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A technique to modulate the signature of a stochastic gravitational wave background

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Abstract

Detecting a stationary, stochastic gravitational wave signal is complicated by the impossibility of observing detector noise independently of the signal. Here we describe a method of identifying this source of systematic error by varying the orientation of one of the detectors, leading to separate and independent modulations of the signal and noise contribution to the cross-correlation. The method can be applied to measurements of a stochastic gravitational wave background by the ALLEGRO/LIGO Livingston Observatory detector pair. We explore—in the context of this detector pair—how this new measurement technique is insensitive to a cross-correlated detector noise component that can confound a conventional measurement.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

One of the goals of the ground-based gravitational wave detectors now operating or under construction [1–8] is to detect or place limits on the amplitude and spectrum of the stochastic gravitational wave background. A single, isolated gravitational wave detector cannot distinguish between instrumental noise and a weak, stationary cosmic gravitational wave background radiation. At least two detectors are needed, in which case the stochastic signal will be apparent in their cross-correlation. Specifically, the two detectors must (i) have an overlapping frequency response, (ii) have a separation shorter than the wavelength of their overlapping response and (iii) both sample the same polarization state of the incident radiation. The LIGO Livingston Observatory (LLO) interferometric detector [5, 9] and the Louisiana State University ALLEGRO cryogenic acoustic detector [1, 10, 11], separated by 42.3 km, constitute such a detector pair, capable of providing an experimental bound on the stochastic gravitational wave background at approximately 900 Hz.

A weak, stationary stochastic signal cannot be distinguished from a similarly weak, stationary noise background that is correlated between the two detectors. Instrumental noise arising from the environment may lead to such a correlated detector noise; consequently, a convincing case must be made that no terrestrial noise source is responsible for any observed correlation. This is a daunting experimental challenge.

The signal contribution to the cross-correlation depends on the relative orientation of the detectors (which determines their sensitivity to the two different polarization states). Changing the orientation of one of the detectors will modulate the signal contribution to the cross-correlation in a predictable way, in principle allowing us to distinguish the signal from correlated noise and thereby leading to a significantly improved estimate or bound on the in-band amplitude of a stochastic gravitational wave background. The technique of introducing a controlled signal modulation in order to identify and eliminate systematic environmental effects was conceived by Dicke [12] as the switching radiometer during the development of radar.

The ALLEGRO group has modified their cryogenic detector to permit reorientation of the detector between data-taking periods [13]. This allows for a modulation of the gravitational wave contribution to the detector noise cross-correlation in exactly the manner described. The modulation can be used to improve the reliability of the estimate or limit that we can place on the amplitude of a stochastic gravitational wave background near 900 Hz.

2. The cross-correlated detector output

The output of each detector—ALLEGRO or LLO—is a single time series, which is the sum of instrumental noise and a projection of the incident gravitational wave strain. Denote the output of LLO as s_{L} and the output of ALLEGRO as s_{A} and define the correlation of s_{L} and s_{A} over an integration time T_{int} by [14–17]

$$C(\Omega_{\mathrm{A}}, \Omega_{\mathrm{L}}) = \langle s_{\mathrm{A}}, s_{\mathrm{L}} \rangle = \int_{-T_{\mathrm{int}}/2}^{T_{\mathrm{int}}/2} \mathrm{d}t \int_{-T_{\mathrm{int}}/2}^{T_{\mathrm{int}}/2} \mathrm{d}t' s_{\mathrm{A}}(t) s_{\mathrm{L}}(t') Q(t-t'; \Omega_{\mathrm{A}}, \Omega_{\mathrm{L}}) \tag{1}$$

where Ω_{A} and Ω_{L} are the angles describing the orientation of ALLEGRO and LLO, respectively.

In [18] we considered the more general case where the noise contribution to the ensemble mean cross-correlation is non-zero. Here we give a summary of the results discussed in that paper. In this case the kernel Q that maximizes the SNR can be conveniently expressed in the frequency domain as

$$\tilde{Q}(f; \Omega_{\mathrm{A}}, \Omega_{\mathrm{L}}) = \frac{\gamma(f; \Omega_{\mathrm{A}}, \Omega_{\mathrm{L}}) \Omega_{\mathrm{GW},0}(f)}{f^3 \left(S_{\mathrm{A}}(f) S_{\mathrm{L}}(f) + S_{\mathrm{AL}}(f)^2 \right)} \tag{2}$$

where S_{A} and S_{L} are the ALLEGRO and LLO noise power spectral densities, respectively; S_{AL} is the ALLEGRO/LLO noise cross-spectral density; $\Omega_{\mathrm{GW},0}$ is the model for the stochastic signal spectrum expressed as a fraction of the closure density in logarithmic frequency (the actual closure denisty is denoted as Ω_{GW}). $\gamma(f; \Omega_{\mathrm{A}}, \Omega_{\mathrm{L}})$ is the *overlap reduction function*, which describes the amplitude of the correlation of the gravitational wave signal between the two detectors as a function of their relative orientation [14–16]

$$\begin{aligned} \gamma(f; \Omega_{\mathrm{A}}, \Omega_{\mathrm{L}}) = {} & \rho_1(\alpha) d_{\mathrm{A}} : d_{\mathrm{L}} + \rho_2(\alpha)\, (\hat{n}_x \cdot d_{\mathrm{A}}) \cdot (d_{\mathrm{L}} \cdot \hat{n}_x) \\ & + \rho_3(\alpha)\, (\hat{n}_x \cdot d_{\mathrm{A}} \cdot \hat{n}_x)\, (\hat{n}_x \cdot d_{\mathrm{L}} \cdot \hat{n}_x)\, . \end{aligned} \tag{3}$$

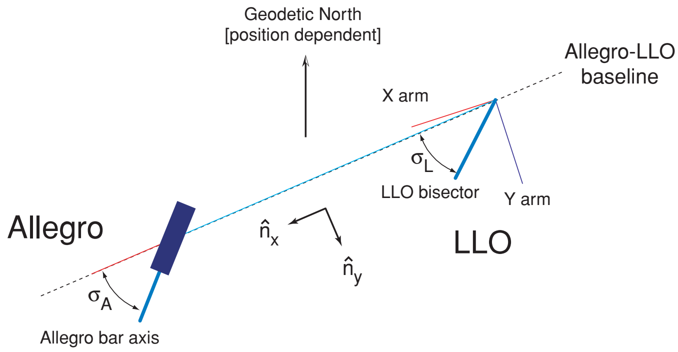

Figure 1. A schematic diagram showing how we characterize ALLEGRO and LLO orientations with respect to geodetic north and the LLO-to-ALLEGRO baseline.

The functions $\rho_k(\alpha)$ characterize the frequency-dependent part of the sensitivity of the detector pair to a stochastic gravitational wave background and are given by

$$\begin{pmatrix}\rho_1(\alpha)\\ \rho_2(\alpha)\\ \rho_3(\alpha)\end{pmatrix} = \frac{1}{\alpha^2}\begin{pmatrix}5\alpha^2 & -10\alpha & 5\\ -10\alpha^2 & 40\alpha & -50\\ 5\alpha^2/2 & -25\alpha & 175/2\end{pmatrix}\begin{pmatrix}j_0(\alpha)\\ j_1(\alpha)\\ j_2(\alpha)\end{pmatrix} \tag{4}$$

where the j_k are the spherical Bessel functions of order k, $\alpha = 2\pi f L/c$, and L is the length of the baseline between the two detectors.

3. Application to ALLEGRO and LLO

The orientation of the ALLEGRO detector may be changed by rotating ALLEGRO in its horizontal plane (cf figure 1). This degree of freedom is described by the angle σ_A. As σ_A varies, $C(\Omega_A, \Omega_L)$ will change through the dependence of γ on σ_A. To express that variation write the output of detector k as the sum of a gravitational wave signal h_k and detector noise n_k: $s_k(t) = h_k(t) + n_k(t)$. We can then write the ensemble average of the correlation $C(\Omega_A, \Omega_L)$ as a function of σ_A

$$\bar{C}(\sigma_A) = (\overline{\langle h_A, h_L\rangle} + \overline{\langle n_A, n_L\rangle}) \tag{5}$$

$$\overline{\langle h_A, h_L\rangle} = T_{\rm int}\int {\rm d}f\,\frac{3H_0^2}{20\pi^2}\frac{\gamma^2(f;\sigma_A)\Omega_{GW}(f)\Omega_{GW,0}(f)}{f^6\left(S_A(f)S_A(f) + S_{AL}(f)^2\right)} \tag{6}$$

$$\overline{\langle n_A, n_L\rangle} = T_{\rm int}\int {\rm d}f\,S_{AL}(f;\sigma_A)\frac{\gamma(f;\sigma_A)\Omega_{GW,0}(f)}{f^3\left(S_A(f)S_L(f) + S_{AL}(f)^2\right)}. \tag{7}$$

Note that S_{AL} may in general depend on the orientation angle σ_A. Since γ depends on the orientation σ_A of the ALLEGRO detector, changing ALLEGRO's orientation changes $\bar{C}$ and allows us to modulate the gravitational wave contribution to C in a predictable way.

We can approximate $\bar{C}(\sigma_A)$ for the ALLEGRO/LLO pair. The ALLEGRO noise power spectral density $S_A(f)$ has a sharp minimum in two narrow bands centred on the two ALLEGRO bar resonances ($f_< = 896.8$ Hz, $f_> = 920.3$ Hz). Correspondingly, the only significant contribution to either of the integrals equation (6) or (7) arises from the respective integrands in the narrow bands about the resonant frequencies. These two resonances are themselves closely spaced; consequently, over those bands the LLO noise power spectral density S_L, and Ω_{GW} will all be approximately constant. Additionally, for the

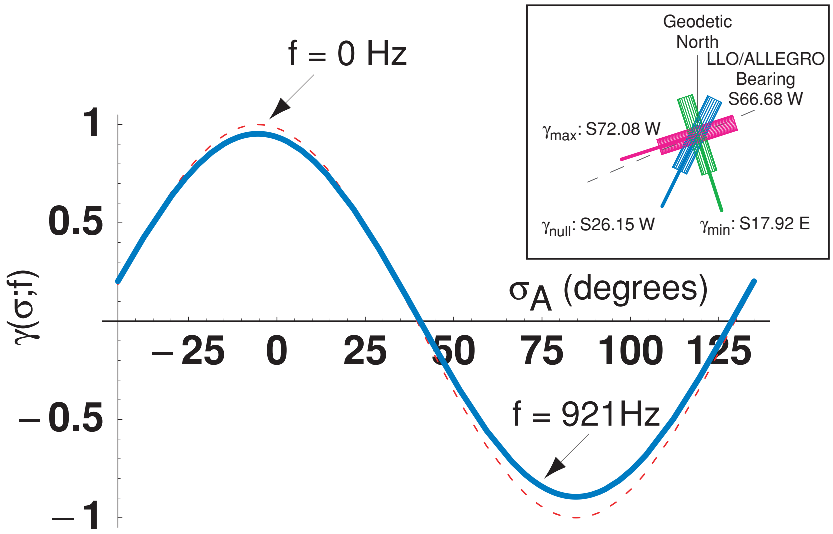

Figure 2. The overlap reduction function γ, which characterizes the dependence of the ALLEGRO–LLO correlation function on an isotropic stochastic signal, depends on the relative orientation of the two detectors. Here we show how this function varies with σ_A, the angle between the ALLEGRO bar axis and the LLO/ALLEGRO baseline (cf figure 1). The bold, solid line shows the variation of γ with σ_A at the operating frequency of the ALLEGRO detector; for comparison, the dashed line shows the same quantity at DC. The inset shows the orientation of the ALLEGRO relative to the LLO/ALLEGRO baseline, when γ vanishes (γ_{null}) and takes on its minimum (γ_{min}) and maximum (γ_{max}) values.

ALLEGRO/LLO detector pair α (cf equation (4)) is small (approximately 0.8) and does not change significantly between the two resonances, so that the overlap reduction function will also be frequency independent in the neighbourhood of the resonances. Further assuming that S_{AL}, the ALLEGRO–LLO instrumental noise cross-spectral noise density, is independent of σ_A and much smaller than either S_L or S_A, we can approximate both integrals to obtain

$$C(\sigma_A) \simeq T_{int}\Delta f \frac{\Omega_{GW,0}(f_0)}{f_0^3 S_A S_L}\left[\gamma^2(f_0;\sigma_A)\frac{3H_0^2}{20\pi^2}\frac{\Omega_{GW}(f_0)}{f_0^3} + \gamma_0(\sigma_A)S_{AL}\right] \tag{8}$$

where

$$f_0 = (f_< + f_>)/2 \qquad \Delta f = \left(\frac{\Delta f_>}{S_A(f_>)} + \frac{\Delta f_<}{S_A(f_<)}\right) S_A \tag{9}$$

$$S_A = [S_A(f_>) + S_A(f_<)]/2 \qquad S_L = [S_L(f_>) + S_L(f_<)]/2 \tag{10}$$

$$S_{AL} = [S_{AL}(f_>) + S_{AL}(f_<)]/2 \qquad \gamma_0(\sigma_A) = \gamma(f_0;\sigma_A). \tag{11}$$

As σ_A varies, the contribution of the stochastic signal to C is quadratic in γ, and varies differently from the contribution of the instrumental noise which is linear in γ. Figure 2 shows, as a solid line, the dependence of γ on σ_A. For reference, the dotted line shows the dependence of γ on σ_A at zero frequency. The inset in figure 2 shows a schematic of the ALLEGRO bar orientation corresponding to the extrema and null of γ.

Since the two terms in equation (8) depend on even and odd powers of $\gamma(f_0, \sigma_A)$, respectively, varying σ_A modulates the contribution to C of correlated noise differently from how it modulates the contribution of a real signal. We can use this differential modulation to eliminate the contribution of any correlated noise S_{AL} that is independent of σ_{A}. Denote the angle σ_{A} for which γ is maximized as $\sigma_{\mathrm{A,max}}$; similarly, denote the angle σ_{A} for which γ is minimized as $\sigma_{\mathrm{A,min}}$. Suppose we make an observation of duration $T_{\mathrm{int,max}}$ with ALLEGRO oriented at angle $\sigma_{\mathrm{A,max}}$, and another observation of duration $T_{\mathrm{int,min}}$ at angle $\sigma_{\mathrm{A,min}}$. The expectation value of C for these two observations is

$$\bar{C}(\sigma_{\mathrm{A,max}}) \simeq T_{\mathrm{int,max}} \left(\Delta f \frac{\Omega_{\mathrm{GW},0}}{f_0^3 S_{\mathrm{A}} S_{\mathrm{L}}}\right)\left[\left(\frac{3H_0^2}{20\pi^2 f_0^3}\right)\gamma_{\mathrm{max}}^2 \Omega_{\mathrm{GW}} + \gamma_{\mathrm{max}} S_{\mathrm{AL}}\right] \tag{12}$$

$$\bar{C}(\sigma_{\mathrm{A,min}}) \simeq T_{\mathrm{int,min}} \left(\Delta f \frac{\Omega_{\mathrm{GW},0}}{f_0^3 S_{\mathrm{A}} S_{\mathrm{L}}}\right)\left[\left(\frac{3H_0^2}{20\pi^2 f_0^3}\right)\gamma_{\mathrm{min}}^2 \Omega_{\mathrm{GW}} + \gamma_{\mathrm{min}} S_{\mathrm{AL}}\right] \tag{13}$$

where $\gamma_{\mathrm{max}} = \gamma(\sigma_{\mathrm{A,max}})$, $\gamma_{\mathrm{min}} = \gamma(\sigma_{\mathrm{A,min}})$. A straightforward linear combination of these two observations

$$C_0 = \frac{\gamma_{\mathrm{max}} T_{\mathrm{int,max}} C(\sigma_{\mathrm{A,min}}) - \gamma_{\mathrm{min}} T_{\mathrm{int,min}} C(\sigma_{\mathrm{A,max}})}{|\gamma_{\mathrm{max}}|\, T_{\mathrm{int,max}} + |\gamma_{\mathrm{min}}|\, T_{\mathrm{int,min}}} \tag{14}$$

thus has an expectation value that is independent of the correlated noise S_{AL}

$$\bar{C}_0 = \left(\Delta f \frac{\Omega_{\mathrm{GW},0}}{f_0^3 S_{\mathrm{A}} S_{\mathrm{L}}}\right)\left(\frac{3H_0^2}{20\pi^2 f_0^3}\right)\Omega_{\mathrm{GW}} \frac{T_{\mathrm{int,max}} T_{\mathrm{int,min}} \gamma_{\mathrm{max}} \gamma_{\mathrm{min}} (\gamma_{\mathrm{min}} - \gamma_{\mathrm{max}})}{|\gamma_{\mathrm{max}}|\, T_{\mathrm{int,max}} + |\gamma_{\mathrm{min}}| T_{\mathrm{int,min}}}. \tag{15}$$

In the present circumstance, $\gamma_0 = \gamma_{\mathrm{max}} \simeq -\gamma_{\mathrm{min}}$. If we also make the observations of equal duration, $T_{\mathrm{int,max}} = T_{\mathrm{int,min}} = T_{\mathrm{int}}/2$, then, following past convention and defining the signal-to-noise ratio ρ_0 of the observation C_0 as the ratio of C_0 to its ensemble variance we have[3]

$$\bar{\rho}_0 \approx \frac{3H_0^2}{10\pi^2}\sqrt{T_{\mathrm{int}}\Delta f}\, \frac{\gamma_0 \Omega_{\mathrm{GW}}(|f_0|)}{f_0^3 \sqrt{S_{\mathrm{A}} S_{\mathrm{L}} + S_{\mathrm{AL}}^2}}. \tag{16}$$

4. Numerical results

Consider, for example, the current generation of ALLEGRO and LLO detectors jointly observing in the presence of a correlated noise $|S_{\mathrm{AL}}| = 10^{-4}(S_{\mathrm{A}} S_{\mathrm{L}})^{1/2}$. Assume that the stochastic gravitational wave signal amplitude is much smaller: Ω_{GW} equal to 10^{-9}. Suppose first that we ignore the possibility that the correlated noise component (represented by S_{AL}) may be present. Then we would leave the ALLEGRO detector orientation fixed in such a manner as to maximize the overlap with LLO. The dashed lines in figure 3(A) show, as a function of observing time, the 90% confidence interval (following the construction of [19]) associated with an observed cross-correlation C (cf equation (1)) equal to the ensemble mean $\bar{C}$. After approximately 0.25 year this most likely observation is clearly no longer consistent with the actual stochastic gravitational wave background amplitude, owing to the systematic error made by excluding the possibility of a correlated noise background. As the observation time increases, the confidence interval on Ω_{GW} shrinks, asymptoting on the amplitude of the correlated noise (S_{AL}) interpreted as a stochastic gravitational signal.

On the other hand, suppose we admit the possibility of a correlated noise background, of unknown cross-spectral density, changing the orientation of the ALLEGRO detector midway through the observation in order that we can construct C_0 (cf equation (14)), which is independent of S_{AL}. Again referring to figure 3(A), the thin grey line shows the 90%

[3] Here we assume that both S_{A} and S_{L} are much greater than either $|S_{\mathrm{AL}}|$ or the corresponding power spectral density of the stochastic signal S_h. Were this not the case we would likely be able to identify the origin of the correlated noise and either isolate the detector pair from it, or regress it from the data during analysis.

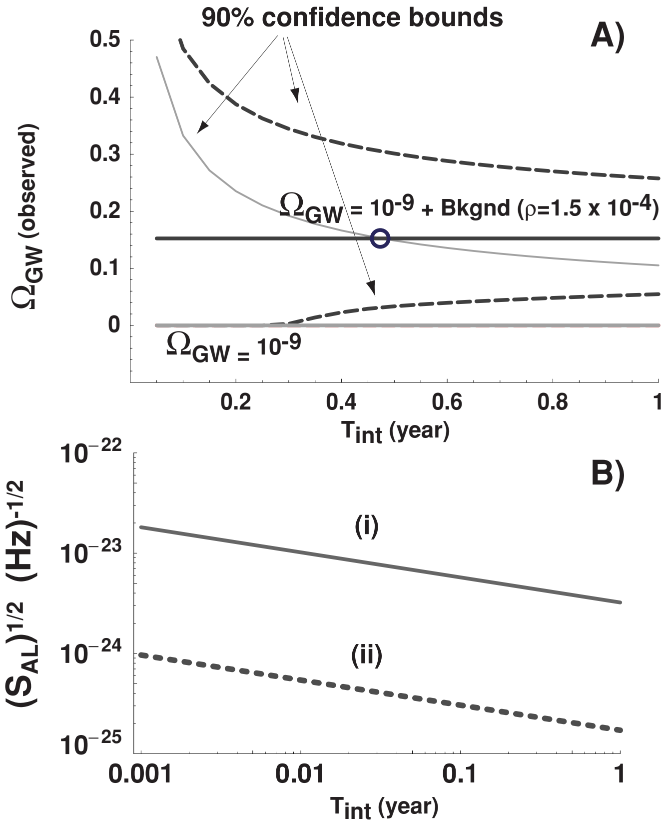

Figure 3. (A) The dashed lines mark the expected 90% confidence interval, as a function of the observing time, on a stochastic gravitational wave background when a much larger, but unaccounted for, correlated terrestrial noise source is present with a cross-spectral density amplitude just 10^{-4} the (geometric) mean noise power spectral density in the ALLEGRO and LIGO I detectors. The heavy solid line is the amplitude of the terrestrial noise, (mis)interpreted as a stochastic gravitational wave signal. Note how, after approximately 3 months, the observations are no longer consistent with stochastic gravitational wave amplitude significantly less than the amplitude of the correlated terrestrial noise. The thin line marks the upper limit on the stochastic signal, again as a function of time, when the modulation technique described in this paper is used to make the measurement. The measurement is no longer biased by the terrestrial noise and the upper limit is less than the correlated terrestrial noise amplitude, in this example, in 0.45 year. (B) The integration time needed for the upper limit, estimated by the modulation technique described here, to be less than the amplitude of the correlated terrestrial noise amplitude (i.e., to reach the crossing point marked by the bold circle in panel (A) as a function of the cross-spectral density. Curve (i) is for LIGO I + ALLEGRO; (ii) is for *advanced* LIGO + *upgraded* ALLEGRO.

confidence interval (following the construction of [19]) on Ω_{GW} when the observed C_0 is equal to its ensemble mean. The confidence interval is, in this case, *always* consistent with a stochastic gravitational wave background amplitude Ω_{GW} of 10^{-9}. Additionally, in less than 0.45 year this 90% bound limits the signal amplitude to less than the correlated background noise amplitude.

In this example the modulation technique described here provides, in approximately 0.45 year, a bound on the stochastic signal below the correlated noise background amplitude, interpreted as a stochastic gravitational wave signal. Figure 3(B) shows the integration period required, using this technique, to limit the stochastic background to an amplitude less than the correlated noise background as a function $|S_{\mathrm{AL}}(f_0)|^{1/2}$. The solid line corresponds to the LIGO I/ALLEGRO detector pair while the dashed line, labelled (ii), corresponds to the *advanced* LIGO/*upgraded* ALLEGRO detector pair. Since, with fixed detectors, the upper limit on the stochastic signal strength is always above the amplitude of the correlated noise, figure 3(B) shows that, after 1 year of observation with LIGO I + ALLEGRO, an unaccounted for correlation in the background at the level of $\sqrt{S_{\mathrm{AL}}(f)} \approx 3 \times 10^{-23} 1/\sqrt{\mathrm{Hz}}$ compromises the measurement. Similarly, after 1 year of observation with *advanced* LIGO + *upgraded* ALLEGRO, a correlated background with a strain spectral density of $\sqrt{S_{\mathrm{AL}}(f)} \approx 2 \times 10^{-25} 1/\sqrt{\mathrm{Hz}}$ will compromise a simple correlation measurement that does not account properly for environmental correlations.

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