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Proposed Source of Gravitational Radiation from a Torus around a Black Hole

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Long gamma-ray bursts (GRBs) could be emitted from rapidly spinning black-hole–torus systems, resulting from either hypernovae or black-hole–neutron-star coalescence. We show that a nonaxisymmetric torus may also radiate gravitational radiation, powered by the spin energy of the black hole. The coupling to the spin energy of the black hole operates by equivalence in poloidal topology to pulsar magnetospheres. Results calculated in the suspended-accretion state indicate that GRBs are potentially the most powerful LIGO/VIRGO burst sources in the Universe, with an expected duration of 10–15 s on a horizontal branch of 1–2 kHz in the $\dot{f}(f)$ diagram.

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Cosmological gamma-ray bursts (GRBs) are the most enigmatic events in the Universe. Their emissions are characteristically nonthermal in the range of a few hundred keV. The BATSE catalog shows a bimodal distribution of GRB durations, with short bursts of about 0.3 s and long bursts of about 30 s [1]. The afterglow phenomenon—broad-band secondary emissions generally towards lower energies—has revolutionized our understanding of GRBs as internal shocks in baryon poor (leptonic) outflows and external shocks in interaction with the interstellar medium at a distance away from the source [2–4]. This gracefully circumvents the original compactness problem. The energetics and rapid temporal variability observed in GRBs suggest an association with failed supernovae/hypernovae [5–7] and black-hole–neutron star coalescence [8] stars. Relics of GRBs might be found in soft x-ray transients which show chemical abundances in α nuclei [9], such as GRO J1655-40 [11] and V4641 Sgr [12].

Catastrophic events from high-angular-momentum compact sources such as hypernovae or black-hole–neutron-star coalescence are expected to result in black hole plus disk or torus systems (see [13] for a review). Short/long bursts may be identified with hyperaccretion/suspended-accretion onto slowly/rapidly spinning black holes [14]. This further points towards a positive correlation between fluence and the spin rate of the black hole, consistent with the distinct values of $\langle V/V_{\max}\rangle$ for long and short bursts [15]. The spin energy of the black hole can safely account for powerful baryonic collimating winds from the torus [16,17], and high mass ejections onto the companion star in the scenario of [9].

In this Letter, we focus on gravitational radiation from the torus powered by the spin energy of the black hole. We shall find that this represents a major fraction of the black-hole luminosity, presently emitted as "unseen" emissions, whenever the torus becomes nonaxisymmetric. True calorimetry of gamma-ray bursts, therefore, may be obtained from measuring the fluence in these gravitational-wave emissions [17]. The remainder will be emitted in various ways, such as Poynting flux winds, baryonic collimating winds and, when sufficiently hot, neutrino emissions. Note that this prediction for as yet unseen emissions invalidates the often stated suggestion that the total energy budget is generally reduced for beamed emissions, in the case of black-hole inner engines with rapid spin. Furthermore, the emissions from the torus in low-frequency radio waves (modulated) are of potential interest to the planned Low Frequency Radio Antenna (LOFAR) and the Square Kilometre Array (SKA), suggesting to consider correlated LIGO/VIRGO-LOFAR/SKA searches.

Gravitational radiation from a torus features several aspects which suggest considering long GRBs as potential sources for LIGO/VIRGO. Namely, the torus is strongly coupled to the spin energy of the black hole; lumpiness in the torus will produce gravitational radiation at twice the

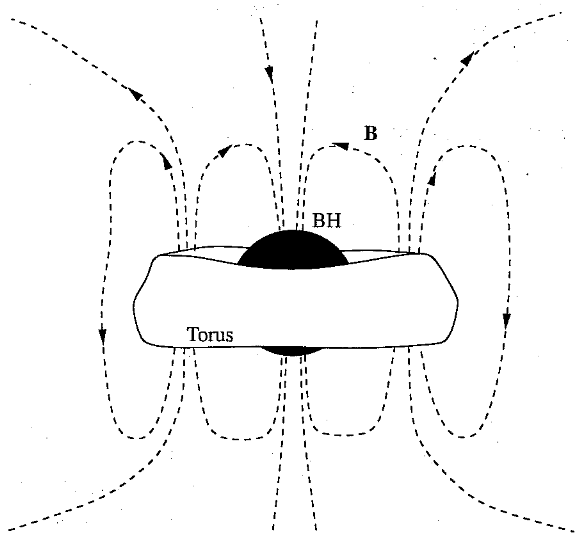

FIG. 1. Cartoon of a rapidly rotating black-hole–torus system in suspended accretion. The black hole assumes an equilibrium magnetic moment in its lowest energy state. The torus around the black hole supports a similarly shaped magnetosphere. Equivalence in poloidal topology to pulsar magnetospheres indicates a high incidence of the black-hole luminosity on the inner face of the torus. The torus reradiates this input in gravitational radiation, Poynting flux winds and, possibly, neutrino emissions. (Reprinted from Ref. [13], ©2001, Elsevier B.V.)

Keplerian angular frequency, i.e., in the range of 1–2 kHz; the emission in gravitational radiation should dominate over emissions in radio waves (see young pulsars [18] and below); the true rate of GRBs should be frequent as inferred from their beaming factor 520 $\pm$ 85 [19]. This process for gravitational wave emissions from a torus powered by the spin energy of the black hole is distinct from emissions in neutron-star–neutron-star mergers [20,21] or by fragmentation in collapse towards supernovae [22].

The torus is likely to possess dynamical and, potentially, radiative instabilities. A geometrically thick torus is consistent [17] with the recent indication that long GRBs may be standard [19]. A thick torus is generally unstable [23]. If the torus reaches an appreciable mass fraction of the central black hole, it will be unstable to self-gravity (see [24]). Similar to rapidly rotating neutron stars, the torus may be subject to the Chandrasekhar-Friedman-Shutz instability. Since lumps of matter radiate preferentially on inner orbits, a quadrupolar, radial deformation of the torus might also be radiatively unstable. It would be of interest to study these radiative instabilities in further detail. We note that some of the quasiperiodic oscillations (QPOs) in accretion disks in x-ray binaries have been attributed to general relativistic effects in orbital motions [25].

An equivalence in poloidal topology to pulsar magnetospheres shows a high incidence of black-hole luminosity into the torus when magnetized by the remnant flux from the progenitor star—a massive star in hypernovae or a neutron star in coalescence onto a black hole. The black hole thus surrounded by a torus magnetosphere will adjust to its lowest energy state by developing an equilibrium magnetic moment [13]

$$\mu_H^e \approx aBJ_H\,, \tag{1}$$

where B denotes the average poloidal magnetic field in the vicinity of the black hole, and $a = J_H/M$ the specific angular momentum of a black hole with angular momentum J_H and mass M. This equilibrium maintains a maximal and uniform horizon flux [26], preserving strong coupling to the torus magnetosphere and, hence, to the surrounding matter. The latter follows by equivalence in poloidal topology to pulsar magnetospheres [13,27], wherein the inner and outer faces of the torus each correspond to a pulsar [28] with an appropriate angular velocity. When the black hole spins sufficiently rapidly, a state of suspended accretion may result ([14] and below). The μ_H^e further serves to support an open magnetic flux tube to infinity which, subject to frame dragging, can account for ultrarelativistic leptonic outflows along the axis of rotation [13,29]. Quite generally, the black-hole luminosity L_T onto the torus *far* exceeds the luminosity L_p into these leptonic outflows [17].

Estimates of the various emissions from the torus can be calculated in a suspended-accretion state [14]. Here, the emissions from the torus are replenished by spin-up Maxwell stresses on the inner face, through the magnetic connection to the black hole: the pulsar equivalent to the inner face has an angular velocity $-(\Omega_H - \Omega_+)$, where Ω_H denotes the angular velocity of the black hole and Ω_+ denotes the angular velocity of the inner face. Thus, the inner face of the torus receives a spin-up torque [10,14]

$$\tau_+ = (\Omega_H - \Omega_+)f_H^2A^2, \tag{2}$$

where f_H denotes the fraction of flux which reaches the horizon of the black hole, of the net poloidal flux $2\pi A$ supported by the torus. In equilibrium with the radiative losses from the torus, a suspended-accretion state will result.

The motion of the torus is to leading order Keplerian, with angular velocity Ω_K. Some deviation away from Keplerian motion is expected, as the competing torques tend to bring the two faces in state of super- and sub-Keplerian motion, with positive radial pressure which promotes a radially slender shape. The interface separating the two faces is expected to be unstable, which favors turbulent mixing into a state of uniform specific energy across the torus. Mixing enhances differential rotation, which drives towards an angular velocity $\Omega(r) \approx \Omega_K[1 - (r - a)/a]^{1/2}$ as a function of radius r for a torus of major radius a; compression into a more slender shape tends to reduce differential rotation. The net result should be that the characteristically Keplerian decrease of angular velocity with radius is approximately preserved. The inner and outer faces will have, respectively, angular velocities $\Omega_\pm \approx \Omega_K(1 \pm 3b/4a)$, where b denotes their radial separation. In what follows, we neglect such perturbations $3b/4a$ to the Keplerian velocity distribution.

Gravitational radiation from a torus surrounding a black hole tends to dominate radio emission of the same frequency. This is generally so for compact systems (of the order of their Schwarzschild radius) in the presence of gravitationally weak magnetic fields. As a torus develops quadruple moments in magnetic moment and mass due to an ellipsoidal deformation, the ratio of its radio-to-gravitational wave emissions can be estimated to be

$$L_{\rm EM}{:}L_{\rm GW} \sim (\Omega M)^{2/3}(E_B/M)(M/m)^2 < 1\,, \tag{3}$$

e.g., when $E_B/M \sim 10^{-6}$ for the relative energy in the magnetic field and $M/m \leq 10^2$.

The suspended accretion state (see Fig. 1) is described by equilibrium conditions for torque and energy:

$$\begin{cases} \tau_+ = \tau_- + \tau_{\rm rad}, \\ \Omega_+\tau_+ = \Omega_-\tau_- + \Omega\tau_{\rm rad} + P_d, \end{cases} \tag{4}$$

where P_d denotes dissipation, $\Omega \approx \Omega_K$ a mean orbital angular frequency, and $\tau_- = A^2f_w^2\Omega_-$ denotes the torque on the outer face of the torus. In (4), we neglect surface stresses due to radiation derived from P_d, notably so in thermal and neutrino emissions. The net magnetic flux $2\pi A$ supported by the torus will partially connect to the black hole and to infinity (by Poynting-flux winds), respectively, with fractions f_H and f_w. Thus, $A \approx ab\langle B_\theta\rangle$ in terms of the average poloidal component B_θ in the torus.

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Generally, $f_H + f_w = 1/2 - 1$ with $f_H \propto (M/a)^2$ for a radially slender torus (which may be thick in the poloidal direction) of major radius a. A remainder $1 - f_H - f_w$ is inactive in closed field lines, whose end points are both on either face of the torus. These field lines extend to the inner light surface and the outer light cylinder, and form toroidal "bags" [27]. Note that for small differential rotation, we have $(\Omega_K\tau_{\rm rad} + P_d)/\Omega_K\tau_{\rm rad} \approx (\Omega_+\tau_+ - \Omega_-\tau_-)/\Omega_K(\tau_+ - \tau_-) \approx 2$, in which limit the efficiency of the radiation is 50%.

The equilibrium conditions (4) are closed after specifying the internal stresses in the torus. We shall assume that the two faces are coupled by magnetohydrodynamical stresses due to radial components B_r of the magnetic field. These stresses are dissipative, by Ohmic heating and magnetic reconnection, which will heat the torus and brings about thermal and, possibly, neutrino emissions. By dimensional analysis

$$P_d \approx A_r^2(\Omega_+ - \Omega_-)^2, A_r = ah\langle B_r^2\rangle^{1/2}, \tag{5}$$

where the second equation denotes the root mean square of the radial flux averaged over the interface between the two faces with contact area $2\pi ah$.

The magnetic stresses on and inside the torus depend differently on the magnetic field. While internal angular momentum transport between the two faces is mediated by $\langle B_r^2\rangle^{1/2}$, the angular momentum transport from the black hole to the torus is by the average $\langle B_\theta\rangle$. The first comprises the spectral density average over all azimuthal quantum numbers m, whereas the second involves only $m = 0$. Indeed, the net flux through the black hole is generated by the coroting horizon charge $q \approx \langle B_n\rangle J$ in magnetostatic equilibrium (1) with the mean external poloidal magnetic field. This averaging process is due to the no-hair theorem. While the exact ratio depends on the details of the magnetohydrodynamical turbulence in the torus, a conservative estimate is that A_r/A is about the square root of the number of azimuthal modes in the approximately uniform infrared spectrum, which should reach up to the first geometrical break at $m = a/b$, i.e., $A_r/A \approx (a/b)^{1/2}$. Substitution of the first into the second equation of (4) gives a luminosity

$$L_{\rm GW} \simeq \Omega_T\tau_{\rm rad} \approx \Omega^2A^2[3(A_r/A)^2(b/a) - 2f_w^2] \simeq \alpha\Omega_T^2A^2, \tag{6}$$

where $\Omega_T = (\Omega_+ + \Omega_-)/2, \Omega_+ - \Omega_- \approx (3/2)(b/a)\Omega_T$, and $\alpha \simeq 3 - 2f_w^2$. Substitution of the right-hand side of (6) in the first equation of (4) gives

$$\frac{\Omega_T}{\Omega_H} \approx \frac{f_H^2}{\alpha + f_H^2 + f_w^2}. \tag{7}$$

The luminosity in gravitational waves, therefore, satisfies

$$L_{\rm GW} \simeq L_H/3; \tag{8}$$

the luminosity in Poynting flux winds is smaller by a factor f_w^2. In the above, the suspended-accretion state is facilitated by magnetohydrodynamical stresses; the results do not depend on the details of the instabilities which give rise to the required nonaxisymmetries in the torus. It would be of interest to study the type of instability by numerical simulations.

The Keplerian frequency will evolve on the secular time scale of spin down of the black hole. This indicates a *horizontal branch* of the frequency dynamics in the $\dot f(f)$ diagram [30]. Lumpiness in the torus will radiate at twice the Keplerian frequency of the torus, and hence we expect gravitational wave frequency of .

$$f_{\rm GW}(t) \sim 1 - 2\ {\rm kHz}/(1 + z), \qquad df_{\rm GW}(t)/dt = {\rm const} \tag{9}$$

for canonical GRB values for a black-hole–torus system at redshift z. Here, the sign of the constant follows the change in Keplerian frequency of the torus; it depends sensitively on the details of the magnetic black-hole-to-torus coupling provided by the inner torus magnetosphere. In particular, it depends on the detailed radial dependence of the horizon flux as a function of the major radius of the torus, which is beyond the scope of the present analysis. If the torus shows violent behavior, the gravitational waves may be episodical, and be correlated with sub-bursts in long GRBs through modulations of the equilibrium magnetic moment (1). In this event, the linear chirp in (9) is more likely to be indicative of an ensemble average over bursts, rather than to hold for individual bursts. The duration should be the intrinsic duration of the gamma-ray burst event, i.e., about 10–15 s as inferred from the mean value of 30s in the BATSE catalog, corrected for redshift.

The black-hole luminosity is partly directed to the torus, and partly in leptonic outflows as input to the observed GRBs. Most of the luminosity is into the torus, since only a small fraction is into the jet through an open flux tube along the axis of rotation. The open flux tube is described by an opening angle θ_H on the horizon and an opening angle θ_j on the celestial sphere. Generally, $\theta_j < \theta_H$ by collimation, e.g., by baryonic collimating winds [16]. A model dependent estimate gives an estimate for the mean geometrical beaming $f_b = \theta_j^2/2 \simeq 1/520$ (for bipolar outflows) and an average GRB fluence of 5×10^{50} [19]. The horizon opening angle θ_H should be sufficient to account for the GRB fluence, yet may leave a dominant fraction of the black-hole luminosity incident on the torus. For a bipolar output, we have [17]:

$$L_p{:}L_T \simeq f_o^2, \tag{10}$$

where $f_o = \theta_H^2/4$ denotes the beaming factor of the flux cone on the horizon. Identifying a long GRB with the spin down of a rapidly spinning black hole, we have $L_p{:}L_T \simeq E_j{:}E_T$, where $E_j = E_{\rm GRB}/\epsilon$ denotes the energy in the jet inferred from the observed GRB-fluence at a (model dependent) efficiency ϵ, and $E_T \simeq \Omega_T/\Omega_H E_{\rm rot}$ denotes the energy input to the torus derived from the rotational energy of the black hole. This gives an estimate $\theta_H \simeq 35°$

[17], which may be standard if the torus is geometrically thick [17]. Thus, the ratio (10) is small.

The stability of the gravitational wave frequency is somewhat uncertain, as it may be variable by the magnetohydrodynamical turbulence in the torus. Nonetheless, it is of interest to consider the possibility of encountering a well-defined secular frequency sweep (upwards or downwards) in case the frequency behavior is quasiperiodic. In this event, a Fourier analysis suffices. The effective amplitude then correlates with the fluence in gravitational waves—derived from an enhancement in gain by a factor $\sqrt{n}$, where n is the number of cycles in the emission. The effective amplitude of the gravitational radiation of a cosmologically nearby source distance D satisfies

$$h_{\rm eff}^{\rm grb} \sim \left(\frac{M}{D}\right)\left(\frac{E_{\rm GW}}{M}\right)^{1/2} \qquad (11)$$

for a net fluence $E_{\rm GW}$ in gravitational waves. By (8), $E_{\rm GW}$ is about one-third of the fraction Ω_T/Ω_H of the spin energy of the black hole, i.e., $E_{\rm GW} \simeq 0.1 M_\odot (M/10M_\odot)$. A geometrical beaming factor of about 500 [19] gives rise to multiple events per year within a distance $D \sim 100$ Mpc with $h_{\rm eff} \sim 10^{-21}$. Combined, this points towards GRBs as potential sources for LIGO/VIRGO.

In summary, a torus around a black hole from hypernovae or black-hole–neutron-star coalescence is a probable transient source of gravitational waves for LIGO/VIRGO. Its gravitational wave emissions are representative for its energetic output, which may be used as a calorimetric constraint on the spin of the black hole. The gravitational wave frequency is expected to be 1–2 kHz on a horizontal branch in the $\dot{f}(f)$ diagram for a duration of about 10–15 s (a redshift corrected mean of long bursts).

Optimal strategies for detecting these gravational wave sources appear to be by Fourier analysis, should the signal be quasiperiodic. Alternatively, LIGO/VIRGO searches might be combined with future radio searches, such as LOFAR/SKA.

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