burst source: the coherent power filter (CPF) algorithm [17, 18]. A number of numerical simulations have been performed with 62.5 ms long signals, consisting of independent realizations of a band-limited white noise process for the plus and the cross polarization waveforms, for a passband going from 125 Hz to 150 Hz. The HLV network was assumed for this analysis, with the simplifying assumption that the Virgo noise was identical to the LIGO noise. All signals were injected along the northern hemisphere normal of the HLV plane with a network SNR of 13.4. Roughly 50% of the trials led to large position errors ($\gtrsim$ 10 degrees), but approximately 25% of the trials led to errors smaller than one degree. More details about these simulations are available in [18].

If we assume that the simulated signal corresponds to a GW carrying a quantity E of energy, a network SNR of 13.4 places the source at a distance of $\sim 70\ \mathrm{kpc}(E/10^{-7}M_\odot c^2)^{1/2}$ for the HLV network. To give a concrete example, the core collapse simulations of [19] give GW signals that are a few milliseconds long, with central frequencies between 100 Hz and 1 kHz, bandwidths of a few hundred Hz, and energies $E \sim 10^{-7} M_\odot c^2$. These signals are not radically different from the ones used in the simulations mentioned above, so that it can be expected that there would be a fair chance for the HLV network to correctly position a core collapse if it occurred within the galaxy. Obviously, it is likely that the EM emission associated with the supernova would be easily detectable without the information provided by the GW detectors. It could be, however, that GW bursts with $E \sim 10^{-7} M_\odot c^2$ that are not necessarily associated with core collapses and that are similar to the signals used in the simulations described above occur at a rate that is large enough that the search of their EM counterparts would be meaningful.

IV. TARGETED UPPER LIMITS

Another interesting application of position estimation with a network of interferometers which is of particular relevance at this time is the targeting of burst searches using a pair of interferometers. Considering interferometers at the two LIGO sites, a number of techniques can be used to estimate accurately the difference in arrival time Δt of a GW burst, especially since the interferometers at both sites are well aligned with respect to each other. In order to reduce the background rate, it is generally asked that $|\Delta t|$ be smaller than the light travel time between the two sites, plus possibly some other delays due to errors in timing. If, however, we are willing to focus our attention on a specific point on the sky, identified by the coordinates (α, δ), a much more stringent requirement on Δt can be applied:

$$|\Delta t - \Delta(\alpha, \delta, t)| < e, \tag{2}$$

where $\Delta(\alpha, \delta, t)$ is the *expected* time delay between the two LIGO sites for a GW source at position (α, δ) and for a burst arriving at the earth barycenter at time t, and where e is a threshold chosen as a function of the error in the measurement of Δt.

This error is known to be inversely proportional to the SNR of the GW signal when it is estimated using a matched filter; for large enough values of the SNR, the same scaling should be expected for burst sources. At least two classes of methods can be used to measure Δt for burst sources. The CPF method is one of them, and it has the advantage of being easily scalable to networks including more than two interferometers. For a network with only two interferometers, however, cross-correlation techniques are likely to be simpler to implement and to offer similar efficiencies [16]. These methods are still in development, and it is not known at this time exactly how well they will perform on realistic data. Preliminary results, however, indicate that timing errors on reasonably large signals will be well below a millisecond, for signal of many different shapes. For illustration purposes, I will assume that the rms error on Δt for the burst signals of interest will be 50 μs, for signals near the detection limit of the analysis pipeline.

Assuming this value for the timing error, a threshold $e = 0.1$ ms will guarantee that a GW signal has a very small probability of failing the test described by Eq.(2). The background, however, will be reduced by this cut by the ratio of the threshold e to the light travel time between the sites, since the noise events are uniformly distributed in Δt. In the LIGO case, the background will thus be reduced by a factor of $\sim 100(0.1 \text{ ms}/e)$. The upper limit on the rate deduced from a burst search scales roughly like the standard deviation of the number of background events, i.e. like the square root of the background. Since the total observation time is not reduced by targeting, a threshold $e = 0.1$ ms should thus allow to measure a targeted upper limit which is an order of magnitude better than the all-sky rate upper limit, assuming that the efficiency of the targeted search is similar to that of the all-sky search.

This is generally a fairly good assumption. To give an example, the sky and polarization rms average of an interferometer's beam-pattern factor is $1/\sqrt{5}$, while its rms average over polarization and over time for the period covered by the S2 Science run has a minimum over sky position of 0.39 and 0.42 for the Hanford and Livingston observatories, respectively. Consequently, even if the targeted source is at the worst possible position with respect to the LIGO instruments, the rotation of the Earth will ensure that the average efficiency is preserved.

It will obviously be important to choose carefully which sources will be scientifically most interesting to target, since the upper limits obtained for two different points on the sky are likely to be highly correlated, given that the two LIGO sites can only locate the source within an annulus on the sky. The standard techniques used in the S1 Science run to determine the noise background by time shifting the data at the two sites in order to remove putative correlations from signals will be directly applicable for targeted searches as well. This will allow the determination of the statistics of the background for the regions of the sky that will be considered to be most interesting.

These considerations therefore indicate that it is quite important to achieve sub-millisecond accuracy for the timing of bursts on the international network of interferometers. I have discussed one application of this for the two sites of the LIGO project, but it should be noted that the addition of a third sensitive and independent interferometer would greatly reduce the size of the source position error box by transforming the two interferometer annulus into one or two small regions on the sky. The reduction in the background rate as compared to an all-sky burst search would then be $\sim 10^4 (0.1 \text{ ms}/e)^2$.

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