

Figure 1: The noise $\tilde{h}(f)$ in several LIGO interferometers plotted as a function of gravity-wave frequency f , and compared with the estimated signal strengths $\tilde{h}_s(f)$ from various sources. The signal strength $\tilde{h}_s(f)$ is defined in such a way that, wherever a signal point or curve lies above the interferometer's noise curve, the signal, coming from a random direction on the sky and with a random orientation, is detectable with a false alarm probability of less than one per cent; see the text for greater detail and discussion.

of signals (a capability absent in the *initial* IFOs). The figure shows noise curves [square root of the spectral density of an IFO’s arm-length difference as a function of frequency, i.e. “strain per root Hertz” $\tilde{h}(f) \equiv \sqrt{S_{\Delta L/L}(f)}$] for two such optimizations: (i) Advanced IFOs optimized to search for waves from inspiraling neutron-star / neutron-star binaries (thick solid curve labeled **WB Sapphire** for “wide-band, Sapphire” and thin solid curve labeled WB Silica for “wide-band, Silica”). (ii) Advanced IFOs optimized to search for pulsars and low-mass X-ray binaries in the vicinity of $f = 600$ Hz (thick dotted curve labeled **NB Sapphire** and thin dotted curve labeled NB Silica, where NB means “narrow-band”). In general, the sapphire IFO is capable of somewhat lower noise than the fused-silica one, because of lower total thermal noise in its test masses. [The fact that the specific Silica IFO in Fig. 1 has better noise performance at low frequencies than the Sapphire one is a price that has been paid in the Sapphire IFO: the noise curve has been shaped to produce the lowest possible noise at its minimum, around 200 Hz, at the price of worsened noise below ~ 50 Hz.] In our discussion of the science, we will describe the performance of the sapphire IFOs, keeping in mind that, if we are forced to the fused-silica fallback, there will be a modest (a few tens of per cent) loss of signal to noise and corresponding factor ~ 2 loss of event rate for typical sources.

Figure 1 shows, along with the noise curves, the estimated signal strengths $\tilde{h}_s(f)$ for various sources. These signal strengths are defined in such a way [1] that the ratio $\tilde{h}_s(f)/\tilde{h}(f)$ is equal to the ratio of signal S to noise threshold T , rms averaged over source directions and orientations, $\tilde{h}_s(f)/\tilde{h}(f) = \langle S^2/T^2 \rangle^{1/2}$, with the threshold being that at which the false alarm probability is one per cent when using the best currently known, practical data analysis algorithm. (For broad-band sources, the algorithm is assumed to integrate over a bandwidth equal to frequency and to use the output from LIGO’s two 4km IFOs and one 2km IFO [2], thereby removing all non-Gaussian noise. For periodic sources such as spinning neutron stars, the algorithm uses data from only one 4 km IFO, usually narrow banded, the noise again is assumed Gaussian, and the signal is integrated for 10^7 sec, except in cases such as Low Mass X-Ray Binaries where there is little gain from integrating so long.) This definition of $\tilde{h}_s(f)$ means that, *wherever a signal point or signal curve lies above the IFO noise curve, the signal, coming from a random direction on the sky and with a random orientation, is detectable with a false alarm probability of less than one per cent.*

In this introductory section, we shall discuss the sources briefly, in turn, and then in subsequent sections we shall discuss them in greater detail, focusing on the likelihood of detection and the science we expect to extract from detected waves. Box 1 gives a brief summary of the various sources and their detectability.

The three arrowed, long-dashed lines in Fig. 1 represent the signal $\tilde{h}_s(f)$ from **neutron-star (NS) and black-hole (BH) binaries in the last few minutes of their inspiral**, assuming masses $M = 1.4M_\odot$ for each NS and $M = 10M_\odot$ for each BH. These sources are best searched for by the method of matched filters [4]. Using matched filters, LIGO’s initial IFOs can detect NS/NS inspirals (with a 1 per cent false alarm probability) out to a distance of 20 Mpc [top arrowed line]; the wide-band advanced IFOs can do so 15 times farther, out to 300 Mpc for NS/NS and out to 650 Mpc for NS/BH [bittin arrowed line]. (The wide-band IFOs can integrate up the signal over a wide range of frequencies, thereby achieving detection even though the signal curve in Fig. 1 is a little below the noise curve.) For BH/BH inspiral, the wide-band IFOs can see so far that cosmological effects are important. For definiteness, throughout this proposal we assume a Hubble expansion rate $H_o = 65$ km/s/Mpc, a cold-matter density 0.4 of that required to close the universe $\Omega_M = 0.4$, and a vacuum energy density (cosmological constant) 0.6 of closure $\Omega_\Lambda = 0.6$ [3]. Then the wide-band IFOs can see $(10M_\odot/10M_\odot)$ BH/BH inspirals out to a cosmological redshift $z = 0.4$. The binary inspiral rates at these advanced IFO distances are likely to be many per year; see Box 1. The middle arrowed line is the signal from BH/BH inspiral at 400Mpc, where the geometric mean of the event-rate estimates for BH/BH field binaries is three per year (third column

Box 1

Brief Summary of Detection Capabilities of Advanced LIGO Interferometers

• **Inspiral of NS/NS, NS/BH and BH/BH Binaries:** The table below [15, 2] shows estimated rates \mathcal{R}_{gal} in our galaxy (with masses $\sim 1.4M_{\odot}$ for NS and $\sim 10M_{\odot}$ for BH), the distances D_{I} and D_{WB} to which initial IFOs and advanced WB IFOs can detect them, and corresponding estimates of detection rates \mathcal{R}_{I} and \mathcal{R}_{WB} ; Secs. 1.1 and 1.2.

	NS/NS	NS/BH	BH/BH in field	BH/BH in clusters
$\mathcal{R}_{\text{gal}}, \text{yr}^{-1}$	$10^{-6} - 5 \times 10^{-4}$	$\lesssim 10^{-7} - 10^{-4}$	$\lesssim 10^{-7} - 10^{-5}$	$\sim 10^{-6} - 10^{-5}$
D_{I}	20 Mpc	43 Mpc	100	100
$\mathcal{R}_{\text{I}}, \text{yr}^{-1}$	$3 \times 10^{-4} - 0.3$	$\lesssim 4 \times 10^{-4} - 0.6$	$\lesssim 4 \times 10^{-3} - 0.6$	$\sim 0.04 - 0.6$
D_{WB}	300 Mpc	650 Mpc	$z = 0.4$	$z = 0.4$
$\mathcal{R}_{\text{WB}}, \text{yr}^{-1}$	1 – 800	$\lesssim 1 - 1500$	$\lesssim 30 - 4000$	$\sim 300 - 4000$

• **Tidal disruption of NS by BH in NS/BH binaries:** First crude estimates suggest WB IFOs can measure onset of disruption at 140Mpc well enough to deduce the NS radius to 15% accuracy (compared to current uncertainties of a factor ~ 2); see table above for rates; Sec. 1.3.

• **BH/BH merger and ringdown:** Rough estimates suggest detectability, by WB IFOs out to the cosmological distances shown in Fig. 2(b); rates for BH/BH total mass $\sim 20M_{\odot}$ are in table above; rates for much larger masses are unknown; Sec. 1.4.

• **Low-Mass X-Ray Binaries:** If accretion's spin-up torque on NS due is counterbalanced by gravitational-wave-emission torque, then WB IFOs can detect Sco X-1, and NB IFOs can detect ~ 6 other known LMXB's; Secs. 1.1, 1.5.

• **Fast, Known Spinning NS's (Pulsars with pulse frequency above 100 Hz):** Detectable by a advanced NB IFO in 3 months' integration time, if NS ellipticity is $\epsilon \gtrsim 2 \times 10^{-8} (1000\text{Hz}/f)^2 (r/10\text{kpc})$, where f is gravity wave frequency (twice the pulsar frequency) and r is distance; actual ellipticities are unknown, but plausible range is $\epsilon \lesssim 10^{-6}$.

• **Fast, Unknown Spinning NS's:** Unknown frequency wandering and doppler shifts degrade the detectable ellipticity ϵ by a factor of a few to ~ 15 , so detection with a NB IFO requires $\epsilon \gtrsim (0.6 \text{ to } 3) \times 10^{-5} (100\text{Hz}/f)^2 (r/10\text{kpc})$; Secs. 1.1 and 1.5.

• **R-Modes in Newborn NS's with Initial Spin Rates Faster than ~ 100 Hz:** Estimates suggest detectability out to $\sim 15\text{Mpc}$ (the Virgo cluster) with WB or NB IFOs; NS birth rate in Virgo is a few per year, but initial spins are unknown; Secs. 1.5 and 1.6;

• **Centrifugally Hung-Up Proto Neutron Stars in White-Dwarf Accretion-Induced Collapse and in Supernovae:** Dynamics of star very poorly understood; if instability deforms star into tumbling bar, may be detectable by WB IFOs to ~ 20 Mpc (the Virgo Cluster), and possibly farther; event rates uncertain but could be enough for detection; Sec. 1.6.

• **Convection of Supernova Core:** May be detectable by WB IFOs, via correlations with neutrinos, for supernovae in our Galaxy and possibly Magellanic Clouds; Sec. 1.6

• **Gamma Ray Bursts:** If triggered by NS/BH mergers, a few per year could be detectable by WB IFOs; if none are seen individually, statistical studies could nevertheless confirm gravity-wave emission by the gamma-burst triggers; Sec. 1.7.

• **Stochastic Background:** Detectable by cross correlating Hanford and Livingston 4km detector outputs, if $\Omega = (\text{gravitational-wave energy in } \Delta f \sim f \sim 40 \text{ Hz}) \gtrsim 8 \times 10^{-9}$; there are many possible sources of such waves in very early universe, all very speculative; Sec. 1.8.

of table in Box 1).

The **tidal disruption of a NS by its BH companion** at the endpoint of NS/BH inspiral should produce gravitational waves that carry detailed information about the NS structure and equation of state. The advanced IFOs may detect these waves and extract their information; see Sec. 3. This tidal disruption is a promising candidate for the **trigger of gamma-ray bursts**, as is the final merger of the two NS's in a NS/NS binary. A gamma-burst / gravitational-wave coincidence would be of great value in revealing the nature of gamma-burst sources; see Sec. 7

For BH/BH binaries much heavier than $\sim 10M_{\odot}/10M_{\odot}$, most of the gravitational signal is likely to come from the **black holes' merger and the vibrational ringdown of the final black hole**, rather than from the inspiral. Rough estimates discussed in Sec. 4 suggest that, if the holes are rapidly spinning (within a few per cent of the fastest spin allowed, i.e. $a/m \gtrsim 0.98$ in the jargon of relativity theorists), then the wide-band IFOs can see the merger waves from two $20M_{\odot}$ holes out to $z = 1$ and two $50M_{\odot}$ holes out to $z = 2$; see the down-sloping, non-arrowed, dashed lines in Fig. 1. The event rates at these distances could well be many per year, and the waves from such mergers will carry rich physical and astrophysical information; see Sec. 4.

The triangles, star, large dots, and up-sloping short-dashed lines in Fig. 1 represent signals from **slightly deformed, spinning neutron stars**. The most interesting of these is a class of objects called **low-mass X-ray binaries (LMXB's)**. These are neutron stars that are being torqued by accretion from a companion, but that seem to be locked into spin periods in the range $\sim 300 - 600$ revolutions per second. The most plausible explanation for this apparent locking is that the accretion is producing an asymmetry that radiates gravitational waves, which torque the star's spin down at the same rate as accretion torques it up [5, 6]. Assuming this to be true, one can deduce an LMXB's wave strength \tilde{h}_s from its measured X-ray flux and its spin frequency [with the frequency inferred, sometimes to within a few Hz but not better, from nearly coherent oscillations (NCOs) in type-I X-ray bursts, or less reliably from frequency splittings of quasiperiodic (QPO) X-ray oscillations[7].] The spin frequency, and thence the gravitational-wave frequency, will wander somewhat due to fluctuations in accretion (which can be estimated by monitoring the X-ray flux) and due to poorly known orbital parameters. As a result, in searching for an LMXB's waves one can only perform coherent integrations for about 20 days; thereafter, one must stack the signals incoherently, allowing for unknown shifts of the wave frequency [8]. When one uses this "stack-slide" method of data analysis, the resulting signal strengths improve only slightly for integration times longer than 20 days [8].

Assuming 20 days of integration using a single 4-km IFO, the estimated signal strengths and frequencies for the strongest known LMXB's are shown by the big dots and the star in Fig. 1. The estimated strengths assume a steady-state balance of accretion torque by gravitational-wave torque, which is expected if density inhomogeneities produce the gravitational waves [5]. However, if sloshing fluid motions ("r-modes") produce the waves [9], then temperature-dependent viscosity could trigger long-term (\gtrsim a few hundred year) oscillations in the gravitational-wave emission, with short epochs of enhanced emission and long epochs of little or no emission [10]; a recent estimate [11] suggests a factor ~ 10 enhancement of wave strength above those shown in Fig. 1 for $\sim 10\%$ of the time. The estimated wave frequencies are also somewhat uncertain: The figure assumes density inhomogeneities, which means the wave frequency is twice the NS rotation frequency; if r-mode sloshing produces the waves, then they will be at $\simeq 4/3$ the rotation frequency (~ 400 Hz rather than 600 Hz). For every LMXB in Fig. 1 except the weakest one, the estimated rotation frequency is based on QPO splittings rather than NCOs, which means that about half of the frequencies might be double these estimates: ~ 1200 Hz for density inhomogeneities and ~ 800 Hz for r-mode sloshing, rather than ~ 600 Hz. Doubling the frequency above 600 Hz reduces the emission amplitude by a factor 2 (at fixed X-ray flux assuming a steady-state torque balance), and increases the amplitude noise in a advanced NB IFO by a factor $\simeq \sqrt{2}$. As a result, at ~ 1200 Hz frequency, Sco X-1 would still be readily detectable in a NB IFO but not in a WB IFO,

and the strongest of the other LMXB's would be marginal. By contrast, if the frequencies are ~ 600 Hz or ~ 400 Hz and the waves are in a steady state, then Sco X-1 should be very easily detected by a WB IFO, and several LMXB's should be detectable by narrow banding (NB).

The large number of caveates and uncertainties in these LMXB estimates (which, however, generally leave at least Sco X-1 detectable) illustrate a very important point: Gravitational-wave observations have the potential to probe a rich range of complex physical processes in neutron stars; see Sec. 5 for further discussion.

The advanced IFOs can perform interesting searches for waves from **known radio pulsars**, such as the **Crab and Vela** for which the current upper limits (based on the pulsars' observed spindown rates) are shown as triangles. A Crab search, using coherent integrations based on the star's observed (slightly wandering) rotation period, could improve the limit on the Crab's wave amplitude by a factor 100 and constrain the star's gravitational ellipticity to $\epsilon \lesssim 7 \times 10^{-6}$ — which is approaching the realm of physically plausible ellipticities, $\epsilon \lesssim 10^{-6}$ [6].

More interesting will be searches for waves from **known, fast pulsars**, since the signal strength scales as $\tilde{h}_s \propto \epsilon f^2 / r$ (where r is distance to the source). The up-sloping short-dashed lines in Fig. 1 show some examples of signal strengths. With a narrow-band IFO tuned to the vicinity of such a fast pulsar, the waves would be detectable when $\epsilon \gtrsim 2 \times 10^{-6} (100\text{Hz}/f)^2 (r/10\text{kpc})$, which is in the realm of plausible ellipticities for pulsars throughout our galaxy so long f exceeds 100 Hz, i.e. the spin frequency exceeds 50 Hz.

Also of great interest will be searches for **previously unknown spinning neutron stars**, for which the signal strengths \tilde{h}_s will be reduced by a factor of a few to ~ 15 by the lack of prior information about the frequency and its evolution and the direction to the source (which determines the time-evolving doppler shift produced by the earth's motion) [12, 8]. A tunable, narrow-band IFO will be crucial to such searches. One can search more deeply by a factor of several, using a narrow-banded IFO that dwells on a given frequency and its neighborhood for a few days or weeks and then moves on to another frequency, than using a broad-interferometer that collects signal at all frequencies simultaneously for a year. Such searches will be in the band of physically plausible ellipticities, for stars throughout our galaxy, if $f \gtrsim 200$ Hz (spin frequency above ~ 100 revolutions per second).

One can search for a **stochastic background** of gravitational waves by cross correlating the outputs of LIGO's two 4 km detectors [13]. For such a search the signal strengths $\tilde{h}_s(f)$ are shown in Fig. 1 as downward-sloping dotted lines, assuming a cross-correlation of 4 months of (not necessarily contiguous) data, and isotropic waves. The lines are labeled by the waves' energy density Ω in a bandwidth equal to frequency and in units of the density to close the universe, $\Omega = (f dE_{\text{GW}}/df) / \rho_{\text{closure}}$. Unfortunately, the frequency of optimal *a priori* sensitivity, $f \sim 70$ Hz (where the noise curve is parallel to the dotted Ω lines), is near the center of a dead band for LIGO. This dead band arises from the fact that $1/70\text{Hz} \simeq 14$ ms is about the round-trip gravity-wave travel time between the two LIGO sites [14]. The result is a net debilitation of the stochastic background sensitivity by a factor of a few: the initial IFOs can detect an isotropic background with Ω down to $\sim 10^{-5}$, while advanced, wide-band IFOs can reach down to $\Omega \sim 5 \times 10^{-9}$, and a factor ~ 2 lower than this if it is reoptimized for low frequencies. These are interesting sensitivities, able to test a wide range of speculations about the physics of the early universe and perhaps even detect waves. Such a detection could have profound implications for physics and cosmology; see Sec. 8.

Other waves that advanced IFOs in LIGO will seek and may detect are those from **the stellar core collapse that triggers supernovae** and the **boiling of the nascent neutron star** and the endpoint of that collapse (Sec. 6), **accretion induced collapse of white dwarfs** (Sec. 6), and **totally unknown sources** (Sec. 9).

We turn, now, to a discussion of each of the above sources, focusing especially on event rate estimates and the information that the waves may bring.

2 Inspiring NS and BH Binaries with $M_{BH} \lesssim 10M_{\odot}$

As we have discussed, wide-band advanced LIGO IFOs can detect the waves from NS/NS inspirals out to 300Mpc, NS/BH out to 650 Mpc, and BH/BH out to $z = 0.4$. The event rates out to these distances can be estimated from observational data in our own galaxy [15], and an extrapolation out through the universe based on the density of massive stars (which can be deduced by several different methods) [16]. The resulting rates are quite uncertain but very promising; see Box 1.

For NS/NS binaries, the event rate in our galaxy is constrained by the results of radio astronomers' searches for binary pulsars that will merge, due to gravitational radiation emission, in less than the age of the universe, and by other aspects of pulsar searches [15]. The resulting constraints, $10^{-6}/\text{yr} \lesssim \mathcal{R}_{\text{gal}} \lesssim 5 \times 10^{-4}/\text{yr}$, extrapolate to a NS/NS event rate for advanced IFOs between 1/yr and 800/yr. Searches in our galaxy for NS/BH binaries in which the NS is a pulsar have failed to find any as yet, so we must turn to much less reliable estimates based on "population synthesis" (simulations of the evolution of a population of progenitor binary systems to determine the number that make NS/BH binaries compact enough to merge in less than the universe's age). Population synthesis gives a NS/BH event rate in our galaxy in the range $\sim 10^{-7}/\text{yr} \lesssim \mathcal{R}_{\text{gal}} \lesssim 10^{-4}/\text{yr}$ [21], though it is possible the rate could be even less than this (hence the $\lesssim 10^{-7}/\text{yr}$ in Box 1). Extrapolating out into the universe, we find a NS/BH rate in WB advanced IFOs between $\lesssim 1/\text{yr}$ and about 1500/yr. A similar analysis for BH/BH binaries based on population synthesis (third column of the table in Box 1) gives a rate between $\lesssim 30/\text{yr}$ and $\sim 4000/\text{yr}$ [17]. Population synthesis ignores the likely role of globular clusters and other types of dense star clusters as "machines" for making BH/BH binaries [18]: single black holes, being heavier than most stars in a globular, sink to the center via tidal friction, find each other, and make binaries; then the BH/BH binaries get "hardened" (made more compact) by interaction with other black holes, reaching sizes where gravitational-wave emission will cause them to merge in less than the age of the universe; and further interactions will often eject the BH/BH binaries from the globular, to interstellar space where they merge. Simulations [18] suggest that each dense cluster will make a number of such BH/BH binaries, and extrapolations into the universe predict an event rate in WB advanced IFOs between ~ 300 and $\sim 4000/\text{yr}$ — though the uncertainties are probably larger than these numbers from the literature suggest [18, 15].

These event rates are very encouraging. They make it seem quite likely that the advanced IFOs will observe tens to thousands of BH and NS inspirals per year, while the initial IFOs will be lucky to observe ~ 1 per year.

The observed inspiral waves will last for between ~ 1000 and $10,000$ cycles depending on the binary's masses, and will carry detailed information about the binary and about general relativistic deviations from Newtonian gravity. This information can be extracted with good precision using the method of matched filters. Specifically (denoting by $M = M_1 + M_2$ the binary's total mass and $\mu = M_1 M_2 / M$ its reduced mass):

(i) The binary's chirp mass $M_c \equiv \mu^{3/5} M^{2/5}$ will typically be measured, from the Newtonian part of the signal's upward frequency sweep, to $\sim 0.04\%$ for a NS/NS binary and $\sim 0.3\%$ for a system containing at least one BH. (ii) If we are confident (e.g., on a statistical basis from measurements of many previous binaries) that the binary's spins are a few percent or less of the maximum physically allowed, then the reduced mass μ will be measured to $\sim 1\%$ for NS/NS and NS/BH binaries, and $\sim 3\%$ for BH/BH binaries. (iii) Because the frequency dependences of the (relativistic) μ effects and spin effects are not sufficiently different to give a clean separation between μ and the spins, if we have no prior knowledge of the spins, then the spin/ μ correlation will worsen the typical accuracy of μ by a large factor, to $\sim 30\%$ for NS/NS, $\sim 50\%$ for NS/BH, and a factor ~ 2 for BH/BH. These worsened accuracies should be improved significantly (though we do not yet know how much) by waveform modulations due to spin-induced precession of the orbit [20], and even without modulational information, a certain

combination of μ and the spins will be determined to a few per cent. (iv) The distance to the binary (angle-effective distance at cosmological distances) can be inferred, from the observed waveforms, to a precision $\sim 1/\rho \lesssim 10\%$, where ρ is the amplitude signal-to-noise ratio in the total LIGO network (which must exceed about 8 in order that the false alarm rate be less than the threshold for detection). (v) With the aid of VIRGO and/or other international partners, the location of the binary on the sky can be inferred, by time of flight between the detector sites, to a precision of order one degree on the sky.

Advanced LIGO will likely produce a catalog of hundreds or thousands of binary inspirals and their inferred parameters; this catalog will be a valuable data base for observational astronomy and cosmology.

Important examples of the general relativistic effects that can be detected and measured with precision, in the inspiral waves, are these: (i) As the waves emerge from the binary, some of them get backscattered one or more times off the binary’s spacetime curvature, producing wave *tails*. These tails act back on the binary, modifying its radiation reaction force and thence its inspiral rate in a measurable way. (ii) If the orbital plane is inclined to one or both of the binary’s spins, then the spins drag inertial frames in the binary’s vicinity (the “Lense-Thirring effect”), this frame dragging causes the orbit to precess, and the precession modulates the waveforms [20]. This precession and modulation should be very strong in a significant fraction of NS/BH binaries [21].

3 Tidal Disruption of a NS by a BH: Measuring the Nuclear Equation of State

As a NS/BH binary spirals inward, its NS experiences ever increasing tidal forces from the BH’s gravitational field (its spacetime curvature). In many cases these tidal forces may tear the NS apart before it begins its final, quick plunge into the hole’s horizon. The gravitational waves from this tidal disruption and from the termination of inspiral should carry detailed information about the NS’s equation of state (the equation of state of bulk nuclear matter at ~ 10 times the density of an atomic nucleus). The disruption waves lie largely in the frequency band $\sim 300\text{Hz} \lesssim f \lesssim 1000\text{ Hz}$, where the wide-band, advanced IFOs have good sensitivity, and where narrow-band or some other optimized IFO can do even better. This suggests that the advanced IFOs may be able to extract new information about the nuclear equation of state from the tidal-disruption waves. A first, crude estimate [22] suggests, for example, that for NS/BH binaries at 140 Mpc distance (where the event rate could be a few per year; see the table in Box 1), tidal-disruption observations may enable the NS radius R to be measured to a precision $\sim 15\%$, by contrast with its present uncertainty (for fixed NS mass) of about a factor 2. From the measured radii would follow the desired equation-of-state information. Detailed numerical-relativity simulations will be required to firm up this estimate, and will be essential as a foundation for interpreting any tidal-disruption waves that are observed.

The merger waves from NS/NS binaries, by contrast with NS/BH, are likely to lie outside the band of good advanced-IFO sensitivity — at frequencies $f \gtrsim 1500\text{ Hz}$. However the onset of NS/NS merger, triggered by a plunge of the two NS’s toward each other, may produce a strong “cliff” in the waves’ spectrum, in a range $f \sim 400 - 1000\text{ Hz}$ of good sensitivity [23], and by measuring the cliff frequency we may learn about the nuclear equation of state.

4 BH/BH Mergers and Ringdown: Observing The Nonlinear Dynamics of Spacetime Curvature

Black holes are made not from ordinary matter, but rather from spacetime curvature. For a single, quiescent black hole, this curvature has a rich structure: a trumpet-horn-like curvature of space, a tornado-like

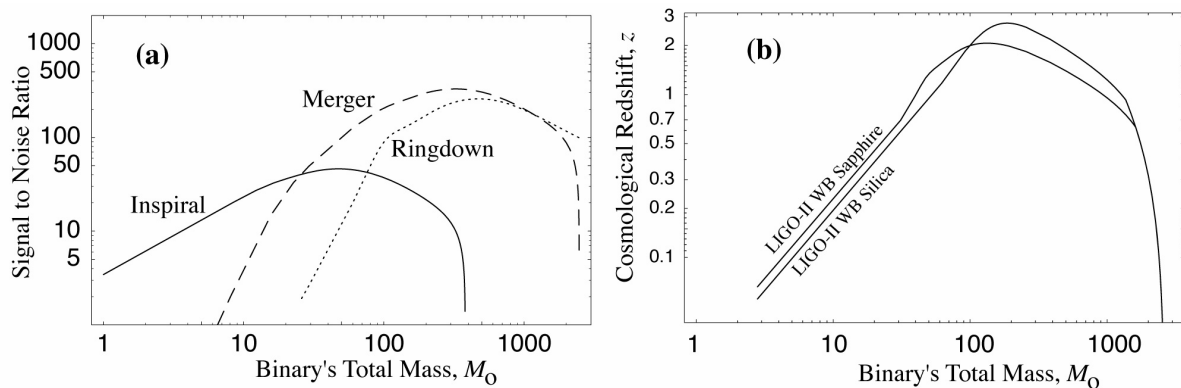


Figure 2: Characteristics of the gravitational waves from the inspiral, merger, and final ringdown of a BH/BH binary with equal masses $M_1 = M_2 = M/2$ and large spins, as computed by combining the source estimates of Ref. [24] with the wide-band noise curves of Fig. 1. (a) The amplitude signal to noise ratio, as a function of the binary’s total mass M for a binary at 300 Mpc distance, and for the three phases of the binary’s evolution: inspiral, merger, and ringdown. (b) The cosmological redshift to which the binary can be seen by wide-band, advanced IFOs, as a function of its total mass M .

whirling of space around the hole’s horizon, and a warpage of time that becomes so strong near the horizon that, in a certain sense, the flow of time grinds to a halt there. Even richer (and as yet only crudely understood) will be the spacetime curvature of a BH/BH binary, as its holes near each other and merge. Among other things, it should entail tornado-like whirlings of space around each of the two black holes caused by their spins, and a third whirl of space around the binary as a whole, caused its orbital angular momentum. In the half dozen years between now and the advanced IFOs’ first operation, numerical relativists will perfect their techniques for simulating such BH/BH mergers, in preparation for comparison with LIGO’s observations. That comparison should bring a clear understanding of the highly nonlinear dynamics of warped spacetime.

The numerical simulations are not yet sufficiently advanced to tell us much about the predicted waves, so we must rely, for now, on crude insights from perturbation theory. Those insights suggest that, if the holes are rapidly spinning, then the wave strengths, quantified by the net signal to noise ratio in wide-band advanced IFOs, are as shown in Fig. 2(a). For total mass $M \gtrsim 30M_\odot$, the merger waves are stronger than the inspiral waves; for $M \sim 100$ to $2000M_\odot$, the ringdown waves are comparable to the merger. Above $M \sim 2000M_\odot$, the waves are at too low frequency to be seen by the advanced IFOs. The distance to which the waves can be detected, expressed in terms of cosmological redshift (assuming $H_o = 65$ km/s/Mpc, $\Omega_M = 0.4$, $\Omega_\Lambda = 0.6$) is shown in Fig. 2(b). For $M \sim 100$ to $1000M_\odot$, the distance exceeds $z = 1$. We know little about what the event rate may be, when $M \gtrsim 40M_\odot$. It is unlikely that main sequence binaries form BH/BH binaries this massive; but they may well be formed by mergers of smaller holes in galactic nuclei in sufficient numbers to produce many mergers per year at $z \sim 1$ [25]. LIGO’s observations may thereby bring us major new insights into the physics of galactic nuclei.

5 Spinning Neutron Stars

In Sec. 1.1 we discussed the signal strengths and search strategies for spinning neutron stars. As we saw, tunable, narrow-band IFOs have good possibilities to detect such stars’ waves, especially for LMXB’s.

For any detected waves, the features that can be measured include their frequency (or frequencies if several lines are seen), the time evolution of the frequencies, the wave amplitudes and their evolu-

tion, and correlations with electromagnetic observations of the same source (if any). These quantities are governed by a wide variety of rich physics in the neutron-star interior, and by the direction to the source. For example: (i) The modulation of the frequency due to the earth’s motion can reveal the source direction to an accuracy of order one arc second [26], enabling identification with electromagnetically observed objects. (ii) The ratio of the gravitational-wave frequency to the electromagnetically observed rotation frequency can reveal the nature of the inhomogeneities that emit the waves: for density inhomogeneities frozen into a spinning neutron star, e.g. those of a deformed crust or core, $f_{\text{GW}} = 2f_{\text{rot}}$. For a free-precessing (wobbling) star with frozen-in inhomogeneities, there will be a second line at $f_{\text{GW}} = f_{\text{rot}} + f_{\text{precess}} \simeq f_{\text{rot}}$. For r-mode oscillations of a star’s fluid mantle driven by gravitational radiation reaction [27], $f_{\text{GW}} \simeq \frac{4}{3}f_{\text{rot}}$. (iii) Depending on the nature of the inhomogeneities, each line’s frequency evolution and amplitude can be influenced by a wide variety of different physical processes in the neutron star, e.g., crust physics (thickness, elasticity moduli, breaking strain, ...), crust-core mechanical and thermal couplings, superfluid transition temperature, magnetic field strength, viscosity, etc. (iv) Correlated gravitational and electromagnetic observations after a frequency “glitch” might be particularly informative about the neutron-star physics. (v) For precessing pulsars, the gravitational-wave observations can determine the orientation of the pulsar angular momentum vector relative to the line of sight, and this can be used to test models of the pulsar emission mechanism, which depend on the angle between the rotation axis and the pulsar beam [28].

6 Supernovae and Accretion-Induced Collapse of White Dwarfs

Type-II supernovae are triggered by the violent collapse of a stellar core to form a NS or BH. The details of the collapse and NS or BH formation are poorly understood, and the optical display sheds little light on them because the photons are emitted $\gtrsim 100,000$ km from the trigger and hours or more afterward. Neutrino observations provide one window on the trigger, with emphasis on the thermal structure of the collapsed core. Gravitational waves provide a complementary window, with emphasis on the compactness and density of the core, on its asymmetry, and on whether the final remnant is a BH or a NS.

Observations show that many type-II supernovae produce neutron stars and give them kicks of magnitude as large as ~ 1000 km/s. Indeed, about half of all radio pulsars are born with a kick larger than 500 km/s [29]. The observed kicks suggest that at least some newborn proto-neutron-stars may be strongly asymmetric, perhaps due to fast rotation, and therefore could produce significant gravitational radiation. Rapidly rotating proto-neutron-stars may also be produced by the accretion-induced collapse of white dwarf stars (AIC) in some cases, depending on the white-dwarf composition, central density and accretion rate [30].

If the newborn proto-neutron-star in an AIC or supernova is spinning so fast that it hangs up centrifugally at a radius large compared to that of the final NS, then it may be dynamically (or at least secularly) unstable to deforming into a bar-shaped object that tumbles end-over-end, emitting gravitational waves in LIGO’s band of good sensitivity [31]. Recent simulations suggest that the bar will be long-lived, rather than just wrapping itself up into an axisymmetric shape and disappearing [32]. The simulations show that the waves from such a bar may sweep upward in frequency, due to a gradual shrinkage of the proto-neutron-star, or may sweep downward due to development of “Dedekind-like” internal circulation [33]. The discovery of the waves from such a proto-neutron-star and observations of their frequency evolutions (which should mirror the bar’s evolution) would teach us much. Though the strengths of the waves and the best signal processing techniques and thresholds are all ill-understood, a rough estimate [34] suggests that the advanced IFOs’ range for detection might be the distance of the VIRGO cluster of galaxies (about 15 Mpc) and conceivably significantly larger, suggesting event rates that could be some

per year but might be far less.

Even if the proto-neutron-star does not hang up centrifugally, if it is born spinning faster than ~ 100 revolutions per second, then its r-modes of oscillation (oscillatory, quadrupolar circulation patterns) may be driven unstable by gravitational radiation reaction. Estimates suggest that the r-modes may radiate for some months, gradually slowing the star’s rotation to ~ 100 revolutions per second, and their waves may be detectable by LIGO’s wide-band or narrow-band IFOs out to the Virgo Cluster; cf. Sec. 5. Theoretical astrophysicists and relativists are struggling to understand the physics that governs and influences the r-modes [35], but it is so complex that a reliable understanding will likely only come from LIGO’s observations of (or failure to observe) the r-modes’ waves.

Numerical models of supernovae suggest that, even if it is slowly rotating, the newborn proto-neutron-star will be convectively unstable, and that the gravitational waves from the convective overturn in the first \sim one second of the star’s life may be detectable throughout our galaxy and its orbiting companions, the Magellanic Clouds [36]. Although the supernova rate is low in our galaxy and its companions ($\lesssim 1/30$ yrs), one observed event could be very valuable scientifically: The bulk of the supernova’s neutrinos are thought to come from the same convecting material as produces the gravitational waves, so there should be correlations between the neutrinos and the waves, which could teach us much about the proto-neutron-star’s dynamics.

If the advanced WB IFOs detect no gravitational waves from a supernova at distance r , during a time T preceding the beginning of the optical outburst or over a time T during a neutrino outburst, then one can thereby place a limit on the emitted gravitational-wave energy. In the band from ~ 100 to 300 Hz this limit is $\Delta E_{\text{GW}} \lesssim 0.05(r/15\text{Mpc})^{1/2}(T/1\text{h})^{1/2}M_{\odot}$. In the case of a neutrino-emitting supernova in our own galaxy, with $T \sim 1$ sec, this limit is very impressive: $\sim 10^{-9}M_{\odot}$.

7 Gamma Ray Bursts

Cosmic gamma-ray bursts, observed \sim once per day by detectors on spacecraft, are believed to arise from shocks in a relativistic fireball, generated by rapid accretion onto a newly formed black hole [37]. The gamma-ray production must occur rather far from the BH ($10^{13} - 10^{15}$ cm), making it difficult to test the BH involvement by conventional astronomical observations. Gravitational waves are more promising: The BH and its accretion flow are thought to form violently, by the collapse of a massive star (perhaps initiated by merger with a companion) [“a hypernova”], or by the merger of a binary made of compact objects: a NS/NS binary, a BH/NS binary, a BH/white-dwarf or a BH/He-core binary [37]. Statistical evidence points to several subclasses of gamma-ray bursts [38], so several of these “triggers” might occur in nature.

Each of these gamma-burst triggers should emit strong gravitational waves that carry detailed information about its source. NS/BH mergers and hypernovae are promising trigger candidates for *long bursts* (duration $\gtrsim 2$ s), for which a number of distances have been measured via afterglows. From the measured distances and the distribution of gamma-burst fluences, one can estimate a long-burst event rate of ~ 1 per year out to 650 Mpc — a distance at which advanced IFOs should be able to detect NS/BH inspirals but will likely not be able to detect the gravitational waves from hypernovae. NS/NS mergers are a promising trigger candidate for *short bursts* (duration $\lesssim 2$ s). The distances to the short bursts are unknown (no after glows have been detected), but they could well be near enough (event rate $\gtrsim 1/\text{yr}$ at 300 Mpc) for the advanced IFOs to detect the inspiral waves from a NS/NS trigger.

If gravitational waves are detected from one or more gamma-burst triggers, the waves will almost certainly reveal the physical nature of the trigger. Moreover, by comparing the arrival times of the gravitational waves and the earliest gamma rays, it should be possible to measure the relative propagation speeds of light and gravitational waves to an accuracy $\sim 1 \text{ sec}/10^{10} \text{ yr} \sim 10^{-17}$.

If no gravitational waves are detected from any individual gamma burst, the correlation between gamma bursts and gravitational waves might nevertheless be established by statistical studies of the advanced IFOs’ gravitational-wave data in narrow time windows preceeding the gamma bursts [39].

8 Sources of Stochastic Background

The most plausible sources of a stochastic gravitational-wave background in LIGO’s frequency band are processes in the very early universe. The current best limit on the strength of such waves is $\Omega \lesssim 10^{-5}$; a wave energy larger than this would have caused the universe to expand too rapidly through the era of primordial nucleosynthesis (universe age \sim a few minutes), thereby distorting the universal abundances of light elements away from their observed values. LIGO’s advanced interferometers would improve on this current limit by a factor $\sim 10,000$, to $\Omega \simeq 5 \times 10^{-9}$ — an improvement enabling LIGO to test a number of current speculations about the very early universe. A positive detection would have profound consequences.

Inflationary models of the early universe predicts that vacuum fluctuations, created in the Planck era when the universe was being born, should have been parametrically amplified, during the first $\sim 10^{-25}$ sec of the universe’s life, to produce a stochastic gravitational wave background in the LIGO band. Unfortunately, if “standard” inflation theory is correct, then the amplified waves are much too weak for LIGO to detect, $\Omega \lesssim 10^{-15}$ [40]. The most plausible modifications of standard inflation push Ω downward from this [40], but some less plausible modifications push it upward, to the point of detectability [41].

The first, tentative efforts to combine superstring theory with inflationary ideas have produced a new description of the very early universe called the “pre-big-bang model”, in which string effects cause the gravitational-wave spectrum to rise steeply at high frequencies — most likely at frequencies above LIGO’s band, but quite possibly in or below that band [42]. The result could be waves strong enough for LIGO to detect. A non-detection would significantly constrain the pre-big-bang model.

There are a wide variety of postulated mechanisms that could have produced strong gravitational waves, with wavelengths of order the horizon size, at various epochs in the very early universe. Those waves (if any) produced at (universe age) $\sim 10^{-25}$ sec, corresponding to (universe temperature T) $\sim 10^9$ GeV, would have been redshifted into the LIGO band today and might be detectable. The temperature (energy) region $\sim 10^9$ GeV is *tera incognita*; LIGO’s advanced detectors will provide our first opportunity for a serious experimental exploration of it. Among the speculated wave-generating mechanisms that could operate there, and that LIGO could constrain (or discover!), are these:

- A first-order phase transition in the states of quantum fields at $T \sim 10^9$ K. Such a phase transition would nucleate bubbles of the new phase that expand at near the speed of light and collide to produce gravitational waves; and their collisions would also generate turbulence that radiates waves. If the transition is strongly first order, the waves would be strong enough for LIGO’s advanced IFOs to detect. [43]
- Goldstone modes (coherent, classical excitations) of scalar fields that arise in supersymmetric and string theories. If strongly excited, these modes will entail coherent flows of energy that radiate gravitational waves strong enough for detection. [44]
- Coherent excitations of our 3+1 dimensional universe, regarded as a “brane” (defect surface) in a higher dimensional universe. The excitations could be of a “radion” field that controls the size or curvature of the additional dimensions, or they could be of the location and shape of our universe’s brane in the higher dimensions; in either case, if there is an equipartition of energy between these

excitations, in the very early universe, and other forms of energy, then the excitations will produce gravitational waves easily strong enough for detection by LIGO’s advanced IFOs. LIGO would thereby probe one or two additional dimensions of size or curvature length $\sim 10^{-10} - 10^{-13}$ mm; by contrast, LISA’s lower-frequency observations would probe lengths $\sim 1 - 10^{-5}$ mm. If the number of extra dimensions is larger than 2, the probes reach to much smaller scales [45]

Cosmic strings (not to be confused with superstrings), produced in the early universe, were once regarded as candidates for seeding galaxy formation, but recent cosmological observations have ruled them out as seeds. Nevertheless, it remains possible that a network of vibrating cosmic strings too weak to seed galaxy formation was formed in the early universe. LIGO can search for the presence of such a network in two ways: (i) Via the stochastic background of gravitational waves that the strings’ vibrations produce; this background would be strong enough for the advanced IFOs to detect if the strings’ mass per unit length is $\gtrsim 10^{-8}$. [46] (ii) Via occasional non-Gaussian, strong bursts (“spikes”) of gravitational waves produced by kinks (cusps) in the string shapes. These bursts could be detectable even if the accompanying stochastic background is too weak for detection. [47]

9 Unknown Sources

Each of these cosmological speculations is plausible, though not highly likely. Perhaps their greatest value is to remind us of how terribly ignorant we are of physics and astrophysics in the domain that LIGO’s advanced IFOs will probe. Our ignorance may well be even greater than that of the pioneering radio astronomers of 1940 and X-ray astronomers of 1960; and as there, so also here, the first waves to be discovered may well be from sources that were previously unknown. Advanced LIGO could bring us a revolution of insights into the universe, and even into gravity, comparable to the revolutions wrought by radio and X-ray astronomy.

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- [2] If, in its advanced (“LIGO-II”) incarnation, the 2km interferometer is replaced by a narrow-banded 4km interferometer, then for the most important wide-band sources, inspiraling binaries, the 3-IFO threshold will be lowered (improved) slightly and correspondingly the distances to which the sources can be seen will be increased slightly beyond those in Box 1. If the 2km interferometer is replaced by a wide-band 4km interferometer, then the threshold will be lowered by a factor 1.16, the binaries’ observable distances will be increased correspondingly by 1.16, and the predicted event rates will go up by $1.16^3 \simeq 1.5$.
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- [13] See, e.g., B. Allen and J.D. Romano, Phys. Rev. D **59**, 102001 (1999) and references therein.
- [14] See the discussion of the overlap reduction function in Sec. III.B of Ref. [13], especially Fig. 2.
- [15] The NS/NS numbers in this table are taken from V. Kalogera, R. Narayan, D. Spergel, and J.H. Taylor, Astrophys. J., submitted, astro-ph/0012038 — as augmented slightly by a new channel for NS/NS formation analyzed by K. Belczynski and V. Kalogera, Astrophys. J. Lett submitted, astro-ph/0012172 — and with adjustment of \mathcal{D}_{WB} from 350 Mpc (appropriate for three 4 km interferometers) to 300 Mpc (for one 2 km and two 4 km interferometers; see Ref. [2]). The NS/BH and BH/BH numbers are based on Kalogera’s survey of all estimates of \mathcal{R}_{gal} [V. Kalogera, in *Gravitational Waves, Proceedings of the Third Edoardo Amaldi Conference*, AIP Conference Proceedings Vol. 523, ed. Sydney Meshkov (American Institute of Physics, 2000), astro-ph/9911532; and V. Kalogera in *Astrophysical Sources of Gravitational Radiation for Ground-Based Detectors*, ed. J.M. Centrella (AIP, Woodbury NY), submitted, astro-ph/0101047] — extrapolated into the distant universe by the B-band method of Phinney as updated by Kalogera et. al. [16]. The last column is based on the estimates by Portegies Zwart and McMillan [18]. The numbers preceded by “ \lesssim ” represent Kalogera’s personal best estimate of the lower limit with the “ $<$ ” being a warning that some researchers have argued for lower numbers.
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