

A CALORIMETRIC TEST FOR KERR BLACK HOLES IN GAMMA-RAY BURSTS

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ABSTRACT

Cosmological gamma-ray bursts are potentially associated with systems harboring a rotating black hole. Calorimetry on yet “unseen” emissions in gravitational waves from a surrounding torus may provide a measure for its rotational energy. We introduce a compactness parameter $\alpha = 2\pi \int f dE$ for the fluence E and frequency f in gravitational waves. For black hole–torus systems, $\alpha \approx 2\pi Ef \approx 0.005$ – 0.05 may exceed the upper bound $\alpha^* = 0.005$ for rapidly rotating neutron stars. A duration in gravitational radiation consistent with the redshift-corrected durations of the BATSE catalog defines an association with long gamma-ray bursts. This promises to be a new test for Kerr black holes as objects in nature.

Subject headings: black hole physics — gamma rays: bursts — gamma rays: theory — gravitational waves

1. INTRODUCTION

Gravitation is a vital and universal force in nature. It ultimately leads to black holes as fundamental particles, according to the Einstein theory of general relativity. These objects could be the source of cosmological gamma-ray bursts (GRBs)—the most enigmatic events in the universe. Their emissions are characteristically nonthermal in the few hundred keV range, with a bimodal distribution in durations of short bursts around 0.3 s and long bursts around 30 s (Kouveliotou et al. 1993). These probably derive from the dissipation of the kinetic energy in ultrarelativistic baryon-poor jets (see Piran 1999, 2000 for a review). Redshift determinations from the X-ray afterglows to long bursts indicate a cosmological origin, consistent with their isotropic distribution in the sky and with $\langle V/V_{\max} \rangle$ being distinctly less than the Euclidean value of $\frac{1}{2}$ (Schmidt 1999). They are probably associated with the formation of young massive stars (Paczynski 1998; Bloom, Kulkarni, & Djorgovski 2000) and hence should be most frequent at a redshift $z = 1$ – 2 . Remnants of GRBs are potentially found in some of the soft X-ray transients (SXTs; Brown et al. 2000), with notable candidates GRO J1655–40 (Israeli et al. 1999) and V4645 (Orosz et al. 2001).

The inner engine producing the GRBs should be energetic and compact. Angular momentum forms a canonical energy reservoir, and GRB inner engines are probably no exception. This is consistent with the breaking of spherical symmetry (Woosley 1993). Then hypernovae follow up on this idea by postulating massive young stars in binaries as their progenitors (Paczynski 1998). These proposals have in common the formation of a black hole, which should be rotating (Bardeen 1970) by conservation of angular momentum from the progenitor star and its binary motion. Black holes formed in prompt collapse, furthermore, are subject to a minimum mass on the order of $10 M_{\odot}$ (van Putten & Ostriker 2001), consistent with the observed masses of 3– $14 M_{\odot}$ in SXTs. We note that high angular momentum is also present in coalescing black hole–neutron star systems. Encysting rotating black holes as the baryon-free energy source in GRBs is perhaps tantamount to understanding how stars shine.

According to the Kerr solution of rotating black holes (Kerr 1963), the horizon surface encloses baryonic matter collapsed under its own gravitational forces, while leaving rotational energy accessible by the Rayleigh criterion. The latter is a consequence of the first law of thermodynamics (Bardeen, Carter,

& Hawking 1973; Hawking 1975, 1976),

$$\delta M = \Omega_H \delta J_H + T_H \delta S_H, \quad (1)$$

where $\Omega_H \leq 1/2M$ denotes the angular velocity of the black hole of mass M , J_H its angular momentum, and T_H and S_H its surface temperature and entropy, respectively. The specific angular momentum $\delta J_H/\delta M$ of a radiated particle, therefore, is at least twice that of the black hole itself. In isolation, rotating black holes are effectively nonradiating due to angular momentum barriers (Unruh 1974; Hawking 1975; Teukolsky 1973; Press & Teukolsky 1973; Teukolsky & Press 1974). However, a surrounding torus may trigger a rotating black hole into violent activity (see van Putten 2001a for a review)—perhaps epitomized in GRBs.

Identifying Kerr black holes as the active nucleus in GRBs may derive from its three defining properties:

1. A compact horizon surface, which causally separates collapsed matter from its progenitor surroundings. The black hole is expected to represent the final outcome of stellar evolution, formed in the accretion-induced collapse of a neutron star, white dwarf, or, alternatively, in prompt collapse. This may produce black holes of intermediate spin in the former (Brown, Lee, & Bethe 1999; Brown et al. 2000) and black holes with rapid spin and above a certain mass threshold in the latter (van Putten & Ostriker 2001).

2. Frame-dragging of spacetime in apparent violation of Mach’s principle, in the sense of a correspondence of zero angular velocity relative to the distant stars with zero angular momentum. A rotating black hole introduces differential rotation in spacetime described by frame-dragging $-\beta$, equal to the angular velocity Ω_H of the black hole on the horizon and zero at infinity. Upon reaching the north pole of a black hole, a ballerina rotating with angular velocity $-\beta$ relative to the distant stars will find her arms naturally pointed down, while one standing still will find her arms spread out.

3. A compact energy reservoir of up to one-third of the total mass of the black hole. Rotation of a Kerr black hole may be parameterized by $\sin \lambda = a/M$, where a denotes the specific angular momentum J_H/M for an angular momentum J_H and mass M . This gives $\Omega_H = \tan(\lambda/2)/2M$ and $E_{\text{rot}} = 2M \sin^2(\lambda/4)$ for the angular velocity and rotational energy of a Kerr black hole, respectively. Thus, the rotational energy may reach about 29% of the total mass of the black hole. This has no counterpart in

baryonic matter. About one-half of the rotational energy is stored in the upper 10% of the angular velocity of a maximally spinning black hole.

The first two properties—a compact horizon and frame-dragging—are potentially the driving agency behind the formation of a macroscopic gap with ultrarelativistic baryon-poor outflows along open magnetic field lines (van Putten 2001a, 2001c), serving as the input to GRBs/afterglows. These field lines find their support in an equilibrium magnetic moment of the black hole in its lowest energy state as a consequence of the no-hair theorem, when surrounded by a torus magnetosphere. The latter is expected to be supported by a disk or torus, from fallback matter in failed supernovae/hypernovae (Woosley 1993; Paczyński 1998) or debris of a neutron star upon tidal breakup (Paczynski 1991).

Establishing the third property requires calorimetry on the output from GRBs in all energy channels. For long bursts from rapidly spinning black holes (van Putten & Ostriker 2001), discussed here, we expect a high incidence of the black hole luminosity onto the inner face of the surrounding disk or torus by the equivalence in poloidal topology to pulsar magnetospheres (van Putten 1999). First, this output into the disk or torus may be processed into several channels, powering gravitational waves, Poynting-flux-dominated winds, and, when sufficiently hot, neutrino emissions. Indeed, evidence for appreciable energy output from the spin of the black hole into the disk may already be found in chemical depositions onto the companion star of GRO J1655–40 and V4641 Sgr, in the hypernovae proposed by Brown et al. (2000). Second, this output may be representative of the total output of the black hole, being at least 2 order of magnitudes more luminous (van Putten & Levinson 2001) than the true output of $(3-5) \times 10^{50}$ ergs in GRBs/afterglows (Frail et al. 2001; Panaitescu & Kumar 2001; Piran et al. 2001). Thus, GRBs could be merely the tip of the iceberg, being accompanied by as yet “unseen” emissions such as in gravitational waves, whose fluence may reach about 1% of the mass of the black hole (van Putten 2001b). Therefore, the upcoming Laser Interferometer Gravitational-Wave Observatory (LIGO; Abramovici et al. 1992) and its French-Italian counterpart VIRGO (Bradaschia et al. 1992) promise a new avenue for calorimetry on the energy reservoir in GRBs and possibly that of rotating black holes.

The association between the fluence in gravitational waves from a torus and the spin energy of a central black hole is described in § 2. A compactness measure for the observed fluence and frequency in gravitational waves is proposed in § 3, which may serve to discriminate black hole–torus systems from rapidly rotating neutron stars. We shall use geometrical units, expressing energy E in terms of the equivalent gravitational radius $EG/c^2 = 1.5 \times 10^5 \text{ cm}(E/M_\odot c^2)$, where G denotes Newton’s constant and c the velocity of light, and frequency f in terms of f/c . For example, in these units, luminosity is dimensionless, given as a rate of change of energy (in units of centimeters) per unit of time (in units of centimeters).

2. PROPOSED SOURCE OF GRAVITATIONAL RADIATION FROM A TORUS AROUND A BLACK HOLE

Long/short GRBs may be associated with a state of suspended/hyperaccretion onto the black hole (van Putten & Ostriker 2001). A suspended-accretion state lasts as long as the black hole spins rapidly, whereas the disk around a slowly rotating black hole evolves by magnetic regulated hyperaccre-

tion toward a finite-time singularity. A bimodal distribution of duration occurs when the ratio of disk mass to torus mass is small. The suspended-accretion state operates by the equivalence in poloidal topology to pulsar magnetospheres (van Putten 1999; Brown, Bethe, & Lee 2001): the inner face of the torus receives energy and angular momentum from the black hole, as does a pulsar when infinity wraps around it (van Putten 2001c). Gravitational radiation may be produced in black hole–torus systems as the torus develops nonaxisymmetric instabilities. With the aforementioned association with GRBs, this features several aspects that suggest considering long GRBs as LIGO/VIRGO sources: the torus is strongly coupled to the spin energy of the black hole; lumpiness in the torus will produce gravitational radiation at twice the Keplerian angular frequency, i.e., in the range of 1–2 kHz; gravitational radiation should dominate over emissions in radio waves; and the true rate of GRBs should be frequent given their beaming factor of a few hundred (Frail et al. 2001; Panaitescu & Kumar 2001; Piran et al. 2001). Powered by the spin energy of the black hole, these emissions are different from radiation produced during the spiral in of neutron-neutron star binaries (Narayan, Paczyński, & Piran 1992; Kochanek & Piran 1993) or the fragmentation in collapse toward supernovae (Bonnell & Pringle 1995).

Nonaxisymmetries in the torus are expected from dynamical and, potentially, radiative instabilities. This can be described by a lumpiness m , which introduces a luminosity according to the Peters & Mathews (1963) formula for quadrupole radiation from two-point masses:

$$L = \frac{32}{5} (\omega \mathcal{M})^{10/3} F(e) \approx \frac{32}{5} (M/R)^5 (m/M)^2, \quad (2)$$

where $\omega \approx M^{1/2}/R^{3/2}$ denotes the orbital frequency with separation R , $\mathcal{M} = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5} \approx M(m/M)^{5/3}$ the chirp mass, and $F(e)$ a factor of order unity as a function of the ellipticity e . The torus may be thick, consistent with the recent indication that long GRBs may be standard (van Putten & Levinson 2001), which is generally susceptible to the Papaloizou-Pringle instability (Papaloizou & Pringle 1984). A massive torus, reaching an appreciable fraction of the mass of the black hole, would moreover be unstable to self-gravity (Woodward, Tholine, & Hachisu 1994). Nonaxisymmetries might also be promoted by gravitational radiation, as in the Chandrasekhar-Friedman-Schutz instability, or by its action preferentially on lumps of matter on inner orbits. Gravitational-wave emissions thus produced could be quasi-periodic oscillations (QPOs), perhaps reminiscent of QPOs in X-ray binaries and possibly also related to general relativistic effects in orbital motions (Stella 2001).

We find that about one-third of the black hole luminosity L_H (most of which is incident onto the torus) is reradiated into gravitational waves (van Putten 2001b):

$$L_{\text{gw}} \approx L_H/3 \quad (3)$$

(the fraction on the right-hand side may range from $\frac{1}{4}$ to $\frac{1}{2}$). For a maximally spinning black hole, therefore, the fluence in gravitational radiation may reach about one-third the rotational energy of the black hole times the efficiency factor given by the ratio of the angular velocity Ω_T of the torus to that of the central rotator. For a torus of radius R around a maximally spinning Kerr black hole of mass M , the latter is given by

(Shapiro & Teukolsky 1983)

$$\Omega_T/\Omega_H = 2/[(R/M)^{3/2} + 1]. \quad (4)$$

Hence, by equations (2)–(4), the characteristic mass m of lumpiness in the torus is required to be

$$m \approx 0.15\% M (R/3M)^{7/4} \quad (5)$$

for a canonical burst duration of 15 s (of long bursts). This estimate is consistent with the condition that the perturbation m is much less than the mass M_T of the torus, while $M_T \ll M$ for there to be a bimodal distribution. Note that the efficiency in equation (4) will be less than 50% for a torus radius $R > 2M$, showing that, generally, most of the rotational energy is dissipated in the horizon of the black hole. Since 90% of the rotational energy of a maximally spinning black hole is contained in the upper 10% of its angular velocity, a black hole–torus system is not expected to evolve significantly during most of its gravitational-wave emissions.

3. COMPACTNESS PARAMETER FOR BURSTS OF GRAVITATIONAL RADIATION

We propose a compactness parameter for gravitational-wave emissions by integration of the frequency f against fluence:

$$\alpha = 2\pi \int_0^E f dE \quad (6)$$

for a net fluence E . An equivalent expression is the integration of the luminosity L against the number of periods n ; i.e., $\alpha = 2\pi \int_0^T L(t) dn(t)$ over the duration T of the burst. The measure in equation (6) is reminiscent of the ratio of the rotational energy E_{rot} to the length scale M of a black hole. Indeed, $\alpha \approx 2\pi E/L$ for a system of size L close to its Schwarzschild radius, whose fundamental angular frequency is of order $2\pi/L$. In geometric units, α is dimensionless.

For a black hole–torus system, equation (6) becomes

$$\alpha \approx 2\pi E f \quad (7)$$

upon neglecting secular evolution while the black hole continues to spin rapidly. For a maximally spinning black hole, equation (3) shows that E may reach about one-third the rotational energy of the black hole times the efficiency factor in equation (4). Hence, we have

$$\alpha \approx \frac{4}{9} [(R/M)^{3/2} + 1]^{-2} \approx 0.005\text{--}0.05, \quad (8)$$

where $R \approx (2\text{--}4) \times M$ denotes a fiducial range of torus radii, corresponding formally to $f = 1.1\text{--}2.5$ kHz for $M = 7 M_\odot$.

Gravitational radiation from a neutron star is commonly expected in its early stages of formation. In the approximation of a neutron star of uniform mass density, rotation about centrifugal breakup gives the upper bound

$$\alpha^* = \frac{4}{15} M_{\text{ns}} R_{\text{ns}}^2 \Omega_{\text{ns}}^3 = \frac{4}{15} (M_{\text{ns}}/R_{\text{ns}})^{5/2} \approx 0.005 \quad (9)$$

for a canonical value $M_{\text{ns}}/R_{\text{ns}} \approx \frac{1}{5}$ of the mass-to-radius ratio

of a neutron star. The true value of α^* may be somewhat less by an (uncertain) efficiency factor and extremal angular velocities below the Newtonian bound $M_{\text{ns}}^{1/2}/R_{\text{ns}}^{3/2}$.

4. CALORIMETRIC TEST FOR KERR BLACK HOLES

Gravitational-wave emissions from a black hole–torus system may be discriminated from those by a neutron star through the test

$$\alpha > \alpha^*. \quad (10)$$

The estimates (eqs. [8] and [9]) show that equation (10) tends to be satisfied *whenever the black hole–torus system is more compact than a neutron star*. Here compactness is now reexpressed in terms of a mass-to-radius ratio: the black hole mass to the radius of the torus in the former and the neutron star mass to its radius in the latter case.

Applying the test (eq. [10]) requires the distance to the source in deriving the fluence from the observed strain amplitude. This may be circumvented using a sample of detections and taking averages to consider

$$\bar{\alpha} > \alpha^* \quad (11)$$

in the approximation

$$\bar{\alpha} \approx 2\pi \bar{E} \bar{f}. \quad (12)$$

A cosmologically nearby sample is well described by a constant GRB event rate \dot{n} per unit volume, e.g., 2 per year within a distance to 100 Mpc. This gives a differential event rate of gravitational-wave bursts $dN \approx \Sigma \dot{n} dr$, where $\Sigma = 4\pi r^2$ denotes the surface area of the sphere reaching the source at radius r . In what follows, we shall assume a source population with standard E and f . Consider a sample of surface energy densities $e = E/\Sigma$ at the detector. Upon averaging, we have $\bar{e} = E \langle \Sigma_{\text{max}}/\Sigma \rangle / \Sigma_{\text{max}} = 3E/\Sigma_{\text{max}}$, where $\Sigma_{\text{max}} = 4\pi r_{\text{max}}^2$ denotes the maximal surface area associated with a source at $r = r_{\text{max}}$ at detection threshold. The assumption, which is that of standard emissions E , may be checked through the relationship

$$d\dot{N} \propto e^{-1/2} d(1/e). \quad (13)$$

The averaged expression in equation (12) for α now becomes

$$\bar{\alpha} = (4\pi/3)^{5/3} (\dot{N}/\dot{n})^{2/3} \bar{e} \bar{f} \quad (14)$$

upon expressing Σ_{max} in the net detection rate \dot{N} .

Sources of gravitational waves in the cosmologically nearby sample can be tested for an association with GRBs by comparing their durations with the redshift-corrected mean duration $\bar{T} = 30 \text{ s} \langle 1/(1+z) \rangle$ of long GRBs in the BATSE catalog (Kouveliotou et al. 1993). This gives $\bar{T} \approx 10$ s if long GRBs are locked to the star formation rate or $\bar{T} \approx 10\text{--}15$ s if working from the measured redshifts z to individual events (as in van Putten 2001b, 2001c). The latter might overestimate \bar{T} because of the extinction of afterglows (see Ramirez-Ruiz, Trentham, & Blain 2001). A typical redshift of about 2 would be consistent with $\langle V/V_{\text{max}} \rangle = 0.282$ for long bursts (Katz & Canel 1996) and the peak in the stellar formation rate (e.g., Connolly et al. 1997). In summary, calorimetry on

gravitational-wave emissions from GRBs could offer a unique opportunity to establish the existence of Kerr black holes as objects in nature and hence to establish the existence of black holes in general.

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