Quadruple Suspension Design for Advanced LIGO

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In this article we describe the conceptual design for the suspension system for the main mirrors (test masses) for Advanced LIGO, the planned upgrade to LIGO, the US laser interferometric gravitational wave detector. The design is based on the triple pendulum design developed for GEO 600 – the German/UK interferometric gravitational wave detector. The GEO design incorporates fused silica fibres of circular cross-section attached to the fused silica mirror (test mass) in the lowest pendulum stage, in order to minimise thermal noise from the pendulum modes. The damping of the low frequency modes of the triple pendulum is achieved by using co-located sensors and actuators at the highest mass of the triple pendulum. Another feature of the design is that global control forces acting on the mirrors, used to maintain the output of the interferometer on a dark fringe, are applied via a triple reaction pendulum, so that these forces can be implemented via a seismically isolated platform. These techniques have been extended to meet the more stringent noise levels aimed at in Advanced LIGO. In particular the Advanced LIGO baseline design requires a quadruple pendulum with a final stage consisting of a 40 kg sapphire mirror, suspended on fused silica ribbons or fibres. The design is chosen to aim to reach a target sensitivity corresponding to a displacement sensitivity of 10^{-19} m/√Hz at 10 Hz at each of the test masses.

I INTRODUCTION

The sensitivity of the US LIGO[1] interferometric gravitational wave detector is expected to be limited by thermal noise associated with the suspensions of its mirrors at frequencies in the region ~40 Hz to ~150 Hz. The LIGO suspension design[2,3] for the main mirrors has the following features.

- The fused silica mirrors (10.7 kg) are hung as single pendulums on a single loop of steel music wire.
- The sensing and actuation for damping of the low frequency pendulum modes is carried out at the mirror itself, with the magnets for actuation attached to the back and side of the mirrors via metal standoffs.

- Actuation for global control, required to hold the interferometer at its correct operating position, is also carried out via the magnets attached to the mirrors.

In GEO 600[4], the German/UK interferometric gravitational wave detector, the approach to the design of the suspension system has been deliberately more aggressive than in LIGO, in particular in terms of the reduction of thermal noise associated with the suspension of the mirrors. The GEO design incorporates fused silica fibres of circular cross-section to suspend the fused silica mirror in the lowest stage of a triple pendulum, the damping of whose low frequency modes is achieved by using co-located sensors and actuators at the highest mass of the triple pendulum. Global control forces are applied via a triple reaction pendulum, so that these forces can be implemented from a seismically isolated platform. These design features have been discussed in previous papers.[5,6,7,8,9,10] Figure 1 shows the first triple pendulum to be assembled with a monolithic fused silica final stage, hanging *in situ* in one of the GEO tanks.

The more advanced suspension design has been used in GEO to compensate for its shorter arm length (600 m compared to 4 km), in order to achieve a similar strain sensitivity to LIGO. Operating these detectors at their design sensitivities will be an exciting step forward in the quest for detecting gravitational waves, and may lead to their first detection. However to begin to open up the possibility of carrying out serious astronomy using gravitational waves, further improvement in sensitivity is required. An obvious step is to adapt the more advanced suspension design of GEO in the planned upgrade to LIGO, and this has been proposed in the white paper[11] put forward to the National Science Foundation describing the next generation of LIGO. The GEO team, in collaboration with LIGO and other members of the LIGO Science Collaboration has been developing the suspension design to meet the requirements for Advanced LIGO. In particular we are developing the design of a quadruple pendulum suspension for the main mirrors, which is an extension of the GEO design. The key features of the proposed design are as follows.

- Sapphire mirrors (40 kg) will form the lowest stage of a quadruple pendulum, and will be suspended on 4 vertical fused silica fibres or ribbons to reduce suspension thermal noise.
- The fibres will be welded to fused silica “ears” or prisms which are silicate bonded to the flat sides of the penultimate mass and the mirror below. This technique ensures that the low loss of the mirror itself is preserved
- Included in the quadruple pendulum are three stages of cantilever blade springs made of maraging steel to enhance the vertical seismic isolation.
- The damping of all of the low frequency modes of the quadruple pendulum will be carried either by using 6 co-located sensors and actuators at the highest mass of the pendulum (as in GEO), or by using eddy current damping applied at this mass. To achieve adequate damping the design has to be such that all the modes couple well to motion of the highest mass.
- DC alignment of mirror yaw and pitch will be done by applying forces to the actuators at the highest mass, or at the mass below. The masses hanging below the highest mass are each suspended by four wires, two on each side, so that the system behaves like a marionette from the highest mass downwards.

- Global control forces including auto-alignment forces, will be applied via a reaction pendulum, essentially identical in mechanical design to the main pendulum, but with wires replacing the silica fibres.
- The global control will be carried out using a split feedback system, with large low frequency motions applied magnetically between the penultimate masses, and small higher frequency signals applied electrostatically between the mirror and the corresponding lowest reaction mass which will be made of silica with a patterned gold coating. Alternatively photon drive may be used for the higher frequency signals, in which case the lowest reaction mass is not required.

Figure 2 shows a schematic diagram of what the quadruple pendulum suspension may look like. We discuss the features of the proposed design in more detail below, addressing the various issues, and giving predictions of the performance of the suspension system.

II THERMAL NOISE ISSUES

A. Some general considerations

The thermal noise performance of the suspension is the paramount design driver. The main contribution comes from the dissipation in the fused silica fibres used to suspend the mirror, giving a direct horizontal noise component. To minimise this noise, the baseline design currently incorporates ribbons rather than fibres of circular cross-section, so that the dilution factor, by which the pendulum loss factor is reduced from the value of the intrinsic loss factor of the suspension material, is increased. The word “fibre” will continue to be used below to describe either type of suspension. The choice between ribbon and cylindrical fibre is discussed more fully below.

Another strong contributor to the thermal noise spectrum arises from the flexing of the lowest set of blade springs, giving a vertical noise component which will couple into horizontal motion. In general thermal noise arising further up the pendulum chain is filtered by the stages below. However the vertical frequency of the final stage is necessarily higher than the horizontal frequency, since no blades are included at that stage, and thus there is less vertical filtering.

Since it is desirable from astrophysical arguments to extend the working frequency of the detector downwards as far as is experimentally practicable, we are considering a baseline design for Advanced LIGO which has a cut-off at 10 Hz, so that the required noise level at each of the test mirrors is 10^{-19} m/$\sqrt{}$ Hz at 10 Hz, falling off at higher frequencies. To achieve such a target requires that the highest vertical mode of the multiple pendulum should be kept below 10 Hz – otherwise a peak in the spectrum will occur in the operational frequency band of the detector. The highest mode essentially corresponds to relative vertical motion of the mirror with respect to the penultimate mass. To push this frequency down, we use a combination of several factors:

a) the fibre length is chosen as long as practicable consistent with ease of production. The current design target is 60 cm.

b) the fibre cross-section is chosen to be as small as practicable, consistent with working at least a factor of 3 away from the breaking stress.
c) the penultimate mass is chosen to be as heavy as possible, consistent with the overall design characteristics of the multiple pendulum. In the baseline design we have chosen to make this mass approximately double the mass of the mirror.

To achieve a penultimate mass which can be bonded, we are considering the use of heavy glass (glass doped with lead or other heavy metals).

We will return to these design factors after consideration of the choice of ribbons or cylindrical fibres.

B. Ribbons and Fibres

There are potential advantages to using ribbons rather than cylindrical fibres, and these have already been discussed elsewhere[12]. Not only can the dilution factor be made larger for ribbons, but also reducing the thickness of the flexing element raises the frequency at which the maximum loss due to thermoelastic damping occurs, which can lead to a lower overall level of noise around 10 Hz. Experimental results on losses in ribbons, including the effects of surface losses, have also been carried out[13,14], and these are encouraging. However there are several other factors which need to be considered before a choice can be made.

Firstly, recent work by Cagnoli and Willems[15] has shown that there is a significant thermoelastic effect not previously considered, basically due to the variation of Young's modulus with temperature. This effect, in combination with the more familiar coefficient of thermal expansion, gives rise to an effective coefficient of thermal expansion which can be zero for a particular static stress. Hence under those conditions the thermoelastic damping goes to zero, and hence the overall noise level is reduced. The null condition can in principle be achieved by increasing the cross-section of the silica suspension over that which has been previously indicated as optimum from other design considerations. However simply increasing the cross-section to null the thermoelastic effect has two adverse consequences. Firstly a larger cross-section causes the highest vertical pendulum mode to be above 10 Hz. Secondly it pushes down the violin mode frequencies, thus increasing the number of these resonances which appear below 1 kHz.

An alternative possibility which has recently been suggested[16] is to use circular cross-section fibres of varying cross-section, thicker near the ends and thinner in the middle section, such that the thermoelastic effect is reduced, but also the highest vertical mode is kept below 10 Hz. Similar tailoring of ribbons could also yield enhanced performance. These ideas are being pursued.

Another consideration is the breaking stress of ribbons and cylindrical fibres, and the ease with which they can be made. Measurements on cylindrical fibres have shown that they can be as strong as high tensile steel[17,10], and we now achieve an average value of breaking stress of ~4.5 GPa. Ribbons with breaking stress comparable to the strongest fibres have yet to be developed. However this is an active area of research, and initial results have already shown breaking stresses in excess of 1.8 GPa.

If ribbons are used, it may be necessary to put twists in them near the top and bottom of the suspension to avoid buckling as the mass swings. Again, this is an area of research.

In conclusion, it can be seen there are various issues in the suspension design which as yet are unresolved. The final design choice of ribbons or cylindrical fibres, possibly with varying cross-section, will depend on the results of investigations of such matters as reliability of manufacture, strength and loss measurements. For the purposes of this baseline design we use ribbons of constant cross-section for our estimation of expected thermal noise in a quadruple suspension system.

C. Thermal Noise Estimation for Quadruple Pendulum Suspension

The thermal noise model which has been used for this estimation has been developed using MAPLE. It has subsequently been modified into MATLAB code for inclusion in the BENCH modelling tool[18] which has been developed as a tool for predicting astrophysical range for various potential sources, for varying parameters of detector configuration for Advanced LIGO. Some details of how the thermal noise calculations are carried out are presented in the Appendix. Examples of pendulum thermal noise spectra produced using the MAPLE code are given in section IV.

III ISOLATION, DAMPING AND CONTROL

Modelling for investigation and optimisation of the mechanical design for a quadruple suspension, with particular reference to the isolation and damping properties, has been carried out using an extension of the MATLAB model developed for the GEO 600 triple suspension[5,19]. Some details of the MATLAB model are presented in the Appendix.

The key elements of the design are very similar to GEO, with the addition of another stage. The aim has once again been to develop a model whose resonant frequencies all lie within a band from approximately 0.4 to 4.5 Hz, with the exception of the highest vertical and roll modes which are associated the extension of the silica fibres in the lowest pendulum stage. In addition we aim for good coupling of all the low frequency modes, so that damping of all such modes can be carried out at the top mass in the chain.

A. Mechanical Design

The mass at the top is suspended from 2 cantilever-mounted, approximately trapezoidal pre-curved spring blades and 2 spring steel wires. The blades are made from from Marval 18 (18% Ni) maraging (precipitation hardened) steel, chosen for its high tensile strength and low creep under stress, as used in the French-Italian VIRGO gravitational waves project[20]. The blades lie horizontally when loaded. The mass below this is suspended from 2 cantilever blades and 2 steel wire loops. The top mass (mass 1) and mass 2 have a ‘sandwich-type’ construction with the blades fitting in between, so that the break-off points for wires going both upwards and downwards lie close to the centre of mass of these masses. See figure 2. Mass 3, which may be made of heavy glass, is suspended from 2 cantilever blades and 2 steel wire loops from mass 2. Fused silica ears silicate bonded to flats on the side of this mass form the breakoff points at the mass. Similar ears are bonded to the mirror (mass 4), and the final suspension is made by welding cylindrical fibres or ribbons between the ears of masses 3 and 4, two fibres on each side.

There are several key points which differ from the original GEO design. Firstly, in order to achieve a smaller footprint, all the blades are angled with respect to each other and crossed (as shown in figure 2). In GEO only the top set of blades in the beamsplitter suspension were crossed. Secondly, again due to space considerations, there are two blades rather than 4 at masses 1 and 2, each blade supporting two wires from its end. As stated earlier, the overall choice of number of wires or fibres is such that orientation of the mirror can be carried out from the top mass.

Currently we have chosen to stress the blades to approximately one half of the elastic limit (~800 MPa), which is conservative. However we may choose to increase this value to raise the internal mode frequencies of the blades, as discussed in section V.

There should be strong coupling of all degrees of freedom to motion of sensors/actuators at the top mass. To a first approximation this is satisfied by having approximately the same mass in each stage, approximately the same moments of inertia about equivalent axes, and by suitable choices of wire angles and connection points. In this design thermal noise considerations have necessitated the use of a significantly heavier penultimate mass than the other masses in the chain.

B. Local control

In GEO the active local control damping is applied at the top mass ensuring that the pendulum stages below filter any extra motion caused by electronic noise in the feedback system. However given the more ambitious target noise level for LIGO of 10^{-19}m/√Hz at 10 Hz, the GEO design needs some modification. In particular to sufficiently isolate the mirror from electronic noise, a quadruple suspension is required.

Typical optical shadow sensors with a range of ~1 mm have a noise level of ~10^{-10} m/√ Hz, and at frequencies where there is gain, the sensor noise contributes to the overall noise level. The target sensitivity at 50 Hz at the mirror in GEO is achieved by a combination of the isolation of the two stages below the top mass, and the aggressive roll-off in the electronic gain above the highest frequency of the low frequency pendulum modes. By the target frequency of 10 Hz in LIGO, very little roll-off in gain can be achieved. Thus most of the sensor noise isolation comes from the pendulums themselves. It should be feasible to turn down the gain of the longitudinal damping loop once the overall global control of the interferometer is switched on, thus reducing or indeed removing the noise imposed by the sensors in that direction. However it is not feasible to turn off the gain for some other modes, notably the vertical mode, although it could be turned down, giving a higher residual Q. Even allowing for this, development of very much better sensors would be needed if one wanted to use a triple pendulum. Local control is discussed more fully in section IV.

A quadruple suspension and eddy current damping in 6 degrees of freedom applied at the top mass to give Qs of approximately 10 for the lowest frequency modes (which dominate the impulse response) is a possible alternative. We have estimated that residual motion at the mirror due to the thermal noise force generated by such eddy current damping is approximately 7×10^{-20}m/√Hz at 10 Hz, which just meets the target sensitivity. The final decision on active control versus eddy current damping will be made once more experimental investigations have been carried out.

C. Global Control

The GEO philosophy for global control was briefly described in the introduction. The general idea is to apply forces between the main pendulum chain and an essentially identical reaction chain (which does not include fibre suspensions). The reaction chain is itself locally damped in the same manner as the main chain. In LIGO however, not all the sensitive optics require wide bandwidth global control, and in those cases the reaction chain does not require to have as many stages. In addition, where wide bandwidth is required, the final stage wide bandwidth low signal feedback could be actuated using photon drive, rather than electrostatically as in GEO. In that case also the lowest stage of reaction chain would not be required.

Another issue is the potential need to damp (actively or passively) the very high Q violin modes of the silica suspensions to allow the global feedback to remain stable. Any such damping has to be done in such a way as not to compromise the low frequency thermal noise performance of the suspensions. In GEO we have taken the approach of using small amounts of amorphous PTFE coating on the fibres, suitably placed to damp the first few violin modes to Qs of around 10^6, without compromising the low frequency suspension noise. For GEO we use two coated regions each 5 mm long, one at the centre and one at 1/3 of the way down the fibre. The LIGO situation has to be considered fully once a control philosophy has been decided upon, and there will be some trade-off required between controllability and thermal noise associated both with the low frequency vertical modes and the violin modes.

IV EXPECTED PERFORMANCE

In this section we present various graphs, showing expected overall thermal noise performance, horizontal and vertical isolation performance with and without damping, and transfer functions from which residual sensor noise may be estimated. Key parameters used in the models to generate these graphs are also given. In some cases several curves are given, where there are possible different choices of parameters.

A. Key Parameters

The key parameters used for all the curves presented in this section are as follows (except where otherwise indicated):

Final mass = 40 kg sapphire, 31.4 cm x 13 cm
Penultimate mass = 72 kg (heavy glass)
Upper masses = 36 kg, 36 kg
Overall length (from top blade to centre of mirror) = 1.7 m
Ribbon parameters: length = 60 cm, cross-section = 113 μm x 1.13 mm
Stress in ribbon = 770 MPa

We note in carrying out these analyses that the availability of sapphire pieces of the desired quality with these dimensions, and the availability of heavy glass of suitable density in the required size, are still open questions.

B. Thermal noise performance.

In figure 3 we present the thermal noise for the baseline design. The target figure of 10^{-19} m/√ Hz at 10 Hz is essentially met. We also show what happens if the penultimate mass is made of silica rather than a heavy glass, raising the uppermost vertical mode frequency of the quadruple pendulum to above 10 Hz. Note that for the latter case, the blade designs were altered to keep the other three vertical resonant frequencies at the same values.

Various changes could be made to the baseline design. A marginal improvement to the performance at 10 Hz and above could be made if one lengthened the final stage to say 70 cm. Increasing the cross-section of the fibre could gain some improvement above 10 Hz at the expense of raising the vertical resonant frequency to be closer to 10 Hz, and lowering the violin mode frequencies. This improvement arises since the changing the cross-section changes the position of the thermoelastic peak. Using cylindrical fibres loaded to the same stress as the baseline design (thus keeping the vertical mode frequency at the same value) raises the thermal noise in the 10 Hz region and above – as can be seen from figure 4.

C Isolation performance

The overall isolation in Advanced LIGO will be achieved by a combination of a two-stage active isolation system[21] and the isolation from the quadruple suspension as shown above. The target noise level for the active system is 2×10^{-13} m/√ Hz at 10 Hz in both longitudinal and vertical directions (where longitudinal refers to the horizontal direction along the beam axis). Figures 5 and 6 show the expected isolation performance in longitudinal and vertical directions respectively. The quadruple suspension has an isolation factor (with damping on) of 2×10^{-7} in longitudinal and 4.5×10^{-4} in vertical at 10 Hz. When these numbers are combined with the target noise level including a cross-coupling factor of 10^{-3} from vertical to horizontal (see Appendix A), we see that the target sensitivity level of 10^{-19} m/√ Hz is achieved for both dimensions.

D. Sensor Noise Performance

In figure 7 we show the transfer function from the sensors to the mirror in both longitudinal and vertical directions. From these curves, noise level at the mirror can be calculated from the transfer function multiplied by the sensor noise in m/√ Hz and the fraction of the full gain being used to damp the modes. The longitudinal transfer function is $\sim10^{-7}$ at 10 Hz. Thus to achieve a noise level of 10^{-19} m/√Hz requires the product of sensor noise and fraction of full gain to equal 10^{-12}. With a sensor noise of 10^{-10} m/√ Hz we would require to turn down the gain by a factor of 100, which would correspond to taking a damped Q of 10 to a Q of 1000 for example. If a suitable quieter sensor can be developed, the reduction in gain can be correspondingly relaxed. Alternatively it may be possible to turn the gain down completely for the longitudinal modes once the global control of the interferometer is in operation.

For the vertical direction, the longitudinal noise level at the mirror can be calculated as above, with an extra factor, the cross-coupling factor, in the product. The vertical transfer function at 10 Hz is $\sim 3\times 10^{-4}$, so with a sensor level of 10^{-10} m/√ Hz, a gain fraction of 3×10^{-3} and a cross-coupling factor of 10^{-3} the residual noise level at the mirror is 10^{-19} m/√ Hz.

V CURRENT AND FUTURE WORK

Work towards developing a quadruple pendulum suspension as described above is already underway. Firstly experience is being gained at GEO 600 with constructing and operating triple pendulum suspensions. This should give us information on many of the key aspects of the design, including thermal, isolation, and damping properties and operation of global control.

Regarding thermal noise issues, ribbon and fibre production, including strength, reliability, welding and loss tests are being carried out in Glasgow and at Caltech. Investigation of bonding continues at Glasgow and Stanford, with regard to bonding silica ears to sapphire and to lead or bismuth loaded glass, the latter materials being considered for the penultimate mass in the quadruple chain.

Regarding mechanical and other issues, the first all metal prototype quadruple pendulum and reaction mass was designed using AutoCAD in Glasgow early in 2001, parts were procured and shipped to MIT where they were assembled during summer 2001. Figure 8 shows pictures of the two quadruple pendulums (main chain and reaction chain), hanging in the lab at MIT in the summer of 2001. This suspension mimics a 30 kg sapphire mirror with an identically sized silica penultimate mass, which was a previous baseline design, now superceded with the design as discussed above. This prototype has already given us experience in assembly and handling. Current and future work includes measuring mode frequencies, and investigating transfer functions, damping and global control.

More work on blade design is underway, involving finite element analysis and comparison to experimental results. Another issue being considered is the noise level from the blades when thermally or seismically excited at their internal mode frequencies (in particular the lowest set of blades nearest to the test masses). It is desirable that the peaks at these frequencies do not compromise the sensitivity, and damping may be needed to ensure this. For the design presented in section IV the lowest internal modes were in the range 75 to 120 Hz. Initial calculations suggest damping could be avoided if the frequencies are a little higher than these. Suitable frequencies could be achieved by allowing the maximum stress to be around 1050 MPa.

It should be noted that we have addressed the design of the most sensitive mirrors in Advanced LIGO in this paper, namely the end mirrors in the two cavities. However the tools developed for designing the quadruple suspension can be easily applied for design of other suspensions. As well as the design issues mentioned above which are under investigation, there are several key issues still unresolved for the suspension design, some of which depend on other areas of research for Advanced LIGO. For example, the choice of mirror material and its size and aspect ratio are not yet fixed. Sapphire is presently favoured, and work is underway on investigating growth of large enough pieces and investigating the optical properties such as absorption, inhomogeneity, polishing etc. The fallback position is to use silica. Another area currently under discussion is the choice of lower cut-off frequency for good sensitivity, and this has a large bearing on the final design.

In conclusion, we have presented the current conceptual design of suspension system for Advanced LIGO, which is based on the GEO suspension system. Experience with GEO will be invaluable as a test of the ideas incorporated in this design. However much work has still to be carried out, and is actively underway in several laboratories in Europe and the USA.

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****Statement in here about LIGO work supported by NSF etc ?****

APPENDIX

We include here a brief discussion of the modelling tools used to produce the thermal noise and isolation curves presented in section IV.

A. Thermal Noise Model.

The calculations in the code are carried out in the following way. The pendulum dynamics are simulated by four point-like masses linked by springs for both horizontal and vertical degrees of freedom, with no coupling between the orthogonal degrees of freedom. Suitable values to be used as input for the masses and other necessary parameters to calculate spring constants have previously been established using the MATLAB model of the quadruple pendulum, discussed in the next section. The first three spring stages consist of maraging steel blades in series with steel wires, and the final (lowest) stage consists of silica fibres. The horizontal and vertical transfer functions are calculated separately and then combined to get the effective overall horizontal function, assuming a cross-coupling of vertical into horizontal of 0.1%. This is a figure we have used in GEO[5] as a conservative estimate for cross-coupling, and is larger than the purely geometric effect due the curvature of the earth over the 4 km arms of LIGO. Dissipation in the pendulum is introduced via the imaginary part of the spring constants, and hence using the fluctuation dissipation theorem the resulting thermal noise at the mirror in the horizontal direction is obtained.

Spring constants of the steel stages have been treated differently from the silica stage. For the steel the loss is included, with a dilution factor as appropriate, by including an imaginary term in the spring constant. For silica, the spring constants have been worked out from the solution of the beam equation, following the method used in Gonzalez and Saulson[22], in which case the imaginary part is introduced into the Young's modulus. As a consequence, the programme calculates the violin modes of the silica stage, but not of the steel stages.

Loss angles for the materials arise as the sum of three parts: bulk, surface and thermoelastic effects, including the new thermoelastic effect referred to in section II above, which is included where appropriate. The surface loss is estimated following the work by Gretarsson and Harry[23], which indicated that there is an energy loss proportional to the surface to volume ratio for silica which dominates the bulk dissipation. For steel however the bulk loss dominates. The thermoelastic loss term has been considered in the pendulum motion of all 4 stages and in the vertical motion of the three steel stages in which the restoring force dominantly arises from the bending of the blades.

B. MATLAB Model for Isolation and Control

The MATLAB model (recently extended to work in Simulink) consists at present of 4 uncoupled sets of dynamical equations, corresponding to vertical motion, yaw, longitudinal and pitch (together) and transverse and roll (together). To first order these motions are uncoupled in the GEO design. Forces due to gravity and extension of wires are included, but not due to bending of wires. Cantilevers with wire(s) attached are approximated by taking the series sum of the spring constants of wire(s) and cantilever, noting that this sum is dominated by the softer cantilever blade. The model makes use of presumed symmetries in the design. With the crossed blades in the LIGO design, there will be some coupling between the longitudinal/pitch and transverse/roll modes. As yet the model does not incorporate this coupling. However it is not expected to significantly affect either the isolation or damping properties of the pendulum. In addition the model does not yet take account of the twisting of the blade tips which will occur as the pendulum moves in the various pitch modes. Experimentally we have seen that this effect slightly lowers the pitch modes. However again the isolation and damping should not be significantly affected.

It should also be noted that the violin modes and the internal modes of the blades are not included in this MATLAB model. The violin modes of the final stage are however included in the thermal noise model, and they can be seen in the thermal noise curves shown in section IV. The expected frequencies of the internal modes of the blades can be calculated from the dimensions of the blades, and are specific to each design of blade. Examples of their typical values were given in section IV.

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Figure Captions

FIG. 1. Picture of the first triple pendulum with monolithic final stage hanging *in situ* in one of the GEO tanks.

FIG. 2 Schematic diagram of quadruple pendulum suspension system for Advanced LIGO. Fig 2a shows shows a face view of the main chain on the left, and on the right a side view with main and reaction chains visible. FIG 2b shows a close up of the first two masses (masses 1 and 2), with the top of mass 1 removed so that the cantilever blades for vertical isolation, which are crossed to save space, can be seen more clearly.

FIG. 3. Suspension thermal noise for baseline 40kg quadruple pendulum.
Two suspension curves are shown. The heavy solid line is the baseline design. The light solid line shows the effect of replacing the 72 kg heavy glass penultimate mass with a silica mass of same dimensions (weighing 22.1 kg). The peaks of the resonances are not resolved. Notice the first violin mode at approximately 500 Hz. For comparison we also show the expected internal thermal noise curve for sapphire, dominated by thermoelastic damping (dotted line). Note that the internal thermal noise curve assumes no loss due to coatings, or due to bonding of ears for attaching the suspensions.

FIG. 4. Light solid line is thermal noise for fibres of 200µm radius, stressed to same value as baseline ribbon design. Heavy solid line is baseline, dotted line is internal thermal noise for sapphire.

FIG. 5. Longitudinal isolation for quadruple pendulum, with (heavy solid line) and without (light solid line) local controls on, and with eddy current damping (dashed line).

FIG. 6. Vertical isolation for quad pendulum, with (heavy solid line) and without (light solid line) local controls on, and with eddy current damping (dashed line).

FIG. 7 Longitudinal (left) and vertical transfer function from sensor to mirror for quadruple pendulum with sensor at the top mass.

FIG. 8. Two views of the prototype quadruple suspension assembled at MIT. Fig 8a is an overall view showing the main and reaction chains, suspended from a support frame. Fig 8b is a close-up of the top masses, with some of the local control actuators visible. The construction can be compared to the diagrams in figure 2.

Figure 1

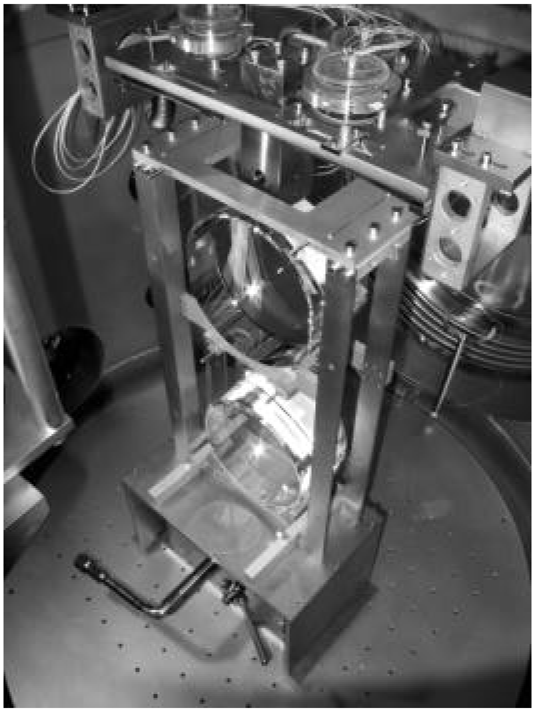


Figure 2a

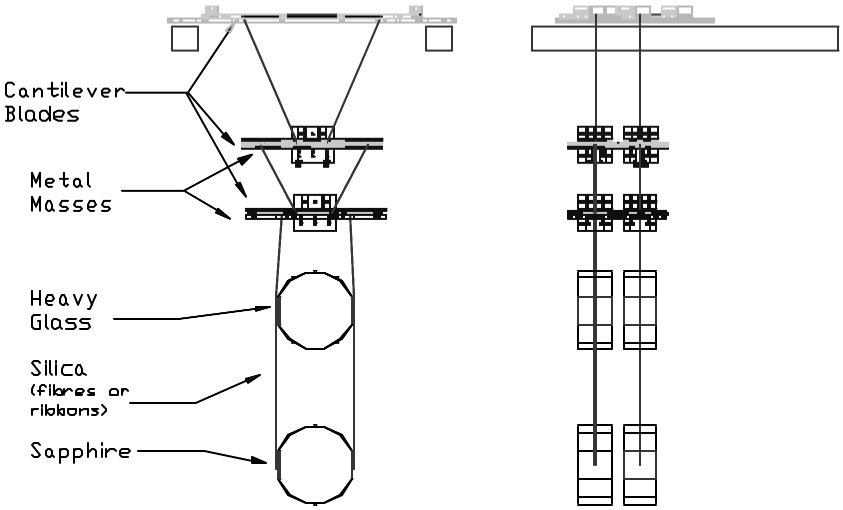


Figure 2b

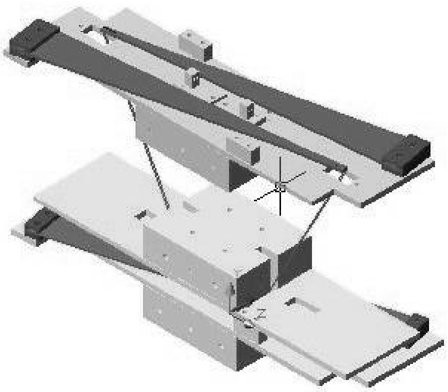


Figure 3

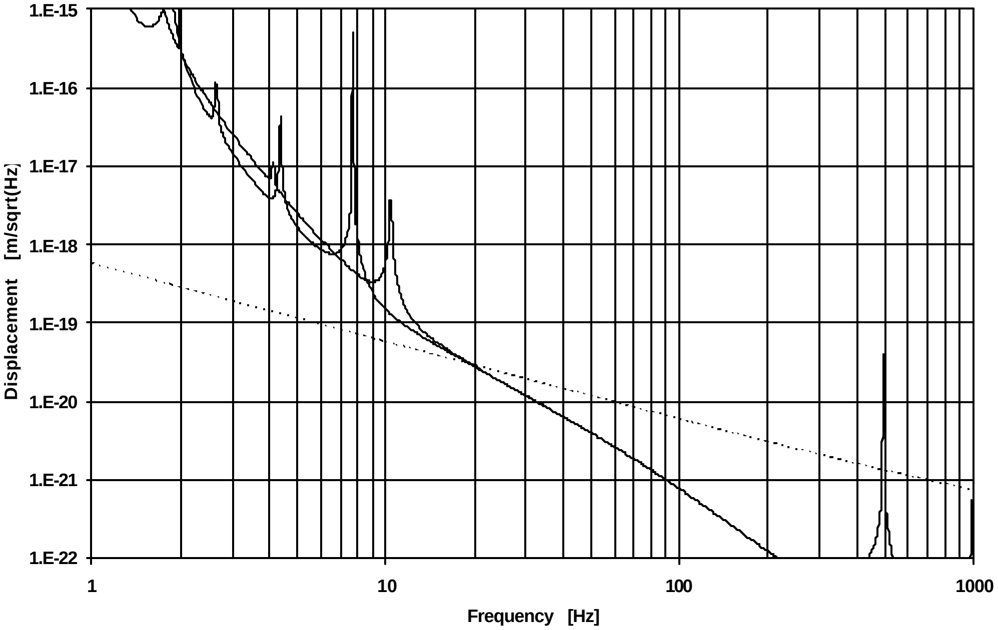

1.E-15
1.E-16
1.E-17
1.E-18
1.E-19
1.E-20
1.E-21
1.E-22
Displacement [m/sqrt(Hz]
1
10
100
1000
Frequency [Hz]

Figure 4

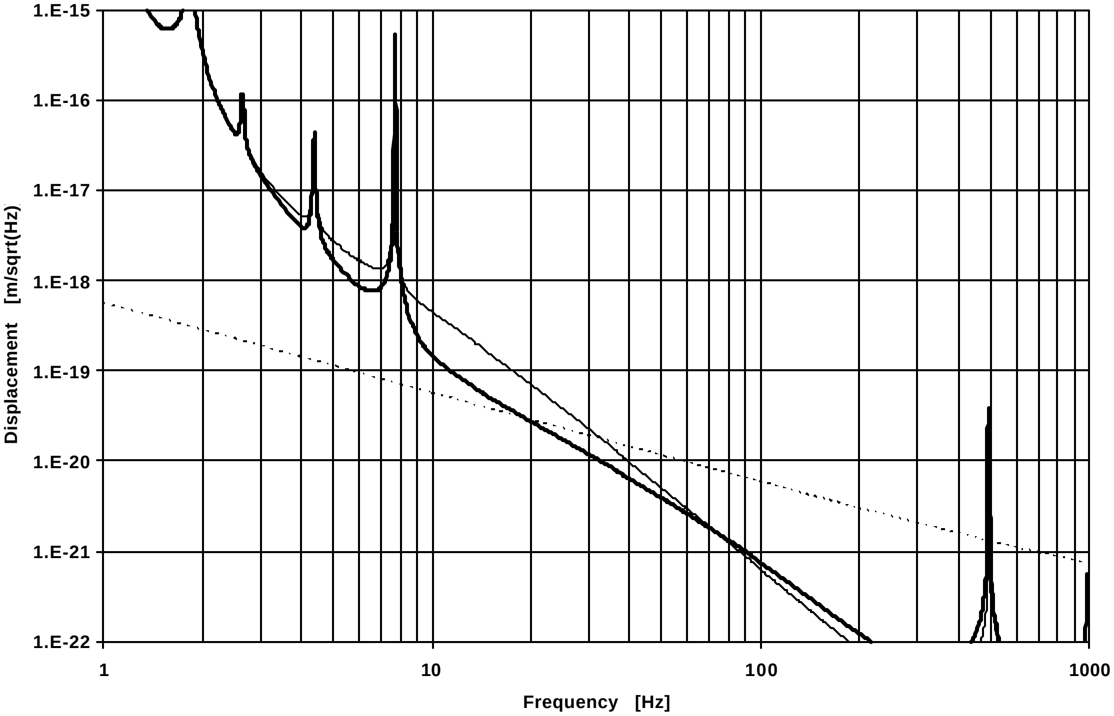

1.E-15
1.E-16
1.E-17
1.E-18
1.E-19
1.E-20
1.E-21
1.E-22
Displacement [m/sqrt(Hz)
1
10
100
1000
Frequency [Hz]

Figure 5

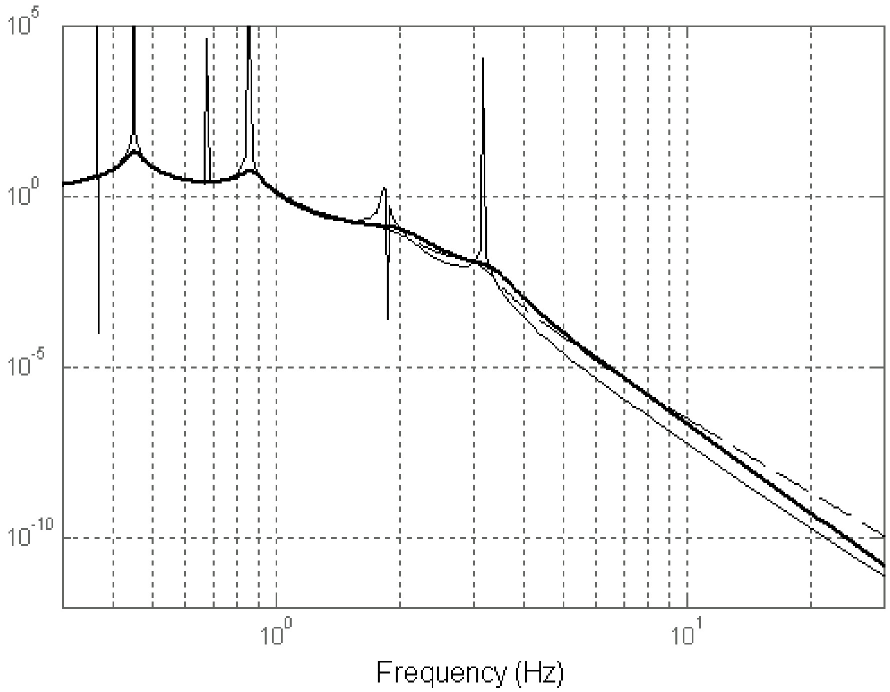


Figure 6

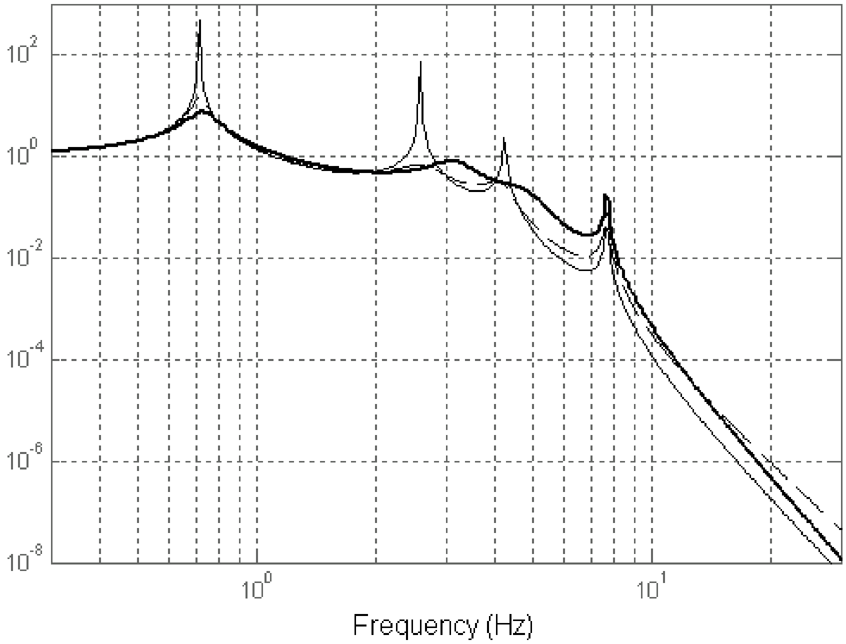
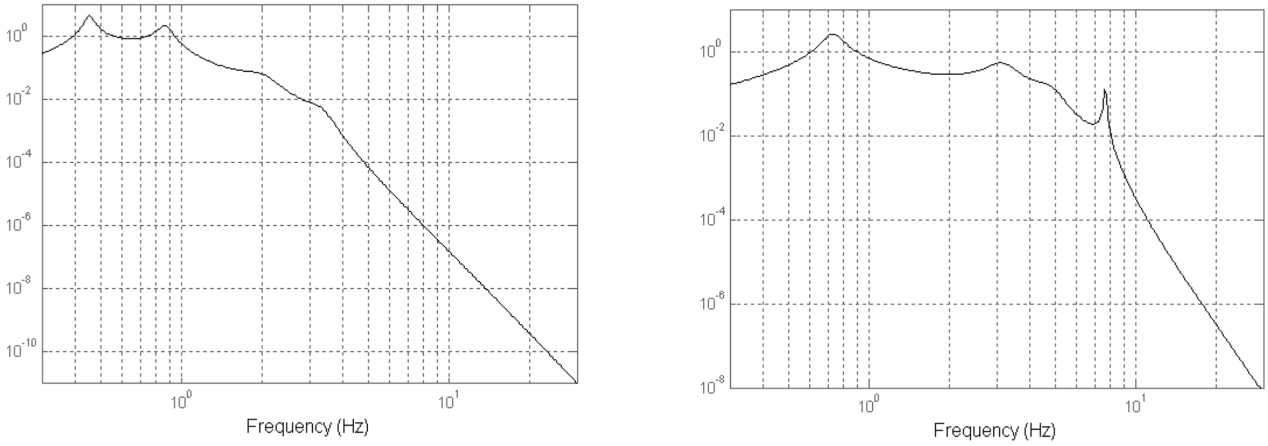


Figure 7



10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{0}
10^{1}
Frequency (Hz)
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{0}
10^{1}
Frequency (Hz)

Figure 8a

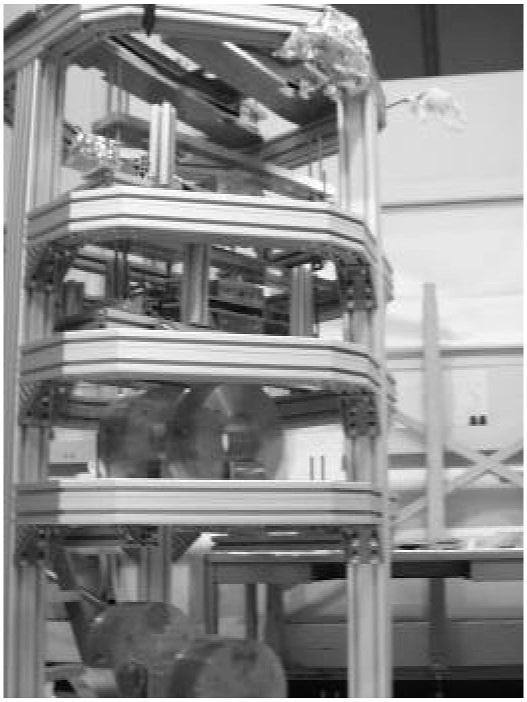


Fig 8b