THE ANNUAL REPORT OF THE MSU GROUP

(Jan.-Dec. 1999)

Contributors: V.B.Braginsky (P.I.), I.A.Bilenko, M.L.Gorodetsky ,

F.Ya.Khalili, V.P.Mitrofanov, K.V.Tokmakov, S.P.Vyatchanin,

some part of the research was done in collaboration with the theoretical group of prof.

K.S.Thorne

Contents

I. SUMMARY

A. The improvements of the Q-factor of the suspension modes

During the 12 months period V.P.Mitrofanov and K.V.Tokmakov carried out a set of measurements of the quality factors Q of torsional-pendulum mode in all-fused-silica suspension of mirror's models. The measurements were performed in a new vacuum chamber which was manufactured one year ago. This chamber was designed specially to provide large mismatching of the impedances to reduce recoil losses. Special precautions were made to reduce the contamination of the fiber by dust and by sedimentation of fused silica vapor on the fiber during the welding process. The laser readout system which recorded the small decrease of the amplitude of the oscillations due to the damping was substantially improved up to the level of resolution which permits to measure the value of $Q \simeq 10^8$ with error less than 10% during time interval $\simeq 10^2$ hours.

The main results of these measurements:

a) Quality factor $1.7 \times 10^8 \pm 10\%$ was obtained in the torsional-pendulum mode (eigenfrequency 0.34 Hz) of the all-fused-silica bifilar suspension of 2 kg mirror's model. The fibers in the suspension were welded to small bumps carved in the fused silica cylinder.

b) Quality factor $2.3 \times 10^8 \pm 10\%$ was obtained in the same type of mode and suspension of 500 gram mirror's model (eigenfrequency 1.17 Hz). This model was backed in vacuum during 6 hours at $t = 100 \div 150°C$. The fibers in this pendulum were welded to fused silica cones which were attached to the cylinder by hydroxide-catalysis bonding. The cylinder of the pendulum with bonded cones was manufactured and provided to us by the Glasgow group (J.Hough and S.Rowan). Details of these experiments are presented in Appendix A.

B. The development and the realization of methods of measurements of the excess noise in all-fused-silica fibers

The initial plan of MSU group from the beginning of this grant was to propose and to realize methods of registering of the excess noise in the violin modes of fused silica fibers and finally to investigate this noise. During the work under the previous grant, the MSU group successfully demonstrated the existence of this noise in tungsten and steel wires and estimated the values of the perturbation of metric which this noise may mimic. The quality factors of the violin modes of steel and tungsten wires are $10^4 \div 10^5$ when for fused silica fibers the quality-factor are $10^7 \div 10^8$ (see the reports of MSU group of the previous grant).

Because the partial variance of the Brownian fluctuations for the time interval of few periods is proportional to $1/\sqrt{Q}$ in the current grant the planned sensitivity of the meter has to be not worse than 10^{-13} cm/$\sqrt{\mathrm{Hz}}$ i.e. two orders better than that realized for metal wires. This tough task is aggravated by one important condition: to preserve the high Q-factors of the fiber's violin modes it is not possible to spoil the fiber's surface by glueing a mirror on it or by depositing any reflecting coating.

After several unsuccessful attempts to implement various readout systems, which have to meet the above conditions, a new concept of such a readout system was proposed by I.A.Bilenko and M.L.Gorodetsky. The idea of the concept is to insert the fiber across the optical beam within Fabry-Perot resonator. If the ratio of the fiber's diameter to the wavelength is close to certain values then the displacement of the fiber relatively to the resonator's mirror produces large phase shift in the output optical wave corresponding to a substantially high finesse. In the first experimental test of this concept the already obtained finesse of such a meter exceeded 60. This value means that this meter may have the sensitivity $\simeq 10^{-15}$ cm/$\sqrt{\mathrm{Hz}}$ if limited only by shot noise of 1mW laser pumping. Theoretical analysis of this concept is presented in the Appendix B.

C. The investigation of the sources of noises produced by electrostatic actuators

V.P.Mitrofanov and graduate student N.A.Styazhkina continued the study of the degradation of mechanical Q produced by the electrical field applied to a model of the mirror. This effect will inevitably appear if electrostatic actuator will be used for the positioning. The damping effect was investigated in a model of electrical actuator which consisted of a set of strip electrodes alternatively at positive and negative electrical potentials. The measurements have shown that for such an actuator there is a substantial damping besides the trivial effects caused by losses in the resistivity in the external circuit of the voltage supply. It was observed that this additional damping depends strongly on the material of electrodes and the process of the electrode fabrication (see details in Appendix C).

D. The analysis of new topologies and new meters for the advanced LIGO

In this area several different results deserve to be mentioned.

D-1. F.Ya.Khalili, V.B.Braginsky, M.L.Gorodetsky, K.S.Thorne have analyzed the so-called energetic quantum limit. In gravitational wave antennae for any topology due to the laws of quantum mechanics there exists minimal optical energy which had to be stored in the Fabry-Perot resonator depending on the chosen sensitivity of the antenna. If one wants to beat the standard quantum limit substantially then for standard topologies this minimal energy becomes unacceptably high. Intracavity readout schemes may substantially reduce this value of energy. In this case the problem of measurement is transposed to the local meter which records the response of intracavity test mass (see details in the Appendix D-1 and also in gr-qc/9907057).

D-2. F.Ya.Khalili and V.B.Braginsky have done the analysis of "artifical" mechanical rigidity which appears due to pondermotive forces in high finesse optical resonator when the pumping frequency is out of resonance. The analysis shows that this mechanical rigidity may have a very low level of noise which is of quantum origin. The usage of this rigidity may substantially reduce technical problems in the implementation of the local meter in the intracavity readout system (see details in Appendix D-2 and in Phys.Lett.A, 257 (1999) 214.

D-3. M.L. Gorodetsky, S.P. Vyatchanin and V.B. Braginsky, have done an analysis in detail of two physical effects which are playing important role in the interferometric part of LIGO when the planned sensitivity is approaching the standard quantum limit.

The first effect (photo-thermal shot noise effect) is the fluctuations of surface due to specific shot noise which appears when optical photon, absorbed in the multilayer reflector, gives birth to a bunch of thermal phonons. The bunches produce fluctuations of the temperature inside the bulk of the mirror. If the thermal expansion of the mirror is nonzero then these fluctuations of temperature cause the fluctuations of the mirror's surface. Numerical estimates show that one megawatt circulating power, which is planed for LIGO-II, is close to the upper value of acceptable power.

The second effect is of pure thermodynamical origin: the fluctuations of temperature in the bulk of mirror (the entropy fluctuations) together with nonzero thermal expansion factor are the source of independent fluctuations of the mirror's surface. Numerical estimates show that for the present planned size of the mirror and for the size of the laser beam spot on it the sensitivity of LIGO-II antenna will not exceed SQL if sapphire will be used to manufacture the mirror.

See details in Appendix D-3 and in *Physics Letters A*, **A264** (1999) 1.

II. APPENDIX

A. The improvement of the Q-factor of the suspension modes

A set of measurements of the quality factor Q of torsion-pendulum mode in all-fused-silica suspension of the model of the mirrors was carried out during the 12 months period. The measurements were performed in a new vacuum chamber which was manufactured one year ago. This chamber was designed specially to provide large mismatching of the impedances to reduce recoil losses. We used our usual technique to suspend a 2-kg fused silica cylinder on two fused silica fibers with a length of about 25 cm and a diameter of about 0.2 mm; the fibers were welded to the small bumps which were carved in the cylinder (see the Annual - 1998 Report of the MSU group). We paid special attention to improvements of various elements in the technique of fabrication of the suspension. Special precautions (screening of the fiber in the process of welding, control of large (more than 10 micron) dust particles on the fiber surface) were made to reduce the contamination of the fibers by dust and by sedimentation of fused silica vapor on the fiber during the welding process. The laser readout system which recorded the small changes of the amplitude due to the damping was substantially improved. The signal/noise ratio at the output of the system was increased approximately by a factor of three. It was possible to measure the value of $Q \simeq 10^8$ with error less than 10% during time interval about 10^2 hours.

Free decay in the amplitude of the 2-kg model of the mirror is shown in Fig.1. An eigen-frequency of the torsion-pendulum mode was 0.34 Hz. The record value of the relaxation time $1.6 \times 10^8\text{s} \pm 10\%$ was obtained for this mode whose loss properties are similar to those of the simple pendulum mode. The quality factor of $Q = 1.7 \times 10^8 \pm 10\%$ was achieved.

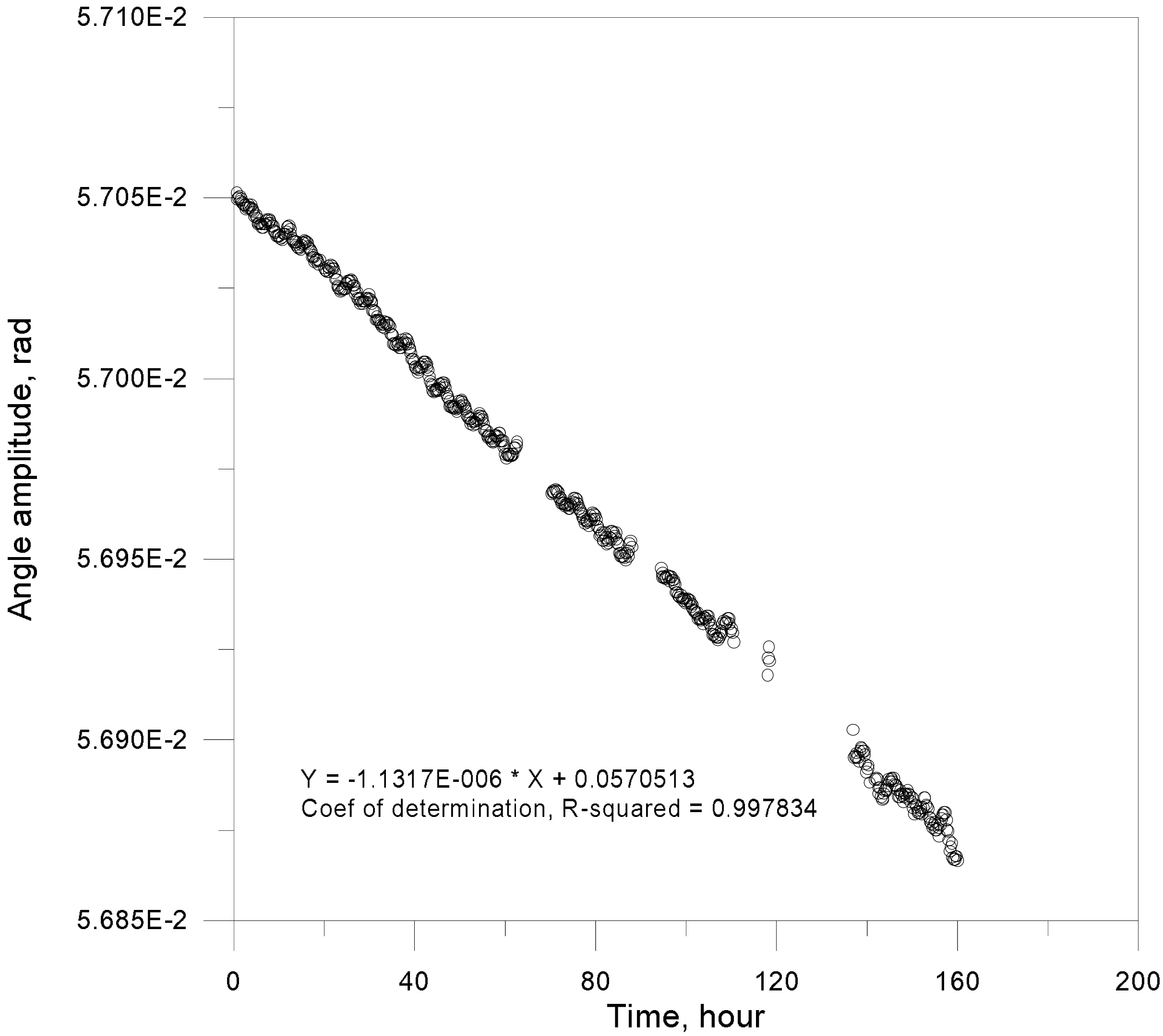

Fig1. Decay in the amplitude of angular motion of 2 kg fused silica pendulum fabricated by welding.

We have also examined dissipation for the other method of attaching the fused silica suspension fibers to the mirrors. In this case the fibers were welded to the fused quartz cones which were attached to the fused quartz cylinder using hydroxide-catalysis bonding technique. This method was investigated in University of Glasgow.

The 0.5 kg fused silica cylinder with bonded cones was manufactured and provided to us by the Glasgow group (J.Hogh and S.Rowan). The construction of such all-fused-silica bifilar suspension is shown in Fig. 2.

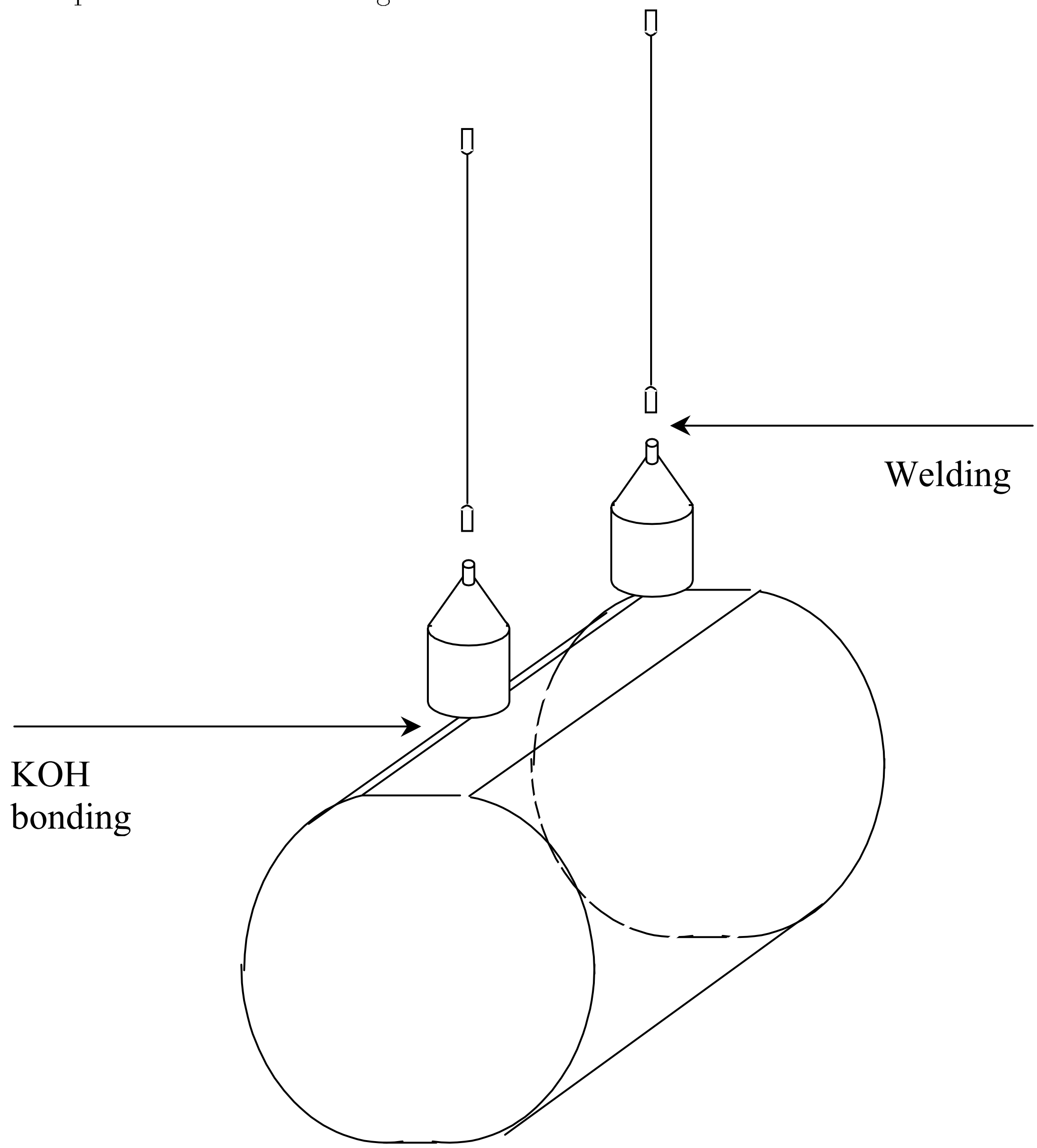

Fig2. All fused silica pendulum constracted use KOH-bonding/welding.

Free decay in the amplitude of 0.5 kg model of the mirror is shown in Fig.3. The eigenfrequency of the torsional-pendulum mode was 1.17 Hz. This pendulum was baked out in vacuum for 6 hours at the temperature $100 \div 150^{\circ}$ C. The Q factor was found to be $2.3 \times 10^{8} \pm 10\%$. This result permits to conclude that the pendulum mode losses associated with hydroxide-catalysis bonding are small in comparison with losses caused by other mechanisms.

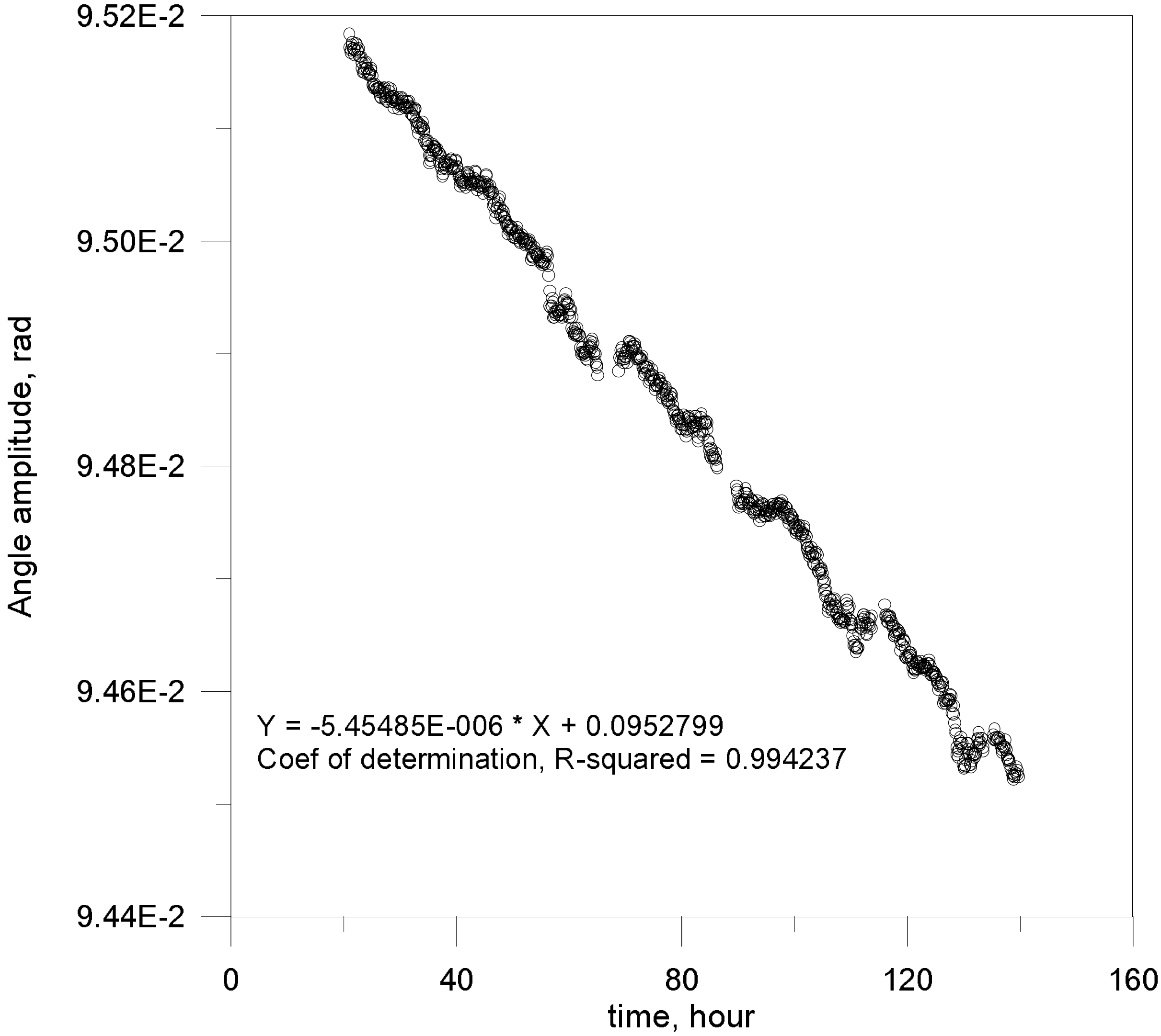

Fig3. Decay in the amplitude of angular motion of 0.5 kg fused silica pendulum with KOH bonded attachments.

The measured Q can be compared with the expected $Q \simeq 1 \times 10^9$, obtained from calculations based on the pendulum dilution factor and the measured loss angle of $\phi = 1.4 \times 10^{-7}$ for the fused silica fibers. The discrepancy may be explained by incomplete elimination of possible sources of excess loss such as recoil losses, contact losses in indium gasket at mounting point, fiber surface contamination and others. The goal of the further research is to reduce substantially the influence of the additional mechanisms of losses in order to reach the Q-factor which is limited by the loss in material - fused silica.

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B. A method for Measuring Small Vibrations of Optically Transparent Objects

The currently designed laser interferometric detectors of gravitational waves LIGO (USA) have the potential sensitivity sufficient not only for detecting radiation splashes arising as a result of the black holes collapse, and other astrophysical catastrophes but also for analyzing the dynamics of these processes [1]. In the first variant (LIGO-I), the minimum detectable perturbance of the metric has a scale of $h \simeq 10^{-21}$ and, in the next variants (LIGO-II and LIGO-III), it can attain $10^{-22} \div 10^{-23}$. To provide such a sensitivity, it is necessary that the variations in test-mass coordinates, which are caused by extraneous factors including thermal fluctuations in the test masses themselves and in the systems of their suspension, would be negligible. As follows from the fluctuation-dissipation theorem, the spectral density of the equilibrium thermal noise can be reduced by increasing the Q-factor for all modes of mechanical vibrations. The Q-factor for the string modes of steel wire suspensions used in the LIGO-I amounts to $Q_m \simeq 10^5$. Recent investigations have shown that this value reaches 1×10^8 [2] for quartz fibers that makes it possible to expect the further increase in the sensitivity. This is explained by the fact that the root-mean square vibrational amplitude variations for times $\tau < Q_m/\omega_m$, which are induced by thermal fluctuations, are equal to

$$\Delta x_T \simeq \sqrt{\frac{4kT\tau}{m^* Q_m \omega_m}}, \tag{1}$$

Here, k is the Boltzmann constant, T is temperature, m^* is the string effective mass, and ω_m is the resonance frequency of the string fundamental mode. At the same time, the existence of an extra (excess) noise of the nonthermal origin is possible. The source of the excess noise may be the development of microcracks, the motion of dislocations and other defects in the suspension. The experiments carried out have shown the presence of such a noise in tungsten wires [3] and steel wires [4] under high tension. The Q-factor of the string modes increases, hence, the effect of thermal noise decreases, and the excess-noise intensity increases with tension. Therefore, for the optimum choice of the construction and the degree of suspension loading, it is necessary to have a possibility to detect the vibrations of the fibers used in suspensions with a spectral coordinate resolution equal to

$$\delta x_\omega \simeq \sqrt{\frac{16\pi^2 kT}{m^* Q_m {\omega_m}^3}} \simeq 4 \times 10^{-15} cm/\sqrt{Hz}. \quad (2)$$

Here, $m^* = 1.1 \times 10^{-3}$g (a quartz fiber 10 cm long and 100 μm in diameter), $\omega_m = 2\pi \times 2.8$ kHz (about 90% of the ultimate tension), and the averaging time is $\tau \simeq 2\pi/\omega_m$. Here we propose a method of the noncontact and nonperturbing coordinate measurements, which provides the desired sensitivity level, and present the results of a preliminary experimental test for a possibility of its realization. The method developed is going to be applied for investigating thermal and excess noise in quartz fibers. Moreover, it can be used directly in a gravitational-wave detector for monitoring the suspension noise. We discuss also a possibility to observe effects of the quantum theory of measurements [5] on the basis of the detector proposed. In order to measure the proper noise in suspension fibers, it is necessary to satisfy several conditions. First, the measuring device must have a sufficient sensitivity; second, in the process of the measurements, the fiber must be isolated from any external action including that of the measuring procedure in itself. Therefore, the noncontact methods are preferable because they require no connection of supplementary details (mirrors, plates of capacitive sensors, etc.). Since the dimension of the fiber-sensor interaction domain is rather small for the fibers used of $10-200\mu m$ in diameter, it is difficult to obtain a high resolution by means of the most sensitive radio-frequency and microwave parametric transducers. In [4], an optical sensor based on the Michelson single-beam interferometer was used in which the surface of a metallic wire played the role of a mirror. It is possible to show that the ultimate possible sensitivity of such a sensor

$$\delta x_{\omega,min} = \frac{\lambda}{4\pi}\sqrt{\frac{\hbar\omega}{W}}, \quad (3)$$

where λ and ω are the wavelength and the frequency of the optical beam, and W is its power, does not allow us to attain desired value (2). The methods of the "knife-and-slit" type also provide nearly the same ultimate sensitivity.

A higher sensitivity is provided by multi-beam interferometers, in particular, a Fabry-Perot interferometer. However, the reflection factor for a pure dielectric surface is low. At the same time, depositing a well-reflecting mirror layer on the surface being measured is associated in our case with a damage of its structure, which decreases a high mechanical Q-factor of vibrations.

The measurement method proposed is also based on using the Fabry-Perot cavity with a high finesse. However, it is free of the disadvantages indicated and provides a required sensitivity level.

We consider a system consisting of the Fabry-Perot interferometer of a length L with mirrors having the reflection factor r_m, (losses for scattering and absorption in these mirrors are ignored) and a dielectric body (it can be a plate, a lens, a microsphere, a fiber, etc.) placed inside the interferometer (approximately, in the middle between the mirrors). Let the body be characterized by amplitude factors of transmission, reflection, and losses t_p, r_p and a_p:

$$|r_p|^2 + |t_p|^2 + |a_p|^2 = 1. \tag{4}$$

In this case, the resonance frequency ω and, thus, the phase ϕ of the transient wave depends on the coordinate of the body. In the case of the optimum choice of the body displacement with respect to the center, we have

$$\frac{\delta\omega}{\delta x} = \frac{2\omega|r_p|}{L}, \tag{5}$$

$$\phi(x) = \frac{2|r_p|}{1 - r_m^2(1 - |a_p|^2)} \sin(4\pi x/\lambda). \tag{6}$$

For a dielectric plate, the reflection factor r_p depending on both the body thickness and the wavelength λ varies from 0 to r_{max}:

$$r_{max} = \frac{n^2 - 1}{n^2 + 1}, \tag{7}$$

where n is the refractive index of the dielectric. Thus, ignoring losses for scattering and absorption in the process of the light reflection from the body and assuming that $|a|^2 \ll 1 - r_m^2$, we obtain that the maximum sensitivity of the sensor proposed, which is limited only by the laser quantum noise, is

$$\delta x_\omega = \frac{\lambda}{16F}\frac{n^2+1}{n^2-1}\sqrt{\frac{\hbar\omega}{W}}, \tag{8}$$

where $F = \pi/(1 - r_m^2)$ -is the cavity finesse. For the laser power $W = 1$ mW and $r_m^2 = 0.99$, we obtain that the spectral sensitivity in the case of the quartz plate (n = 1.45) is $\delta x_\omega \simeq 6 \times 10^{-16} cm/\sqrt{Hz}$. With increasing the reflection factor of the mirrors, the sensitivity increases but the problem of providing stability of the cavity mode arises. For the normal operation of the displacement sensor, it is necessary to provide the stability of the fundamental mode for the optical system formed by two mirrors and two reflecting surfaces of the dielectric body. This can be attained only in the case when all the surfaces coincide with the wave-front surfaces. If R_m and R_p are, respectively, the curvature radii for the mirrors and body surfaces, d is the plate thickness, and $L \gg d$ is the spacing between the mirrors, we arrive at the following conditions:

$$R_m \simeq \frac{L}{2}\left(1 + \frac{2dR_p - d^2}{L^2}\right) \qquad d < 2R_p \tag{9}$$

The most interesting case for many experimental applications (the measurement of motions of fibers, microspheres, and drops), corresponds to the geometrical-optics limit and lies, unfortunately, at the stability boundary. However, it is possible to show that, if a stable configuration of main cavity mirrors is chosen, so that $R_m - L/2 \ll L$, the instability leads only to a small increase in the diffraction losses of the cavity the term a_p in formula (6) and to the corresponding insufficient decrease in the sensitivity (the estimate presented corresponds to the cylindrical geometry):

$$a_p^2 = 2r_p^2\left(\frac{R_m - L/2}{R_m}\right)^2 \text{cm.} \tag{10}$$

Thus, to conserve the sensitivity at the same level for the chosen parameter values, it is necessary to fulfill the relatively weak condition $R_m - L/2 < 0.1R_m$.

To verify the method proposed, we have carried out preliminary measurements. We used a Fabry-Perot interferometer having a 16 cm base with a plane-parallel glass plate 5 mm thick placed in the center of the interferometer. The frequency-tuned He-Ne laser served as a pumping source. The measurements were carried out by observing resonances on the screen of an oscilloscope, whose horizontal-sweep voltage was used for the laser frequency control. In the interferometer, we used spherical dielectric mirrors with a radii of 50 cm and reflection factors $r_m^2 \simeq 0.99$. Initially, the finesse $F = 280 \pm 20$ was measured without the plate. Then, the plate was installed inside the interferometer. In the process of the measurements, the plate position along its axis was varied by the PZT drive. The finesse of the interferometer with the plate amounted to $F = 250 \pm 20$. We measured the resonance frequency as a function of the plate longitudinal displacement. The maximum tuning ratio $\delta f/\delta x \simeq 490 MHz/\mu m$ turns out to be approximately four times as low as that found from formula (5).

The result obtained confirms the possibility of using the scheme proposed for investigating noise in quartz fibers provided that the choice of the mirrors geometry is adequate. The decreased frequency-tuning ratio compared to the theoretical is explained by non-perfect in the adjustment of the optical system and difference between the actual plate thickness and the optimum one for which r_{max} is attained.

It should be noted that even in the case of using mirrors with the reflection factor $R > (1 - 1 \times 10^{-4})$, it is necessary to take into account in calculations quantum effects of interaction of a measuring device with an object. These effects manifest themselves as the energy transfer by quantum fluctuations of the field in the cavity to mechanical degrees of freedom. Thus, in the case of the same parameters as before, the amplitude increment owing to this effect amounts to for one period of vibrations

$$\delta x_{\hbar} \simeq \frac{F}{\omega_m m^*}\sqrt{\frac{\hbar\omega W\tau}{c^2}} = 4.4 \times 10^{-13}\text{cm} \qquad (11)$$

which is comparable with thermal fluctuations (1).

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C. The investigation of the sources of noises produced by electrostatic actuators

The group continued the study of damping produced by electric field applied to a model of the mirror. This effect appears if electrostatic actuator is used for the positioning. The damping effect was investigated in a model of actuator which consisted of a set of strip electrodes having alternatively positive and negative potentials which produced force on fused silica model of the mirror due to the inhomogeneous electric field. Schematic of the electrostatic actuator is shown in Fig.4. The gold film strip electrodes (length of 2 cm, width of 1 cm, spaced by 0.2 cm) were fabricated on the surface of fused silica block by a method of cathode sputtering. The gap between the pendulum and the electrodes was about 0.3 cm. Such actuator produced the force of the order of 1 dyn if the voltage 10^3 V was applied between the electrodes. The damping of the pendulum introduced by the trivial effect caused by the losses in the resistor in the external circuit of the voltage supply was inessential. The measurements have shown that for such an actuator there is an additional damping proportional to the square of the applied voltage (see Fig. 5). We observed that this additional damping depends strongly on the material of electrodes, the process of the electrode fabrication and treatment. It was found also that when AC electric field was applied to the electrodes the additional damping decreased with the frequency of the electric field. For the frequencies higher than 150 Hz this damping was less than the error of measurements. In this way one may expect the significant reduction of the additional thermal noise produced by the electrostatic actuator at frequencies of operation of the gravitational wave detector. The detailed investigation of these problems is now in progress.

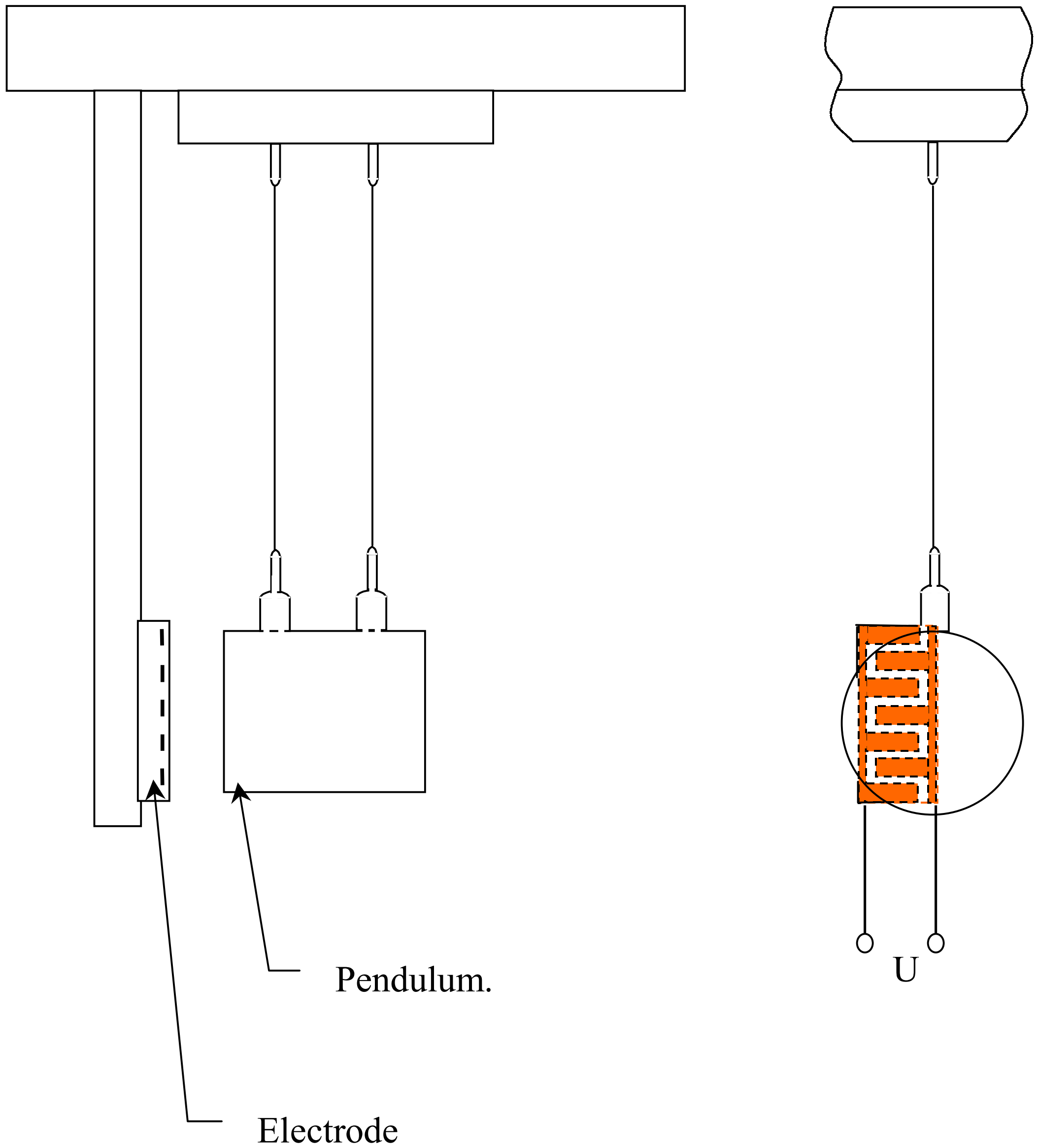

Fig 4. Schematic of the electrostatic actuator.

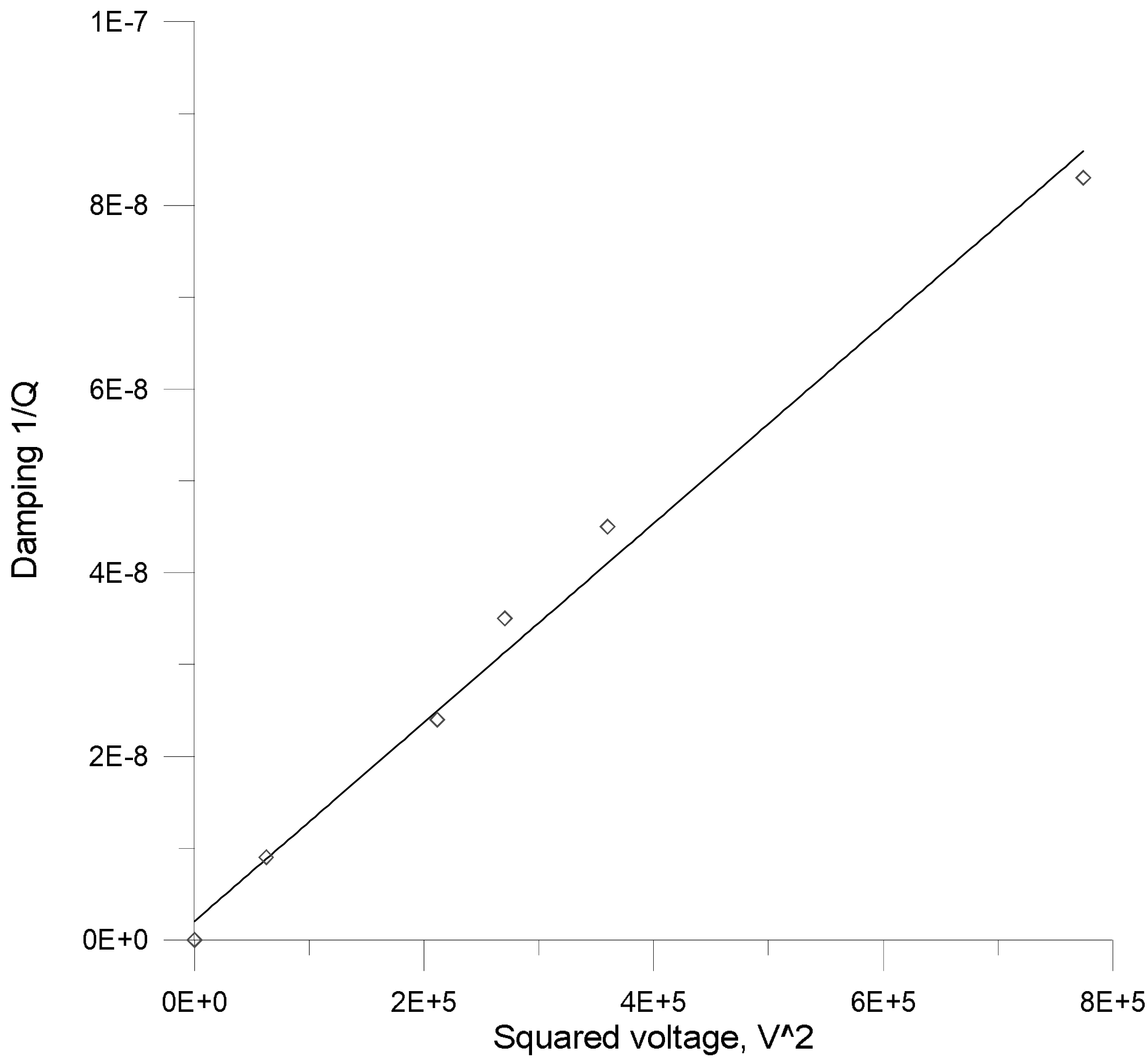

Fig.5 Damping produced by electrostatic actuator

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D. The analysis of new topologies and new meters for the advanced LIGO

1. The Energetic Quantum Limit

The creation of new or the development of existing theoretical models are forcing the experimentalists to invent more sensitive methods of measurements. This trend, in particular, is evident in experiments with test masses in which the task of detecting the signal is the detection of a small force $F(t)$ (an acceleration, a gradient of acceleration) acting on a macroscopic mass m. For example, at the second stage of the project LIGO-II (Laser Interferometer Gravitational Wave Observatory [2]) the experimentalists will be confronted with the so-called Standard Quantum Limit of the sensitivity (SQL) for the force $F(t)$ [3], and on the third stage this limit probably will be overcomed.

The next serious limitation is the Energetic Quantum Limit. Gravitational wave in the interferometric antennae changes the phase of optical field. In order to detect this phase shift, uncertainty of the phase $\Delta\phi$ should be sufficiently small. Hence due to the uncertainty relation

$$\Delta\mathcal{E}\Delta\phi \geq \frac{\hbar\omega_0}{2} \tag{12}$$

(ω_0 - is optical frequency), large uncertainty of the optical energy is required.

This is not a peculiar property of interferometric meters only, but a consequence of a more general principle: In order to detect external action on the quantum object, uncertainty of the interaction Hamiltonian $\hat{\mathcal{H}}_I$ should be sufficiently large [4]:

$$\left\langle \left(\int_{-\infty}^{\infty} \Delta\mathcal{H}_I(t)dt\right)^2 \right\rangle \geq \frac{\hbar^2}{4} \tag{13}$$

It can be shown, that this condition leads to very severe requirements for the optical energy $\mathcal{E}$ stored in the interferometer:

$$\mathcal{E} = \frac{M\omega_{signal}^3 L^2}{4\xi^2\omega_0} = \frac{2\cdot 10^8 erg}{\xi^2} \times \left(\frac{M}{10^4 gr}\right) \times \left(\frac{\omega_{signal}}{10^3 s^{-1}}\right)^3 \times \left(\frac{L}{4\cdot 10^5 cm}\right)^2 \times \left(\frac{\omega_0}{2\cdot 10^{15} s^{-1}}\right)^{-1}, \tag{14}$$

where $\xi < 1$ is the required sensitivity in the units of SQL, M is the test mass, ω_{signal} is the signal frequency, L is the arm length and ω_0 is the optical frequency.

All known methods of overcoming the SQL for the case of position measurement of free test masses [5,6] apply additional restriction on the value of optical energy and pumping power. All of them require correlation between back action noise and output noise of the meter. Due to this requirement all these schemes are limited by the level of intrinsic losses of the e.m. resonator because fluctuations corresponding to these losses can't correlate with the output noise of the meter. It can be shown that sensitivity of these methods cannot exceed the limit

$$\xi = \left(\frac{\tau^*_{load}}{\tau^*_{intr}}\right)^{1/4}, \tag{15}$$

where τ^*_{intr} is the relaxation time which characterize intrinsic losses in the e.m. resonator (i.e. absorption in the mirrors and beam-splitters and nonzero transmittance of end mirrors) and τ^*_{load} is the relaxation time corresponding to coupling of the resonator with e.m. detector which should be close to ω^{-1}_{signal}. Using the best known mirrors and the LIGO value of $L = 4 \times 10^5 cm$ one can achieve the value of $\tau^*_{intr} \sim 10s$. Hence if $\omega_{signal} \simeq 10^3 s^{-3}$ then condition (15) may be satisfied only if $\xi \geq 0.1$. Further increase of sensitivity leads to necessity to decrease τ^*_{load}. In this case

$$\tau^*_{load} \ll \omega^{-1}_{signal} \tag{16}$$

and

$$\mathcal{E} = \frac{M\omega^2_{signal}L^2}{16\xi^6\omega_0\tau^*_{intr}} \tag{17}$$

Such a very strong dependence of $\mathcal{E}$ on ξ it practically impossible to obtain the value of $\xi < 0.1$ using standard topologies.

A new principle of extracting of the information from gravitational-wave antenna was proposed in the article [7]. Instead of measuring the time phase shift of output optical wave it was suggested to detect directly the spatial phase shift of the optical field in the antenna using some QND-type method. Two practical realizations of this idea was considered in the articles [8,9]. The most promising of them is probably the “optical bar” scheme [8].

In this scheme optical fields in two arms of the antenna work as a mechanical springs with rigidity proportional to the optical energy $\mathcal{E}$ stored in each arm. They shift internal mirror placed into the resonator when gravitational wave shifts the end ones. This displacement may be measured relative to an additional reference mass which is not affected by the optical field in the antenna, using some of the measurement methods developed for solid-state gravitational-wave antennae.

The value of the displacement may be close to $Lh(t)/2$ if energy $\mathcal{E}$ is sufficiently high:

$$\mathcal{E} \geq \frac{m\omega_{signal}^3 L^2}{2\omega_0} \tag{18}$$

where m - is the mass of internal mirror. Structure of this expression is similar to expression (14) for traditional scheme, but it contains the mass m of internal mirror instead of the masses of the end mirrors M, and, which is more important, it doesn't depend on ξ. Hence in this case it is possible to rise sensitivity beyond the level of SQL without necessity to rise $\mathcal{E}$. If, for example, $m = 10^3 gr$ and the values of all other parameters are the same as above, then $\mathcal{E} \simeq 4 \cdot 10^7 erg$.

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2. Low noise rigidity produced by pondermotive force

It is evident that in case of intracavity schemes the focus of the problem shifts to the local meter of the position of the central mirror. Intracavity schemes require higher precision to exceed the SQL. During the last twenty five years several possible methods of such a measurement were proposed.

The experimentalist has a choice in this type of experiments: to detect $F(t)$ acting on a free mass m or on a mass coupled with rigidity $K = m\omega_m^2$, in other words on an oscillator with eigen frequency ω_m which is close to ω_F - characteristic frequency of $F(t)$. At first glimpse the choice of free mass does look more attractive because it is easier to isolate the mass from the heat bath. The free mass and the oscillator "behave" as quantum objects if the following conditions for them are satisfied:

$$\frac{2kT\tau^2}{\tau^*_{H.B.}} \leq \hbar, \tag{19}$$

$$\frac{2kT\tau}{Q_{H.B.}} \leq \hbar, \tag{20}$$

where τ is the averaging time, k is Boltzmann constant, T is the temperature of the heat-bath, $\tau^*_{H.B.}$ is the relaxation time for the free mass, $Q_{H.B.}$ is the quality factor of the oscillator. For $T = 300K$ and $\tau = 10^{-2}sec$ it is necessary to reach $\tau^*_{H.B.} \simeq 10^{10}sec.$ to satisfy condition (19). The value $\tau^*_{H.B.} \simeq 10^8 sec$ was already reached [1], new methods of the "subtraction" of the heat-bath action on the free mass are also proposed. These methods permit to reach the equivalent $\tau^*_{H.B.} \simeq 10^{10} sec$ [2]. For the same values of T and τ it is necessary to have $Q_{H.B.} \geq 10^{12}$ (note that at room temperature the highest obtained $Q \simeq 4 \cdot 10^8$ [3]) to satisfy the condition (20).

If conditions (19,20) are satisfied then to circumvent the SQL it is necessary to use some methods of quantum measurements, for example Quantum-Non-Demolition methods (see [4]) or methods based on continuous monitoring of coordinate with the use of spectral or time domain features of noises of the meter [5,6]. All proposed schemes for a free mass based on these last two methods are in essence parametrical converters (the $x(t)$ is transferred into the modulation of some parameter of an e.m. resonator coupled to the mass m). Hence they limited by the condition (15) for the relaxation times of the resonator. In some specific cases of this class of meter even more strict limitations can exist. For example, in the case of QND speed meter [7, 8] it is necessary to have

$$\xi \geq (\omega_{signal} \tau^*_{intr})^{-1/4}, \tag{21}$$

so if $\xi = 0.1$ and $\omega_{signal} = 10^2 s^{-1}$ then must be $\tau^*_{intr} \geq 10^2 s$ (which corresponds to the quality factor $Q_{intr} \geq 10^{12}$ even in the case of microwave resonator).

In contrast with the free mass in the case of harmonic oscillator it is possible to overcome the SQL without using the correlation of the meter noises because during free unitary evolution quantum state of the oscillator is periodic. This feature permits to evade the confrontations with conditions (15,21).

Pondermotive force which acts on two mirrors of Fabry-Perot resonator (in microwave or optical bandwidth) creates mechanical rigidity K_{pond} which may be large if the reflectivity of the mirrors is high and its losses are low and if this resonator is pumped with the frequency ω which is detuned from the eigen frequency $\omega_e = \omega - \gamma$ (see e.g. [9]). If the detuning is large: $\gamma \tau^*_e \gg 1$ (this is necessary to get a low level of noises, see below), then this rigidity is equal to

$$K_{pond} = m\omega_m^2 = \frac{2\omega_e \mathcal{E}}{l^2 \gamma} = \frac{2\omega_e W}{l^2 \gamma^3 \tau^*_e}. \tag{22}$$

Here

$$\tau^*_e = \frac{l\mathcal{F}}{\pi c} \tag{23}$$

- is the relaxation time of the resonator, l is the distance between the mirrors, c is the speed of light, $\mathcal{E}$ -is the energy between the mirrors.

On the other hand the finite value of $\mathcal{F}$ will produce a fluctuating force which will act on the masses coupled to the mirrors. The spectral density of this "back action" force is equal to

$$S_{B.A.} = \frac{\hbar\omega_e\mathcal{E}}{l^2\gamma^2\tau_e^*} = \frac{\hbar\omega_e W}{l^2\gamma^4(\tau_e^*)^2}. \tag{24}$$

The "quality" of rigidity may be characterized by the minimal detectable force in units of SQL. We may use the same parameter ξ as in formula (15-21), but now we take into account only the noise from the rigidity. In this case

$$\xi^2 = \frac{S_{B.A.}}{\hbar m\omega_m^2} = \frac{1}{2\gamma\tau_e^*}. \tag{25}$$

Substituting this value of ξ into the formula (22) results in

$$K_{pond} = \frac{16\omega_e W\mathcal{F}^2\xi^6}{\pi^2c^2} \simeq 10^{10}dyn/cm \times \left(\frac{W}{10^4 erg/s}\right) \times \left(\frac{\mathcal{F}}{5\cdot 10^5}\right)^2 \times \left(\frac{\omega_e}{2\cdot 10^{15}}\right) \times \xi^6. \tag{26}$$

Thus for $\xi = 0.3$ and $W = 10^7 erg/s$ one may obtain the rigidity $K_{pond} \simeq 10^{10}dyn/cm$ which will "convert" a 10^4 gram mass into a mechanical oscillator with eigen frequency $\omega_m \simeq 10^3 s^{-1}$. For $\xi < 0.3$ the value of W will be inaccessible high. However for smaller masses (e.g. $m \simeq 10gr$) this type of artifical rigidity seems attractive. It is worth noting that this method may be realized only because very high values of $\mathcal{F}$ were recently obtained [10].

Another scheme with the pondermotive rigidity has been considered in the article [11]. If mirror with transmittance $\mathcal{T}$ is situated inside the Fabry-Perot resonator equidistant from two end mirrors with high finesse $\mathcal{F}$ then this internal mirror splits the eigen modes of the resonator into doublets whose frequencies are apart from each other by

$$\Omega = \frac{c\mathcal{T}}{l}. \tag{27}$$

where l is the distance between the internal mirror and the end ones.

If the upper frequency component of the doublet is excited then the pondermotive force which acts on the internal mirror will strongly depend on the displacement of the mirrors due to the redistribution of the e.m. energy $\mathcal{E}$ in the two parts of the resonator. In other words mechanical rigidity K_{pond} between the internal mirror and the end mirrors will be created. The value of K_{pond} may be presented as

$$K_{pond} = \frac{4\omega_e \mathcal{E}}{l^2 \Omega} = \frac{4\omega_e W \mathcal{F}}{\pi c^2 \mathcal{T}} \simeq 10^{10} dyn/cm \times \left(\frac{W}{3 \cdot 10^7 erg/s}\right) \times \left(\frac{\omega_e}{2 \cdot 10^{15}}\right) \times \left(\frac{\mathcal{F}}{10^6}\right) \times \left(\frac{\mathcal{T}}{10^{-2}}\right)^{-1}. \quad (28)$$

Here $\mathcal{E}$ is the energy stored in each half of the resonator. This estimate show that for the mass of the mirror $m = 10^4 g$ the mechanical eigen frequency of the internal mirror will be equal to $10^3 s^{-1}$ if the pumping power equals to $W = 3 \cdot 10^7 erg/s$. This relatively modest value of W for such a large K_{pond} is the first substantial advantage of this type of artifical rigidity.

Spectral density of the back action force in this case is equal to

$$S_{B.A.} = \frac{2\hbar\omega_e W}{l^2 \Omega^2}, \quad (29)$$

hence

$$\xi^2 = \frac{S_{B.A.}}{\hbar m \omega_m^2} = \frac{\pi}{2\mathcal{F}\mathcal{T}} = 1.5 \cdot 10^{-4} \times \left(\frac{\mathcal{T}}{10^{-2}}\right)^{-1} \times \left(\frac{\mathcal{F}}{10^6}\right)^{-1}. \quad (30)$$

Thus the second advantage is a very low level if intrinsic quantum noise. This level is approximately equal to the level of noise of mechanical oscillator with quality factor $Q_m \simeq 2 \cdot 10^{16}$ in the heat-bath with temperature 300K. The "fee" which experimentalist has to pay for such a large value of K_{pond} and small value of $S_{B.A.}$ is the level of symmetry in the positioning of inside mirror $\delta l/l$ and the phase difference in the two arms of the resonator $\delta\phi$. which may be quite severe if the value of $\xi < 0.1$ is required.

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3. Photo-thermal shot noise and thermodynamical fluctuations in gravitational wave antennae

The aim of this Appendix is to present the results of the analysis of the two effects which are "responsible" for random fluctuations of temperature in the bulk of the mirrors and which may mimic at certain level the gravitational wave if $\alpha \neq 0$: photo-thermal shot noise (due to random absorption of optical photons in the surface layer of the mirror) and thermodynamical fluctuations of temperature.

As in both effects we are interested in the fluctuations of the coordinate averaged over the spot on mirror surface with radius $r_0 \simeq 1.5$ cm of laser beam, which is much smaller than the radius of mirror $\simeq 15$ cm, we replace the mirror by half-space: $0 \leq x < \infty$, $\infty < y < \infty$, $\infty < z < \infty$. We consider also for simplicity this half-space to be isotropic (the anisotropy of different constants for sapphire is not very large). The role of these two effects for the condition to reach the standard quantum limit of frequency stability was analyzed more than 20 years ago [1].

As at room temperature the power radiated from the surface (Stephan-Boltzmann law) is much lower than the heat exchange due to thermal conductivity (Fourier law), below we use the simplified boundary condition:

$$\left.\frac{\partial u(x,y,z,t)}{\partial x}\right|_{x=0} = 0, \tag{31}$$

where u is the deviation of temperature from the mean value T.

For the calculations we use approximation in which we neglect the effects of limited speed of sound but take into account thermal relaxation.

Photo-thermal shot noise in the mirrors of the antennae

The multilayer coating of the mirrors absorbs a fraction of power W of the optical beam, circulating between the mirrors of Fabry-Perot resonator. This fraction for the best existing today coatings has not been measured with good precision yet, but for estimates the usually accepted value of this fraction is approximately equal to $5 \times 10^{-7} \div 10^{-6}$. The value of W sufficient to reach $h \simeq 10^{-22}$ (which is close to Standard Quantum Limit (SQL) of sensitivity) is approximately $W \simeq 1 \times 10^6$Watt $= 10^{13}$erg/sec or $\dot{N}_0 \simeq 5 \times 10^{24}$ photon/s. It is reasonable to estimate that each mirror absorbs $W_{abs} \simeq 1$ Watt $= 10^7$ erg/s or $\dot{N} \simeq 5 \times 10^{18}$ photon/s. Each absorbed optical photon having energy $\hbar\omega_0 \simeq 2 \times 10^{-12}$ erg gives birth to a bunch of approximately 50 thermal phonons. Each of these phonons has a relatively short free path l^* (for SiO_2 and Al_2O_3 the values are $l^*_{SiO_2} \simeq 8 \times 10^{-8}$ cm and $l^*_{Al_2O_3} \simeq 5 \times 10^{-7}$ cm. After a very short time interval which is a few times longer than $l^*/v_s \simeq 10^{-12}$ s (where v_s is the speed of sound) these phonons will produce a local jump of temperature inside the coating $(5 \div 10) \times 10^{-4}$ cm thick. As we want to know the variance of temperature during the averaging time $\pi/\omega_{\rm grav} \sim 5 \times 10^{-3}$ s which is several orders longer than l^*/v_s and as during this time a large number ($\dot{N}\pi/\omega_{\rm grav} \sim 2.5 \times 10^{16}$) of photons are creating bunches of thermal phonons — it is reasonable to use the model of shot noise. As the length l_T is much smaller than the radius of laser beam $r_0 \simeq 1.5$ cm for semi-qualitative analysis one can use one dimensional model, described by thermal conductivity equation:

$$\frac{\partial u(x,t)}{\partial t} - a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{w}{\rho C \pi r_0^2}\, 2\delta(x), \tag{32}$$

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = 0, \quad 0 \le x < \infty, \tag{33}$$
$$\langle w(t)\, w(t')\rangle = \hbar\omega_0 W_0 \delta(t-t'), \quad w(t) = W_{\rm abs} - W_0, \quad W_0 = \langle W_{abs}(t)\rangle,$$

where $a^2 = \lambda^*/(\rho C)$, λ^* is thermal conductivity, ρ is density and C is specific heat capacity.

Assuming that displacement $X_{\rm 1D}$ (which is of interest) is proportional to temperature u averaged along the axis x

$$X_{\rm 1D} = \alpha \int_0^\infty dx\ u(x,t). \tag{34}$$

it is not difficult to obtain the spectral density of $X_{\rm 1D}$ [1]:

$$S_{\rm TS1D}(\omega) = 2\alpha^2\, \frac{\hbar\omega_0 W_0}{(\rho\, C\, \pi r_0^2)^2}\, \frac{1}{\omega^2}. \tag{35}$$

For the exact calculations for half-space (see details in [2]) we introduce the displacement $\bar X$ averaged over the beam spot on the surface

$$\bar X = \frac{1}{\pi r_0^2} \int\!\!\int_{-\infty}^{\infty} dy\, dz\, v_x(x=0,y,z)\, e^{-(y^2+z^2)/r_0^2}, \tag{36}$$

where $v_x(x=0,y,z)$ is x-component of vector of deformation on the surface, and find the spectral density of $\bar X$:

$$S_{\rm TS}(\omega) = 2\alpha^2 (1+\sigma)^2\, \frac{\hbar\omega_0 W_0}{(\rho\, C\, \pi r_0^2)^2}\, \frac{1}{\omega^2}, \tag{37}$$

where σ is the Poisson coefficient.

Here and below we use the fact that for materials at room temperature the thermal relaxation time of the spot $\tau_T \simeq r_0^2/a^2$ is very large:

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[1]We use "one-sided" spectral density, defined only for positive frequencies, which may be calculated from correlation function $\langle X(t) X(t+\tau)\rangle$ using formula

$$S_X(\omega) = 2\int_{-\infty}^{\infty} d\tau\, \langle X(t) X(t+\tau)\rangle\, \cos(\omega\tau).$$

$$\omega \gg \frac{a^2}{r_0^2}. \tag{38}$$

It is easy to see that the exact solution (37) differs from the approximate one (35) only by Poisson coefficient.

Thermodynamical fluctuations of temperature, thermoelastic damping and surface fluctuations

It is known from thermodynamics that the total variance of fluctuations of temperature in volume V is described by the following simple formula:

$$\langle \delta T^2 \rangle = \frac{\kappa T^2}{\rho C V}, \tag{39}$$

where κ is the Boltzmann constant. In classical solid state thermodynamics these fluctuations are not correlated with fluctuations of volume (which are responsible for the Brownian noise) but this is not the case for nonzero thermal expansion coefficient. To find the influence of the fraction of this type of fluctuations in the vicinity of ω_{grav} on the vibration of the surface along the x-axis in direct way we use the Langevin approach and introduce fluctuational thermal sources $F(\vec{r}, t)$ added to the right part of the equation of thermal conductivity:

$$\frac{\partial u}{\partial t} - a^2 \Delta u = F(\vec{r}, t). \tag{40}$$

These sources should be normalized in a way to satisfy formula (39). By substituting the solution of this equation for the temperature u into the equation of elasticity and averaging displacement (36) over the spot on the surface, we find spectral density of its fluctuations in frequency range of interest (see details in [2]):

$$S_{\mathrm{TD}}(\omega) \simeq \frac{8}{\sqrt{2\pi}} \alpha^2 (1+\sigma)^2 \frac{\kappa T^2}{\rho C} \frac{a^2}{r_0^3} \frac{1}{\omega^2} \qquad \text{if} \quad \frac{r_0^2 \omega}{a^2} \gg 1. \tag{41}$$

We can also find the spectral density of displacement using Fluctuation-Dissipation theorem (FDT) if we assume that the only dissipation mechanism in the mirror is thermoelastic damping. Elastic deformations in a solid body through thermal expansion lead to inhomogeneous distribution of temperature and hence to fluxes of heat and losses of energy. To calculate the fluctuations in this approach we should apply a periodic pressure p distributed over the beam spot on the surface:

$$p(y,z,t) = \frac{F_0}{\pi r_0^2}\, e^{-(y^2+z^2)/r_0^2}\, e^{i\omega t} \tag{42}$$

and we should find susceptibility $\chi = \bar{X}/F_0$. After that in accordance with FDT the spectral density of displacement $\bar{X}$ must be proportional to imaginary part of susceptibility χ:

$$S_x(\omega) = \frac{4\kappa T}{\omega} |\mathrm{Im}(\chi(\omega))|. \tag{43}$$

Calculations (see details in [2]) show that FDT gives the same result as formula (41). Therefore, we can conclude that thermodynamical fluctuations of temperature are the physical source of fluctuations deduced from FDT based on thermoelastic damping.

Calculations of $\mathrm{Im}(\chi)$ are rather bulky but Zener suggested a very simple approach allowing to estimate thermoelastic loss angles and hence the imaginary parts of elastic coefficients. It allows to find $\mathrm{Im}(\chi)$ only from formula for χ in zero order approximation:

$$\chi^{(0)}(\omega) = \frac{(1-\sigma^2)}{\sqrt{2\pi}\, E\, r_0}, \tag{44}$$

where E is the Young's modulus.

The imaginary parts of elastic coefficients due to thermoelastic damping may be calculated in Zener's approach according to the formulas:

$$\begin{aligned} \frac{\mathrm{Im}(\sigma)}{\sigma} &= \frac{\sigma_S - \sigma_T}{\sigma_T}\, \frac{\omega\tau_T}{1+\omega^2\tau_T^2} = \frac{\alpha^2\, T E}{C\rho}\, \frac{1+\sigma}{\sigma}\, \frac{\omega\tau_T}{1+\omega^2\tau_T^2}, \\ \frac{\mathrm{Im}(E)}{E} &= \frac{E_S - E_T}{E_T}\, \frac{\omega\tau_T}{1+\omega^2\tau_T^2} = \frac{\alpha^2\, T E}{C\rho}\, \frac{\omega\tau_T}{1+\omega^2\tau_T^2}, \end{aligned} \tag{45}$$

where E_T, E_S are correspondingly iso-thermic and adiabatic Young's moduluses ($E_T \simeq E_S \simeq E$) and σ_T, σ_S are iso-thermic and adiabatic Poisson coefficients ($\sigma_T \simeq \sigma_S \simeq \sigma$), τ_T is the time of thermal relaxation of our spot. By substituting these values in (44) and then in (43) and taking into account that $\tau_T = 2r_0^2/a^2 \gg 1/\omega$ (this relaxation time may be found relatively easy from the solution of the problem of thermal relaxation of the heated spot) we finally obtain the same formula (41).

It is really worth noting that all the three methods give precisely the same result.

Numerical estimates

Now we want to compare the fluctuations of the mirrors' surface caused by the described above mechanisms with other known sources of noise in LIGO antennae.

The main determining source of noise associated with the mirrors' material is usually described by the losses in the model of structural damping (we denote it as Brownian motion of the surface). In this model the angle of losses ϕ does not depend on frequency and analogously by substituting $E = \bar{E}(1 + i\phi)$ in (44) and then in (43) we obtain:

$$S_{\rm SD}(\omega) \simeq \frac{4\kappa T}{\omega} \frac{(1-\sigma^2)}{\sqrt{2\pi} E r_o} \phi. \tag{46}$$

For simplicity here we neglected the unknown imaginary part of Poisson coefficient.

The sensitivity of gravitational wave antenna to the perturbation of metric may be recalculated from noise spectral density of displacement X using the following formula:

$$h(\omega) = \frac{\sqrt{2 S_X}}{L}, \tag{47}$$

where we used the fact that antenna has two arms (with length L) and in each arm only one mirror adds to noise (on end mirrors the fluctuations are averaged over the larger beam spot).

The LIGO-II antenna will reach the level of SQL, so we also compare the noise limited sensitivity to this limit in spectral form:

$$h(\omega) = \sqrt{\frac{8\hbar}{m\omega^2 L^2}}. \tag{48}$$

On figures 1 and 2 we plot the sensitivity limitations for the perturbation of metric caused by the described thermal mechanisms and SQL (formulas 37,41,46,47,48) for fused silica (Fig.6) and sapphire (Fig.7) mirrors. We used the following numerical values for the parameters of the two materials — fused silica and sapphire (we should note that the actual figures, which various sapphire manufactures provide, significantly vary – tens of percents, especially for the thermal expansion coefficient):

$$\omega = 2\pi \times 100\ \mathrm{s}^{-1}, \quad r_0 = 1.56\ \mathrm{cm}, \quad T = 300\ \mathrm{K},$$

$$\omega_0 = 2 \times 10^{15}\ \mathrm{s}^{-1}, \quad L = 4 \times 10^5\ \mathrm{cm}, \quad W_0 = 10^7 \mathrm{erg/s}; \tag{49}$$

$$\text{Fused silica:} \quad \alpha = 5.5 \times 10^{-7}\ \mathrm{K}^{-1}, \quad \lambda^* = 1.4 \times 10^5\ \frac{\mathrm{erg}}{\mathrm{cm\ s\ K}}, \tag{50}$$

$$\rho = 2.2\ \frac{\mathrm{g}}{\mathrm{cm}^3}, \quad C = 6.7 \times 10^6\ \frac{\mathrm{erg}}{\mathrm{g\ K}}, \quad m = 1.1 \times 10^4\ \mathrm{g},$$

$$E = 7.2 \times 10^{11} \frac{\mathrm{erg}}{\mathrm{cm}^3}, \quad \sigma = 0.17, \quad \phi = 5 \times 10^{-8};$$

$$\text{Sapphire:} \quad \alpha = 5.0 \times 10^{-6}\ \mathrm{K}^{-1}, \quad \lambda^* = 4.0 \times 10^6\ \frac{\mathrm{erg}}{\mathrm{cm\ s\ K}}, \tag{51}$$

$$\rho = 4.0\ \frac{\mathrm{g}}{\mathrm{cm}^3}, \quad C = 7.9 \times 10^6\ \frac{\mathrm{erg}}{\mathrm{g\ K}}, \quad m = 3 \times 10^4\ \mathrm{g},$$

$$E = 4 \times 10^{12}\ \frac{\mathrm{erg}}{\mathrm{cm}^3}, \quad \sigma = 0.29, \quad \phi = 3 \times 10^{-9}. \tag{52}$$

From these two figures it is evident that the effects associated with thermal expansion in no case may be neglected. Especially unfortunate result is obtained for sapphire – the noise from thermodynamical fluctuations of temperature dominates for frequencies of interest and is several times larger than the SQL. While for fused silica the situation is the opposite. The SQL is one order larger than TD fluctuations. It is important to note, that unlike structural damping for which angles of losses at low frequencies were not yet directly measured, our mechanism has fundamental nature and is calculated explicitely without model assumptions.

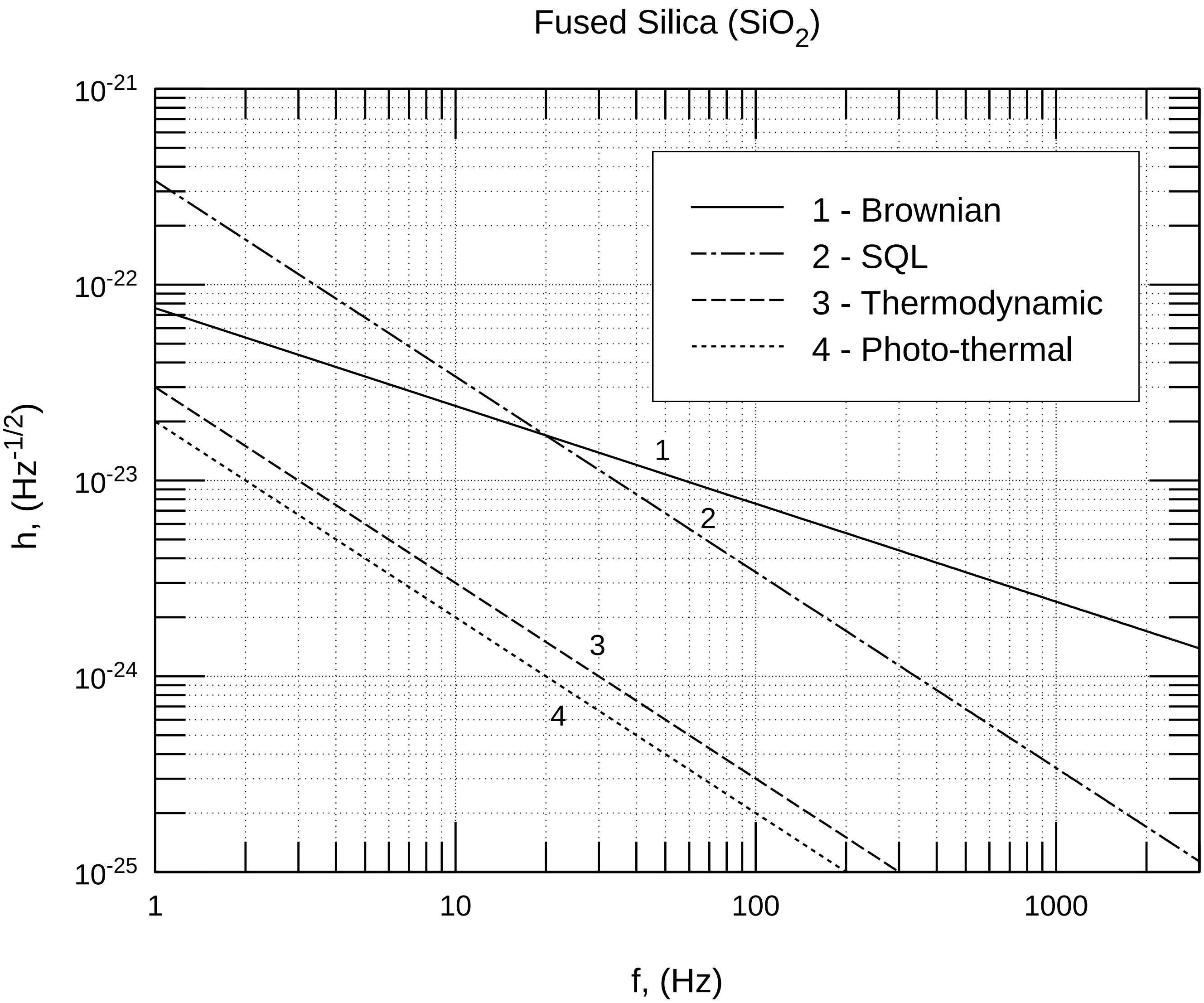

Fig.6 The sensitivity limitations for the perturbation of metric caused by the SQL and by thermal fluctuations in fused silica mirrors.

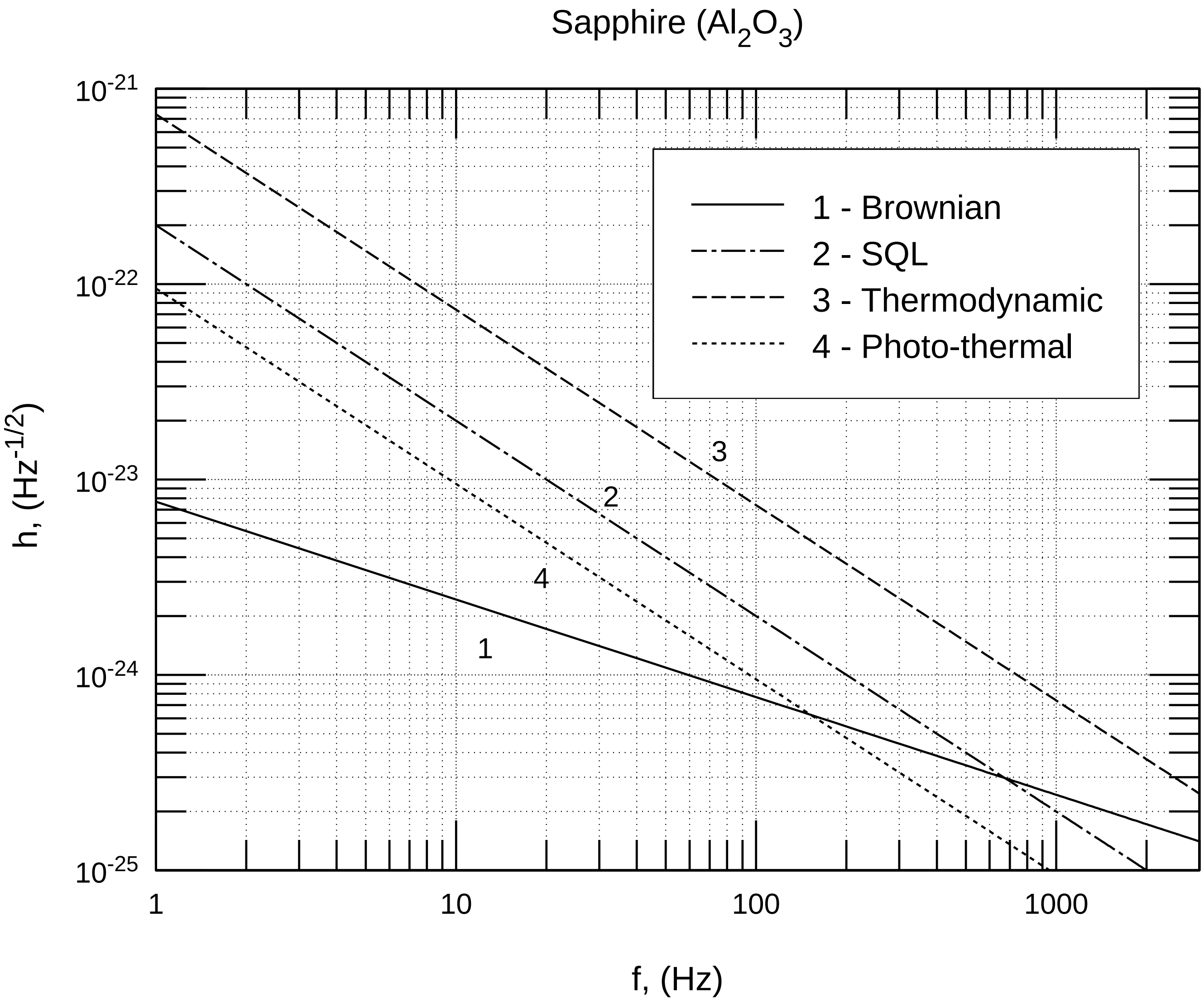

Fig.7 The sensitivity limitations for the perturbation of metric caused by the SQL and by thermal fluctuations in sapphire mirrors.

CONCLUSION

The analysis and numerical estimates presented above for two effects allow to give some recommendations for the strategy of upgrading laser interferometric gravitational wave antennae.

1. It is important to emphasize that contrastingly to the gravitational wave, both effects

do not displace the centers of masses of the mirrors. Thus, in principle it is possible to subtract these effects as well as usual Brownian fluctuations (with certain level of accuracy) in a way similar to the procedure suggested for the subtraction of thermal noise in suspension fibers [3]. However, till now to our knowledge nobody presented the scheme of such subtraction.

2. Potentially there exists a possibility to decrease substantially these effects by choosing materials for the mirrors with substantially smaller values of α. There also exists the possibility to use special cuts of monocrystals without center of symmetry for which α turns to zero (crystal quartz for example) as in secondary frequency standards.

3. Thermodynamic fluctuations decrease with growing radius of the beam spot as $r_0^{3/2}$ even stronger then for the Brownian fluctuations. In this way, some optimization of the geometry of the resonators may help.

4. The "brute force" method for the improvement of the sensitivity by increasing the circulating optical power W does not look promising, taking into account photo-thermal shot noise. It seems that few mega-Watts is the upper limit for W.

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